(e) The carrier power is, assuming a sufficiently high carrier frequency,

\[
P = \frac{1}{2} A_c^2 = \frac{1}{2} (100)^2 = 5000 \text{ Watts}
\]

**Problem 3.33**
The frequency deviation in Hertz is the plot shown in Fig. 3.76 with the ordinate values multiplied by 25. The phase deviation in radians is given

\[
\phi(t) = 2\pi F_d \int^t m(\alpha) d\alpha = 50\pi \int^t m(\alpha) d\alpha
\]

For \(0 \leq t \leq 1\), we have

\[
\phi(t) = 50\pi \int^t 2\alpha d\alpha = 50\pi t^2
\]

For \(1 \leq t \leq 2\)

\[
\phi(t) = \phi(1) + 50\pi \int^t_1 (5 - \alpha) d\alpha = 50\pi + 250\pi (t - 1) - 25\pi (t^2 - 1)
\]

\[
= -175\pi + 250\pi t - 25\pi t^2
\]

For \(2 \leq t \leq 3\)

\[
\phi(t) = \phi(2) + 50\pi \int^t_2 3d\alpha = 225\pi + 150\pi (t - 2)
\]

For \(3 \leq t \leq 4\)

\[
\phi(t) = \phi(3) + 50\pi \int^t_3 2d\alpha = 375\pi + 100\pi (t - 3)
\]

Finally, for \(t > 4\) we recognize that \(\phi(t) = \phi(4) = 475\pi\). The required figure results by plotting these curves.

**Problem 3.34**
The frequency deviation in Hertz is the plot shown in Fig. 3.77 with the ordinate values multiplied by 10. The phase deviation is given by

\[
\phi(t) = 2\pi F_d \int^t m(\alpha) d\alpha = 20\pi \int^t m(\alpha) d\alpha
\]

For \(0 \leq t \leq 1\), we have

\[
\phi(t) = 20\pi \int^t_0 \alpha d\alpha = 10\pi t^2
\]
3.1. PROBLEMS

For $3 \leq t \leq 4$

$$\phi(t) = \phi(3) + 10\pi \int_{3}^{t} (2\alpha - 8) \, d\alpha = 5\pi + 10\pi(t^2 - 9) - 10\pi(8)(t - 3)$$

$$= 10\pi(t^2 - 8t + 15.5)$$

Finally, for $t > 4$ we recognize that $\phi(t) = \phi(4) = -5\pi$. The required figure follows by plotting these expressions.

Problem 3.36
(a) The peak deviation is $(12.5)(4) = 50$ and $f_m = 10$. Thus, the modulation index is $\frac{50}{10} = 5$.
(b) The magnitude spectrum is a Fourier-Bessel spectrum with $\beta = 5$. The $n = 0$ term falls at 1000 Hz and the spacing between components is 10 Hz. The sketch is that of Figure 3.24 in the text.
(c) Since $\beta$ is not $\ll 1$, this is not narrowband FM. The bandwidth exceeds $2f_m$.
(d) For phase modulation, $k_p(4) = 5$ or $k_p = 1.25$.

Problem 3.37
The results are given in the following table:

<table>
<thead>
<tr>
<th>Part</th>
<th>$f_d$</th>
<th>$D = 5f_d/W$</th>
<th>$B = 2(D + 1)W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>20</td>
<td>0.004</td>
<td>50.2 kHz</td>
</tr>
<tr>
<td>b</td>
<td>200</td>
<td>0.04</td>
<td>52 kHz</td>
</tr>
<tr>
<td>c</td>
<td>2000</td>
<td>0.4</td>
<td>70 kHz</td>
</tr>
<tr>
<td>d</td>
<td>20000</td>
<td>4</td>
<td>250 kHz</td>
</tr>
</tbody>
</table>

Problem 3.38
From

$$x_c(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + \omega_m) t$$

we obtain

$$\langle x_c^2(t) \rangle = \frac{1}{2} A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

We also know that (assuming that $x_c(t)$ does not have a significant dc component - see Problem 3.24)

$$\langle x_c^2(t) \rangle = \langle A_c^2 \cos^2[\omega_c t + \phi(t)] \rangle$$
which, assuming that $\omega_c \gg 1$ so that $x_c(t)$ has no dc component, is

$$\langle x_c^2(t) \rangle = \frac{1}{2} A_c^2$$

This gives

$$\frac{1}{2} A_c^2 = \frac{1}{2} A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

from which

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

**Problem 3.39**

Since

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-jn(\beta \sin x)} \, dx = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \, dx$$

we can write

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos (\beta \sin x - nx) \, dx + j \frac{1}{2\pi} \int_{-\pi}^{\pi} \sin (\beta \sin x - nx) \, dx$$

