

Binary trees

```
; a database is  
; - false  
; - (make-db-node person database database)  
(define-struct db-node (person left right))
```

Lookup for binary trees

; lookup : number database → person or false
; to find the person with `ssn' in `bt' if possible

```
(define (lookup ssn bt)
  (cond
    [(boolean? bt) false]
    [else
     (cond
       [(ssn-matches? ssn (db-node-person bt))
        (db-node-person bt)]
       [else
        (pick (lookup ssn
                     (db-node-left bt))
              (lookup ssn
                     (db-node-right bt)))))]))
```

Helper function for binary trees

; pick : person-or-false person-or-false → person-or-false
; to pick whichever, p1 or p2, that is a person,
; or false if none are

```
(define (pick p1 p2)
  (cond
    [(and (boolean? p1) (boolean? p2))
     false]
    [(and (person? p1) (boolean? p2))
     p1]
    [(and (boolean? p1) (person? p2))
     p2]
    [(and (person? p1) (person? p2))
     p1]))
```

; ssn-matches? : ssn person → boolean

```
(define (ssn-matches? ssn person)
  (= ssn (person-ssn person)))
```

```
(lookup
```

```
  5
```

```
  (make-bst
```

```
    (make-person 4 'tim)
```

```
    (make-bst
```

```
      (make-person 2 'jon)
```

```
      (make-bst
```

```
        (make-person 1 'anna)
```

```
        false
```

```
        false)
```

```
      (make-bst
```

```
        (make-person 3 'ellen)
```

```
        false
```

```
        false))
```

```
    (make-bst
```

```
      (make-person 6 'jane)
```

```
      (make-bst
```

```
        (make-person 5 'george)
```

```
        false
```

```
        false)
```

```
      (make-bst
```

```
        (make-person 7 'emily)
```

```
        false
```

```
        false))))
```

```
=>
```

```

(lookup
  5
  (make-bst
    (make-person 4 'tim)
    (make-bst
      (make-person 2 'jon)
      (make-bst
        (make-person 1 'anna)
        false
        false)
      (make-bst
        (make-person 3 'ellen)
        false
        false))
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))

```

```

=> (pick
    (lookup
      5
      (make-bst
        (make-person 2 'jon)
        (make-bst
          (make-person 1 'anna)
          false
          false)
        (make-bst
          (make-person 3 'ellen)
          false
          false)))
    (lookup
      5
      (make-bst
        (make-person 6 'jane)
        (make-bst
          (make-person 5 'george)
          false
          false)
        (make-bst
          (make-person 7 'emily)
          false
          false))))

```

```
<= (pick
    (lookup
        5
        (make-bst
            (make-person 2 'jon)
            (make-bst
                (make-person 1 'anna)
                false
                false)
            (make-bst
                (make-person 3 'ellen)
                false
                false)))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                (make-person 5 'george)
                false
                false)
            (make-bst
                (make-person 7 'emily)
                false
                false))))))
```

```

(pick
  (pick
    (lookup
      5
      (make-bst
        (make-person 1 'anna)
        false
        false))
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))

```

```

<= (pick
    (lookup
      5
      (make-bst
        (make-person 2 'jon)
        (make-bst
          (make-person 1 'anna)
          false
          false)
        (make-bst
          (make-person 3 'ellen)
          false
          false)))
    (lookup
      5
      (make-bst
        (make-person 6 'jane)
        (make-bst
          (make-person 5 'george)
          false
          false)
        (make-bst
          (make-person 7 'emily)
          false
          false))))

```

```
(pick
  (pick
    (lookup
      5
      (make-bst
        (make-person 1 'anna)
        false
        false))
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))
```

=>


```

(pick
  (pick
    (lookup
      5
      (make-bst
        (make-person 1 'anna)
        false
        false)))
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false))))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))))

```

```

=> (pick
    (pick
      (pick
        (lookup 5 false)
        (lookup 5 false))
      (lookup
        5
        (make-bst
          (make-person 3 'ellen)
          false
          false))))
    (lookup
      5
      (make-bst
        (make-person 6 'jane)
        (make-bst
          (make-person 5 'george)
          false
          false)
        (make-bst
          (make-person 7 'emily)
          false
          false))))))

