Optimal power-delay trade-offs in fading channels - small delay asymptotics

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Abstract

When transmitting stochastically arriving data over fading channels there is an inherent trade-off between the required average transmission power and the average queueing delay experienced by the data. This trade-off can be exploited by appropriately scheduling the transmission of data over time. In this paper, we study the behavior of the optimal power-delay trade-off for a single user in the regime of asymptotically small delays. In this regime, we first lower bound how much average power is required as a function of the average queueing delay. We show that the rate at which this bound increases as the delay becomes asymptotically small depends on the behavior of the fading distribution near zero, as well as the arrival statistics. We characterize this rate for two different classes of fading distributions, one class that requires infinite power to minimize the queueing delay and one class that requires only finite power. We then show that for both classes, this rate can essentially be achieved by a sequence of simple "channel threshold" policies, which only transmit when the channel gain is greater than a given threshold. We also consider several other transmission scheduling policies and characterize their convergence behavior in the small delay regime.

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I. INTRODUCTION

In many wireless communication scenarios energy management is an important issue for reasons such as reducing the size and cost of communication devices and/or extending a device's usable life-time. Examples range from communication satellites to micro-sensors. Often, the required transmission power is one of the main energy consumers in a wireless devices; consequently, there has been much interest in approaches for efficiently utilizing this resource. A basic technique for accomplishing this is through transmission power control, i.e. adapting the transmission power over time in an attempt to not use any more energy than needed to communicate reliably. With data traffic, in addition to adjusting the transmission power used to send each packet of data, energy efficiency can be further improved by adjusting the transmission rate or equivalently the transmission time per packet, for example, by using adaptive modulation and coding. Such approaches exploit the well-known fact that the required energy per bit needed for reliable communication is decreasing in the number of degrees of freedom used to send each bit; for fixed bandwidth, the available degrees of freedom increase with the transmission time. In a fading channel, another benefit of adapting the transmission rate and power is that it enables the transmitter to be "opportunistic" and send more data during good channel conditions, which again reduces the required average energy per bit.

Recently, a number of energy-efficient transmission scheduling approaches have been studied including [1]–[15]. In these approaches transmission rate and/or power are adjusted over time based in part on the offered traffic as well as any available channel state information. In each case, the goal is to effectively balance some cost related to packet delay (e.g. the average queueing delay or a deadline by which all packets must be transmitted) with some cost related to power or energy (e.g. the total energy over a finite horizon or the long-term average power). Clearly, there is a fundamental trade-off between such concerns, i.e., packet delay can be reduced by transmitting at a higher rate, but this requires more energy per bit.

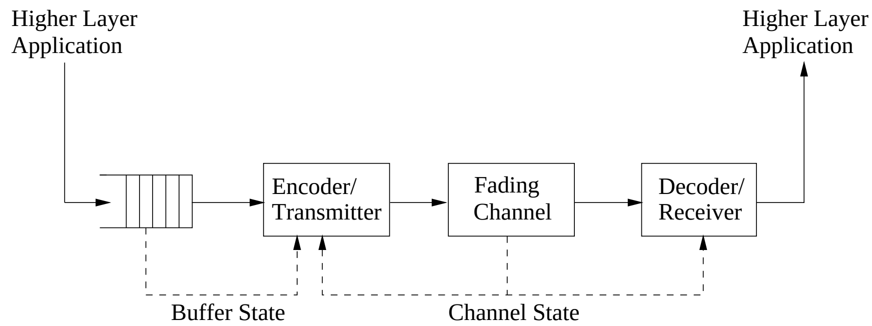

Fig. 1. System Model.

