Do the following problems:

1. Problem 8.1 in C&T.
2. Problem 8.4 in C&T.
3. Problem 8.6 in C&T.
4. Problem 8.9 in C&T.
5. Problem 8.12 in C&T.

6. **Random Source Coding:** This problem develops the idea of using a random coding approach to prove a coding theorem for lossless fixed-to-fixed length source codes. Let \( X_1, X_2, \ldots \) be an i.i.d. source with p.m.f. \( p(x) \). For any \( \varepsilon > 0 \), let \( A^{(n)}_\varepsilon \) be the set of typical sequences of length \( n \). Let \( x^n \in A^{(n)}_\varepsilon \) be a particular typical sequence.

   a. Let \( Y^n \) be a length \( n \) i.i.d. sequence with p.m.f. \( p(x) \). Find a good lower bound on the probability \( q \) that \( x^n = Y^n \).

   b. Let \( C \) be a random codebook containing \( M \) i.i.d. sequences drawn according to \( p(x) \). Find an exact expression for the probability that \( x^n \notin C \). Convert this to an upper bound using (a).

   c. Find the least rate \( R^* \) such that for a random codebook \( C^{(n)} \) with \( M = 2^nR^* \) codewords, the probability that \( x^n \notin C \) approaches 0 as \( n \to \infty \).

   d. Argue that for sufficiently large \( n \) there exists a codebook with \( M = 2^nR^* \) codewords that noiselessly encodes the i.i.d. source with arbitrarily low probability of failure.