Combining Interior-Point and Active-Set Approaches for Nonlinear Optimization in KNITRO

Richard A. Waltz, Northwestern University
Ziena Optimization, Inc.

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KNITRO Overview

General purpose optimization software package:

- Unconstrained problems
- Bound constrained problems
- Equality constrained problems
- Linear programs (LP)
- Quadratic programs (QP)
- General non-linear optimization problems
KNITRO Overview

Motivation

• Diversity of nonlinear optimization probs. requires diversity of algorithms/features

Key Features

- Interior-point and Active-set algorithms
- Iterative and Direct approaches
- 1st and 2nd derivative options
- Feasible/honor bounds options
- Adaptive techniques
KNITRO 4.0 (New October 2004)

New Features

- new Active-set (SLQP) algorithm
- Automatic algorithm selection
- Improved performance on LPs and QPs
- Improved infeasibility detection
- Improved robustness and handling of degenerate problems
- Improved API
- threadsafe
KNITRO 3.x vs 4.0 (CUTER test set)

968 probs.

Robustness
4.0: 87%
3.1: 81%
3.0: 79%
Three Algorithms

1. **KNITRO/InteriorCG:**
   - Interior-point iterative approach
   - Good for large problems with dense Hessians

2. **KNITRO/InteriorDirect**
   - Interior-point direct approach
   - Good for large ill-conditioned problems

3. **KNITRO/Active (new October 2004!):**
   - SLQP active-set approach
   - Good for infeasibility detection and warm starts
CG-Direct Comparison

- **CVXQP2**
  - n=10,000,
  - m=2,500,
  - nnzH = 40,000
  - 99.6% time spent on factorization

- **BQPGAUSS**
  - n = 2003
  - bound-constrained
  - Hessian ill-conditioned but not dense

<table>
<thead>
<tr>
<th>Code</th>
<th>iters</th>
<th>time</th>
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KNITRO Algorithms

- Function evals.
- 616 probs

Robustness

Auto: 91%
IPDirect: 91%
IPCG: 88%
Active: 79%
Any: 94%
- CPU time

**Robustness**
- Auto: 91%
- IPDirect: 91%
- IPCG: 88%
- Active: 79%
- Any: 94%
Other problem classes

<table>
<thead>
<tr>
<th>Prob class</th>
<th>InteriorCG</th>
<th>InteriorDirect</th>
<th>Active</th>
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<tr>
<td>Uncon</td>
<td>Newton-CG</td>
<td>Newton</td>
<td>Newton-CG</td>
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<td>Levenb-Marq.</td>
<td>Gauss-Newt</td>
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<tr>
<td>Bound Con</td>
<td>Interior-iterative</td>
<td>Interior-direct</td>
<td>Gradient projection</td>
</tr>
<tr>
<td>Equal Con</td>
<td>SQP-iterative</td>
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<td>SQP-iterative</td>
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<tr>
<td>LP</td>
<td>Interior-iterative</td>
<td>Interior-direct</td>
<td>simplex</td>
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<tr>
<td>QP</td>
<td>Interior-iterative</td>
<td>Interior-direct</td>
<td>SLQP</td>
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Second Derivative options

- **Exact** 2\textsuperscript{nd} derivatives
- **Quasi-Newton**
  - SR1 (dense)
  - BFGS (dense)
  - Limited memory BFGS (large-scale)
- **Hessian-vector products** (InteriorCG/Active)
  - exact Hessian-vector products
  - via finite-differencing of gradients
Second Derivative options

-548 small-med. probs -fevals

Robustness:
Exact: 92%
FD: 92%
BFGS: 89%
SR1: 88%
LBFGS: 87%
Second Derivative options

- 548 small-med. probs
- CPU time

Robustness:
Exact: 92%
FD: 92%
BFGS: 89%
SR1: 88%
LBFGS: 87%
Feasible Option

- By default constraints may be violated during optimization process
- **Feasible option**: enforces feasibility with respect to inequalities given initial point satisfying inequalities
  - Only available for InteriorCG
- **Honor bounds**: special case of feasible option
  - Available for all algorithms
Crossover/Warm Starts

HS66

Solve 1: KNITRO/InteriorDirect (11 iterations)

c1 = 6.0105e-09  lambda1 = 0.665464
c2 = 1.92786e-08  lambda2 = 0.2
c3 = 0.184127  lambda3 = 2.1702e-08
c4 = 1.20217  lambda4 = 3.28748e-09
c5 = 3.32732  lambda5 = 6.03145e-10
Crossover/Warm Starts

HS66

Solve 2: KNITRO/Active (3 iterations)

c1 = 0  lambda1 = 0.665464

\(c2 = 4.44089e-16\)  lambda2 = 0.2

\(c3 = 0.184126\)  lambda3 = 0

\(c4 = 1.20217\)  lambda4 = 0

\(c5 = 3.32732\)  lambda5 = 0
## Infeasibility detection

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Future Work

- More work on crossover, adaptive rules, automatic tuning
- Mixed Integer NLP
- MPECS
- Preprocessing NLP’s
Interfaces/Partners

- AMPL
- GAMS (beta release, official release coming soon)
- Frontline Systems (Excel interface)
- NEOS (free version used via internet)
- Tomlab (Matlab interface)
- Callable library (C/C++/Fortran API)
  - Artelys (Europe) www.artelys.com
  - Ziena (US and non-Europe)

www.ziena.com/knitro.html (student version)
www.ece.northwestern.edu/~rwaltz