Do the following problems:

1. For a BSC with cross over probability $q$, using the definitions from the supplementary notes to Lecture 14, show that

$$R_0 = 1 - \log_2 \left(1 + 2 \sqrt{q(1-q)}\right).$$

Also, show that $(0.5C) < R_0 < C$, where $C$ is the capacity of the BSC.

2. Problem 9.1 in C&T.

3. Problem 9.2 in C&T.

4. Problem 9.3 in C&T.

5. Problem 9.6 in C&T.

6. We have seen that differential entropy is not invariant to transformations, i.e., in general $h(X) \neq h(g(X))$ for any 1-1 transformation $g(x)$. Is the same true for mutual information, i.e. does $I(g(X);Y) = I(X;Y)$ for any 1-1 transformation? Give a proof or a counter example.