Finger Exercises Determine the value of the following expressions in DrScheme.

**Hand Evaluation** Determine the value of this expression by hand. Show all of the steps that you took. (There is more than one way to do this problem.)

$$(-(*(sqrt\ 36)(/\ 1\ 2))(+\ 1\ 2))$$

**Solution** There is more than one path to take to solve this problem. Here are all of them, represented with arrows and boxes. Any of these paths is okay.

**Algebraic Notation vs Scheme Notation** Translate the following expressions from algebraic notation to prefix parenthesized notation. Evaluate them in DrScheme. (Do any of these have more than one translation?)

$$f(x) = 1 - x^2$$

$$f(f(\frac{1}{2}))$$

$$g(x) = f(x) * f(x)$$

## **Solution**

$$(+(+1(*23))4)$$

$$(+ 1 (* 2 3) 4)$$

```
(define (f x)
(-1 (* x x)))
(f (f 1/2))
(f (f (/ 1 2)))
(define (g x)
(* (f x) (f x)))
(g 12)
```

**Auxiliary Function Definitions** Imagine the owner of a movie theater who has complete freedom in setting ticket prices. The more he charges, the fewer the people who can afford tickets. In a recent experiment the owner determined a precise relationship between the price of a ticket and average attendance. At a price of \$5.00 per ticket, 120 people attend a performance. Decreasing the price by a dime (\$.10) increases attendance by 15. Unfortunately, the increased attendance also comes at an increased cost. Every performance costs the owner \$180.00. Each attendee costs another four cents (\$0.04). The owner would like to know the exact relationship between profit and ticket price so that he can determine the price at which he can make the highest profit.

Being a careful student of program design, the movie owner writes a series of helper functions to describe the precise relationships between the profit and the ticket price:

```
;; profit-first : number \rightarrow number
;; determines the profit of the program
(define (profit ticket-price)
  (- (revenue ticket-price)
     (cost ticket-price)))
;; revenue : number → number
;; determines the revenue for a particular ticket price.
(define (revenue ticket-price)
  (* (attendees ticket-price)
     ticket-price))
;; cost : number \rightarrow number
;; determines the cost for a particular ticket price.
(define (cost ticket-price)
  (+180
     (* .04 (attendees ticket-price))))
;; attendees : number → number
;; determines the number of attendees for a particular ticket price.
(define (attendees ticket-price)
  (+120
     (* (/ 15.10)
        (- 5.00 ticket-price))))
```

The owner's neighbor also got interested in this problem and wrote a program to calculate the relationship between the profit and the ticket price:

```
(define (profit ticket-price)
(- (* (+ 120
(* (/ 15 .10)
```

```
(- 5.00 ticket-price)))
ticket-price)
(+ 180
(* .04
(+ 120
(* (/ 15 .10)
(- 5.00 ticket-price)))))))
```

Verify that both programs produce the same output for ticket prices of \$4.00, \$5.00, and \$6.00.

**Auxiliary Function Definitions and Program Maintenance** After studying the cost structure of his show, the owner discovered several ways of lowering the cost. As a result of his improvements, he no longer has a fixed cost. He now simply pays \$1.50 per attendee.

Modify both programs to reflect this change. Verify that they still produce the same results for \$4.00, \$5.00, and \$6.00.

## **Solution**

```
;; profit-first : number \rightarrow number
;; determines the profit of the program
(define (profit-first ticket-price)
  (- (revenue ticket-price)
      (cost ticket-price)))
;; revenue : number \rightarrow number
;; determines the revenue for a particular ticket price.
(define (revenue ticket-price)
  (* (attendees ticket-price)) ticket-price))
;; cost : number \rightarrow number
;; determines the cost for a particular ticket price.
(define (cost ticket-price)
  (* 1.50 (attendees ticket-price)))
;; attendees : number \rightarrow number
;; determines the number of attendees for a particular ticket price.
(define (attendees ticket-price)
  (+120
      (* (/ 15.10) (- 5.00 ticket-price))))
;; profit-second : number \rightarrow number
;; determines the profit of the program
(define (profit-second ticket-price)
  (-(*(+120
            (*(/15.10)
               (- 5.00 ticket-price)))
         ticket-price)
      (*1.50 (+120)
                  (* (/ 15.10)
                     (- 5.00 ticket-price))))))
(profit-first 3.00)
(profit-second 3.00)
```

(profit-first 4.00) (profit-second 4.00) (profit-first 5.00) (profit-second 5.00)