In this paper, we re-visit the basic model for transmission scheduling over a fading channel from [15]. In this model, data randomly arrives from some higher layer application and is placed into a transmission buffer as shown in Figure 1. Periodically, some data is removed from the buffer, encoded and transmitted over the fading channel. We focus on the case where each codeword is sent over a fixed number of channel uses, but different codewords may be of different rates. After the codeword is received, it is decoded and passed to the corresponding higher layer application at the receiver. The transmitter can vary the transmission power and rate based on both the channel state and the buffer occupancy. As in [15], we consider the optimal power/delay trade-off, $P^*(D)$. This characterizes the minimum long-term average power under any scheduling policy as a function of the average queueing delay, for a given arrival process and channel fading process. If the traffic arrives at an average rate of $\bar{A}$ bits per second, then for a stable system, the long-term average energy per bit is given by $P^*(D)/\bar{A}$, i.e., $P^*(D)$ also reflects the minimum energy per bit needed for a given average delay. For any system where the channel and arrival processes are not both constant, $P^*(D)$, will be a strictly decreasing and convex function of D. In [15], the behavior of $P^*(D)$ was studied in the asymptotic regime of large delays (low power). In this regime, it was shown that $P^*(D)$ approaches a limiting value of $\mathcal{P}(\bar{A})$ at rate of $\Theta\left(\frac{1}{D^2}\right)$. Here, we use the following notation to characterize the asymptotic behavior a function $g(x)$ as $x \to x^*$:

- $g(x) = O(f(x))$ if $\limsup_{x\to x^*} \frac{|g(x)|}{|f(x)|} < \infty$,
- $g(x) = \Omega(f(x))$ if $\limsup_{x\to x^*} \frac{|f(x)|}{|g(x)|} < \infty$, and
- $g(x) = \Theta(f(x))$ if $g(x) = O(f(x))$, and $g(x) = \Omega(f(x))$.

In [15] it was also shown that, this rate can be achieved with a sequence of "buffer threshold policies" whose only dependence on the buffer occupancy is via a simple threshold rule. Moreover, this weak dependence on the buffer occupancy is required for a sequence of policies to be order optimal (i.e., have the optimal convergence rate).

Here, we focus on the behavior of the power/delay trade-off in the asymptotic regime of small delays (high power). Specifically, we study the optimal rate at which the average delay decreases to its minimum value as the average power increases. The analysis of the large delay asymptotics in [15] is based, in part, on using large deviation bounds on the buffer occupancy, which are asymptotically tight for large buffer sizes. In the small delay regime, these bounds are not very useful and we instead take a different approach to analyze the system. We show that, in this regime, the optimal power/delay trade-off behaves quite differently from the large delay regime. In particular, the convergence rate is shown to depend strongly on the behavior of the fading distribution near zero. We focus on two broad classes of channels: one class ("type A channels") that requires infinite power to minimize the queueing delay, and one class ("type B channels") for which the queueing delay can be minimized with finite power. These classes include most common fading models, such as Rayleigh, Ricean and Nagakami fading. For each class, we first lower bound the convergence rate in the small delay regime that can be achieved by any transmission policy. We then show that this bound is achievable for both classes of channels when using a sequence of "channel threshold policies." These are policies under which the transmission rate only depends on the channel gain through a simple threshold rule. This demonstrates an interesting duality with the large delay regime, where instead buffer threshold policies were order optimal. For a type A channels, we then show that an even simpler fixed-rate channel threshold policy is also order optimal, where this policy does not depend at all on the buffer occupancy. However, such policies are not order optimal for the type B channels. Finally we consider two sequences of sub-optimal policies - *fixed power policies* and *fixed water-filling policies*, which also have no dependence on the buffer occupancy. These are shown to not have optimal convergence rates for either type of channel.

The outline of the rest of the paper is as follows. In Section II, we discuss the problem formulation in more detail and given some preliminary results. In Section III, we give lower bounds on the optimal convergence rate. In Section IV, we analyze several optimal and sub-optimal sequences of policies. We conclude in Section V. Most of the proofs are given in the appendices.

II. PROBLEM FORMULATION

We next give a more precise description of our model for the system shown in Figure 1. First the fading channel model is described. The channel is modeled as a discrete-time, block-fading channel with additive white Gaussian noise and frequency-flat fading [16], [17].[1] Over each block of N consecutive channel uses, the channel gain stays fixed. Let $\sqrt{H_n}$ denote the magnitude of the complex (base-band) channel gain during the nth block and Θ_n denote the phase. Let $\mathbf{X}_n = (X_{n,1}, \dots, X_{n,N})$ and $\mathbf{Y}_n = (Y_{n,1}, \dots, Y_{n,N})$ be vectors in $\mathbb{C}^N$ which denote, respectively, the channel inputs and outputs over the nth block. These are related by:

$$\mathbf{Y}_n = \sqrt{H_n} e^{-j\Theta_n} \mathbf{X}_n + \mathbf{Z}_n. \tag{1}$$