```

```
<= (pick
    (pick
        (pick
            (lookup 5 false)
            (lookup 5 false))
        (lookup
            5
            (make-bst
                (make-person 3 'ellen)
                false
                false)))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                (make-person 5 'george)
                false
                false)
            (make-bst
                (make-person 7 'emily)
                false
                false))))))
```

```

(pick
  (pick
    (pick false (lookup 5 false))
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))

```

```

<= (pick
  (pick
    (pick
      (lookup 5 false)
      (lookup 5 false))
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))

```

```
(pick
  (pick
    (pick false (lookup 5 false))
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))
```

=>

```

(pick
  (pick
    (pick false (lookup 5 false))
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))

```

```

=> (pick
  (pick
    (pick false false)
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))

```

```
<= (pick
    (pick
      (pick false false)
      (lookup
        5
        (make-bst
          (make-person 3 'ellen)
          false
          false)))
    (lookup
      5
      (make-bst
        (make-person 6 'jane)
        (make-bst
          (make-person 5 'george)
          false
          false)
        (make-bst
          (make-person 7 'emily)
          false
          false))))))
```

```

(pick
  (pick
    false
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))

```

```

<= (pick
  (pick
    (pick false false)
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))

```

```
(pick
  (pick
    false
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))
```

=>


```

(pick
  (pick
    false
    (lookup
      5
      (make-bst
        (make-person 3 'ellen)
        false
        false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))

```

```

=> (pick
  (pick
    false
    (pick
      (lookup 5 false)
      (lookup 5 false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))

```

```
<= (pick
    (pick
      false
      (pick
        (lookup 5 false)
        (lookup 5 false)))
    (lookup
      5
      (make-bst
        (make-person 6 'jane)
        (make-bst
          (make-person 5 'george)
          false
          false)
        (make-bst
          (make-person 7 'emily)
          false
          false))))))
```

```

(pick
  (pick
    false
    (pick false (lookup 5 false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))))

```

```

<= (pick
  (pick
    false
    (pick
      (lookup 5 false)
      (lookup 5 false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))))

```

```
(pick
  (pick
    false
    (pick false (lookup 5 false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))
```

=>

```
(pick
  (pick
    false
    (pick false (lookup 5 false)))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))))
```

```
=> (pick
      (pick false (pick false false))
      (lookup
        5
        (make-bst
          (make-person 6 'jane)
          (make-bst
            (make-person 5 'george)
            false
            false)
          (make-bst
            (make-person 7 'emily)
            false
            false))))))
```

```
<= (pick
    (pick false (pick false false))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                (make-person 5 'george)
                false
                false)
            (make-bst
                (make-person 7 'emily)
                false
                false))))))
```

```
(pick
  (pick false false)
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))))
```

```
<= (pick
  (pick false (pick false false))
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))))
```

```
(pick
  (pick false false)
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))))
```

=>


```
(pick
  (pick false false)
  (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))))
```

```
=> (pick
      false
      (lookup
        5
        (make-bst
          (make-person 6 'jane)
          (make-bst
            (make-person 5 'george)
            false
            false)
          (make-bst
            (make-person 7 'emily)
            false
            false))))))
```

```
<= (pick
    false
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                (make-person 5 'george)
                false
                false)
            (make-bst
                (make-person 7 'emily)
                false
                false))))))
```

```

(pick
 false
 (pick
  (lookup
   5
   (make-bst
    (make-person 5 'george)
    false
    false))
 (lookup
  5
  (make-bst
   (make-person 7 'emily)
   false
   false))))

```

```

<= (pick
    false
    (lookup
     5
     (make-bst
      (make-person 6 'jane)
      (make-bst
       (make-person 5 'george)
       false
       false)
      (make-bst
       (make-person 7 'emily)
       false
       false))))

```

```
(pick
 false
 (pick
  (lookup
   5
   (make-bst
    (make-person 5 'george)
    false
    false))
 (lookup
  5
  (make-bst
   (make-person 7 'emily)
   false
   false))))
```

