Sampling-Based Methods for Stochastic Optimization

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The model

Objective is to solve

$$\min_{x \in X} \{ g(x) := \mathbb{E}G(x, \xi) \}$$

where $\xi$ is a random vector representing the “uncertainty” in the model (e.g. future demand). Denote by $F$ the (joint) distribution of $\xi$.

We are interested in the case where $\xi$ either has a continuous distribution or a huge number of scenarios, so sampling is required.
Two approaches

1. **External Sampling:**

   Let $\xi^1, ..., \xi^N$ be an iid random sample drawn from $F$.

   Solve
   
   $$\min_{x \in X} \left\{ \hat{g}_N(x) := N^{-1} \sum_{j=1}^{N} G(x, \xi^j) \right\}.$$

   Let
   
   $x^* = \text{optimal solution of original problem}$

   $\nu^* = \text{optimal value of original problem}$

   $\hat{x}_N = \text{optimal solution of approximating problem}$

   $\hat{\nu}_N = \text{optimal value of approximating problem}$

   Many convergence properties exist: under proper assumptions,
   
   - $\hat{x}_N \rightarrow x^*$ w.p.1, $\hat{\nu}_N \rightarrow \nu^*$ w.p.1

   • For any given $\varepsilon > 0$, $P(\|\hat{x}_N - x^*\| \geq \varepsilon)$ approaches zero exponentially fast as $N \rightarrow \infty$.

   • etc.

   **ISSUES:** How to choose $N$, quality of solution for given $N$. 
Two approaches (cont.)

2. *Internal Sampling:*

   Idea is to *incorporate* sampling into an optimization algorithm.

   In our context, this means the following:

   (i) Let $\xi^1, \ldots, \xi^N$ be an iid random sample drawn from $F$.

   (ii) Compute $\hat{g}_N(x) := N^{-1} \sum_{j=1}^N G(x, \xi^j)$.

   (iii) Do some optimization steps with $\hat{g}_N(x)$.

   (iv) Repeat (i)-(iii) above.

**ISSUES:**

- Convergence
- Choice of sample sizes
- New samples vs. cumulative samples
- Testing optimality conditions
- etc.

Statistical tests are often used as auxiliary tools.
My interests

- Development of efficient sampling-based methods, especially for *non-smooth* stochastic optimization
  - A stochastic bundle method?

- Use of variance-reduction techniques from simulation literature
  - Guarantees of efficiency, practical performance

- Applications
  - Manufacturing, logistics, *revenue management*