The imaginary part of $J_n(\beta)$ is zero, since the integrand is an odd function of $x$ and the limits $(-\pi, \pi)$ are even. Thus

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos (\beta \sin x - nx) \, dx$$

Since the integrand is even

$$J_n(\beta) = \frac{1}{\pi} \int_{0}^{\pi} \cos (\beta \sin x - nx) \, dx$$

which is the first required result. With the change of variables $\lambda = \pi - x$, we have

$$J_n(\beta) = \frac{1}{\pi} \int_{0}^{\pi} \cos [\beta \sin (x) - nx + \pi] \, (-1) \, d\lambda$$

$$J_n(\beta) = \frac{1}{\pi} \int_{0}^{\pi} \cos [\beta \sin (\pi - \lambda) - n(\pi - \lambda)] \, d\lambda$$

Since $\sin (\pi - \lambda) = \sin \lambda$, we can write

$$J_n(\beta) = \frac{1}{\pi} \int_{0}^{\pi} \cos [\beta \sin \lambda + n\lambda - n\pi] \, d\lambda$$
3.1. PROBLEMS

Problem 3.41
The required spectra are given in Figure 3.10. The modulation indices are, from top to bottom, $\beta = 0.5$, $\beta = 1$, $\beta = 2$, $\beta = 5$, and $\beta = 10$.

Problem 3.42
We wish to find $k$ such that

$$P_r = J_0^2(10) + 2 \sum_{n=1}^{k} J_n^2(10) \geq 0.80$$

This gives $k = 9$, yielding a power ratio of $P_r = 0.8747$. The bandwidth is therefore

$$B = 2k f_m = 2 \times (9) \times (150) = 2700 \text{ Hz}$$

For $P_r \geq 0.9$, we have $k = 10$ for a power ratio of 0.9603. This gives

$$B = 2k f_m = 2 \times (10) \times (150) = 3000 \text{ Hz}$$

Problem 3.43
From the given data, we have

$$f_{c1} = 110 \text{ kHz} \quad f_{d1} = 0.05 \quad f_{d2} = n(0.05) = 20$$

This gives

$$n = \frac{20}{0.05} = 400$$

and

$$f_{c1} = n(100) \text{ kHz} = 44 \text{ MHz}$$

The two permissible local oscillator frequencies are

$$f_{0.1} = 100 - 44 = 56 \text{ MHz}$$
$$f_{0.2} = 100 + 44 = 144 \text{ MHz}$$

The center frequency of the bandpass filter must be $f_c = 100$ MHz and the bandwidth is

$$B = 2(D+1)W = 2(20+1)(10^3)$$

or

$$B = 420 \text{ kHz}$$
Problem 3.44
For the circuit shown
\[ H(f) = \frac{E(f)}{X(f)} = \frac{R}{R + j2\pi f L + \frac{1}{j2\pi f C}} \]
or
\[ H(f) = \frac{1}{1 + j \left( 2\pi f \tau_L - \frac{1}{2\pi f \tau_C} \right)} \]
where
\[ \tau_L = \frac{L}{R} = \frac{10^{-3}}{10^3} = 10^{-6}, \]
\[ \tau_C = RC = (10^3)(10^{-9}) = 10^{-6} \]
A plot of the amplitude response shows that the linear region extends from approximately 54 kHz to 118 kHz. Thus an appropriate carrier frequency is
\[ f_c = \frac{118 + 54}{2} = 86 \, \text{kHz} \]
The slope of the operating characteristic at the operating point is measured from the amplitude response. The result is
\[ K_D \cong 8 \left( 10^{-6} \right) \]

Problem 3.45
We can solve this problem by determining the peak of the amplitude response characteristic. This peak falls at
\[ f_p = \frac{1}{2\pi \sqrt{LC}} \]
It is clear that \( f_p > 100 \, \text{MHz} \). Let \( f_p = 150 \, \text{MHz} \) and let \( C = 0.001 \left( 10^{-12} \right) \). This gives
\[ L = \frac{1}{(2\pi)^2 f_p^2 C} = 1.126 \left( 10^{-3} \right) \]
We find the value of \( R \) by trial and error using plots of the amplitude response. An appropriate value for \( R \) is found to be 1 \( \text{M}\Omega \). With these values, the discriminator constant is approximately
\[ K_D \approx 8.5 \left( 10^{-9} \right) \]

Problem 3.46