=>

```
(pick
 false
 (pick
  (lookup
   5
   (make-bst
    (make-person 5 'george)
    false
    false)))
 (lookup
  5
  (make-bst
   (make-person 7 'emily)
   false
   false))))
```

```
=> (pick
     false
     (pick
      (make-person 5 'george)
      (lookup
       5
       (make-bst
        (make-person 7 'emily)
        false
        false))))))
```

```
<= (pick
    false
    (pick
        (make-person 5 'george)
        (lookup
            5
            (make-bst
                (make-person 7 'emily)
                false
                false))))))
```

```
(pick
 false
 (pick
  (make-person 5 'george)
  (pick
   (lookup 5 false)
   (lookup 5 false))))
```

```
<= (pick
     false
     (pick
      (make-person 5 'george)
      (lookup
       5
       (make-bst
        (make-person 7 'emily)
        false
        false))))
```

```
(pick
 false
 (pick
  (make-person 5 'george)
  (pick
   (lookup 5 false)
   (lookup 5 false))))
```




```
(pick
 false
 (pick
  (make-person 5 'george)
  (pick
   (lookup 5 false)
   (lookup 5 false))))
```

```
=> (pick
     false
     (pick
      (make-person 5 'george)
      (pick false (lookup 5 false))))
```

```
<= (pick
    false
    (pick
        (make-person 5 'george)
        (pick false (lookup 5 false))))
```

```
(pick
 false
 (pick
  (make-person 5 'george)
  (pick false false)))
```

```
<= (pick
     false
     (pick
      (make-person 5 'george)
      (pick false (lookup 5 false))))
```

```
(pick  
false  
(pick  
  (make-person 5 'george)  
  (pick false false)))
```



```
(pick
 false
 (pick
  (make-person 5 'george)
  (pick false false)))
```

```
=> (pick
     false
     (pick
      (make-person 5 'george)
      false))
```

```
<= (pick  
    false  
    (pick  
      (make-person 5 'george)  
      false))
```

```
(pick  
false  
  (make-person 5 'george))
```

```
<= (pick  
    false  
    (pick  
      (make-person 5 'george)  
      false))
```

```
(pick  
false  
(make-person 5 'george))
```




```
(pick  
false  
(make-person 5 'george))
```

```
=> (make-person 5 'george)
```

(that took forever!)

Binary search trees

```
; a database is  
; - false  
; - (make-db-node person database[left] database[right])  
; where all people in left have smaller ssns than person, and  
; all people in right have larger ssns than person.  
(define-struct db-node (person left right))
```

Lookup for binary search trees

; lookup : number database → person or false
; to find the person with `ssn' in `bt' if possible

```
(define (lookup ssn bt)
  (cond
    [(boolean? bt) false]
    [else
     (cond
       [(ssn-matches? ssn (db-node-person bt))
        (db-node-person bt)]
       [(ssn-less? ssn (db-node-person bt))
        (lookup ssn (db-node-left bt))]
       [(ssn-more? ssn (db-node-person bt))
        (lookup ssn (db-node-right bt))])])])
```

Helper functions for binary search trees

```
; ssn-less? : ssn person → boolean  
(define (ssn-less? ssn person)  
  (< ssn (person-ssn person)))
```

```
; ssn-more? : ssn person → boolean  
(define (ssn-more? ssn person)  
  (> ssn (person-ssn person)))
```

```
(lookup  
  5  
  (make-bst  
    (make-person 4 'tim)  
    (make-bst  
      (make-person 2 'jon)  
      (make-bst  
        (make-person 1 'anna)  
        false  
        false)  
      (make-bst  
        (make-person 3 'ellen)  
        false  
        false))  
    (make-bst  
      (make-person 6 'jane)  
      (make-bst  
        (make-person 5 'george)  
        false  
        false)  
      (make-bst  
        (make-person 7 'emily)  
        false  
        false))))
```

=>

```

(lookup
  5
  (make-bst
    (make-person 4 'tim)
    (make-bst
      (make-person 2 'jon)
      (make-bst
        (make-person 1 'anna)
        false
        false)
      (make-bst
        (make-person 3 'ellen)
        false
        false))
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))

```

```

=> (lookup
  5
  (make-bst
    (make-person 6 'jane)
    (make-bst
      (make-person 5 'george)
      false
      false)
    (make-bst
      (make-person 7 'emily)
      false
      false)))

```

```
<= (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false)))
```



```
(lookup
  5
  (make-bst
    (make-person 5 'george)
    false
    false))
```

```
<= (lookup
    5
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false)))
```

```
(lookup  
  5  
  (make-bst  
    (make-person 5 'george)  
    false  
    false))
```

=>

```
(lookup  
  5  
  (make-bst  
    (make-person 5 'george)  
    false  
    false))
```

```
=> (make-person 5 'george)
```

(much quicker that time, no?)