Here the additive noise $\mathbf{Z}_n$ is a complex, circularly symmetric Gaussian random vector with zero mean and covariance matrix $\sigma^2 I$, where I denotes a $N \times N$ identity matrix. Furthermore, the sequence $\{\mathbf{Z}_n\}$ is independent and identically distributed (i.i.d.). The sequence of channel gains, $\{H_n\}$, are also modeled as a sequence of random variables; for simplicity, we assume that these are also i.i.d., which is appropriate when the time

[1]Most of the following will also generalize directly to a frequency-selective, block fading channel.

for one block of N channel uses is on the order of one coherence-time. For all n, H_n is assumed to take values in $\mathcal{H} = \mathbb{R}^+$ and have a continuous probability density function $f_H(h)$ and probability distribution $F_H(h)$. For simplicity, we assume that $f_H(h) > 0$ for all $h > 0$, which is true for most channel models of interest.[2] This implies that $F_H(h)$ is strictly increasing over $\mathcal{H}$. We assume that both the transmitter and receiver have perfect channel state information (CSI), *i.e.*, during the nth block, both the transmitter and receiver know the value of H_n and Θ_n. Since both the transmitter and receiver know Θ_n, we will ignore it in the following.

To model the buffer, we consider a discrete-time "fluid" buffer model in which time is slotted and the length of each time-slot corresponds to a block of N channel uses. Let A_n be the number of bits that arrive between time n and $n-1$, and let S_n be the buffer size at the start of the nth time-slot. Denote by U_n the number of bits removed from the buffer at the start of each time-slot, encoded and transmitted over the fading channel during the time-slot. This is a fluid model because we do not restrict the amount of "bits" that arrive to or are removed from the buffer during a time-slot to be an integer. The resulting buffer dynamics are given by:

$$S_{n+1} = \max\{S_n + A_{n+1} - U_n, A_{n+1}\}, \tag{2}$$

which ensures that the arriving data (A_{n+1}) waits in the buffer for at least one time-unit. The buffer size is assumed to be infinite and we denote the buffer state space by $\mathcal{S} = \mathbb{R}^+$. We consider the case where the arrival process $\{A_n\}$ is a sequence of i.i.d. random variables taking values in a compact set $\mathcal{A} = [a_{min}, a_{max}] \subset \mathbb{R}^+$ with probability distribution $F_A(a)$. Here, a_{max} and a_{min} are, respectively, upper and lower bounds on the amount of data that can arrive per time-unit. This process is assumed to be independent of both the channel fading process and the noise process. Let $\bar{A} = \mathbb{E}(A_n)$ denote the expected amount of data that arrives per time-slot.

[2]This can be relaxed to assuming that $f_h(h) > 0$ in some small interval $(0, \epsilon)$.

We assume that for the transmitter to reliably transmit at a rate of r bits per channel use during a given time-slot requires a received signal-to-noise ratio (SNR) given by a function $S(r)$. The main example we will consider is where

$$S(r) = 2^r - 1, \tag{3}$$

which is the received SNR required for the Gaussian channel in (1) to have a capacity of r bits per channel use. More generally, the following analysis will hold for any function $S(r)$ that satisfies the following regularity property:

Definition 1: A SNR function $S(r)$ is ***regular*** if $S(r)$ is increasing, differentiable, and strictly convex with $S(0) = 0$, $S'(0) > 0$, and $\lim_{r\to\infty} S'(r) = \infty$.[3]

In addition to (3), most practical modulation and coding schemes will satisfy this definition (e.g., see [4]). During a time-slot when the channel gain is h, the received SNR is given by $\frac{hP}{\sigma^2}$, where P is the transmission power. Thus, the required transmission power to send u bits during this time-slot is given by

$$P(h, u) := \frac{\sigma^2}{h} S(u/N). \tag{4}$$

In the case of (3), this becomes

$$P(h, u) = \frac{\sigma^2}{h}\left(2^{u/N} - 1\right), \tag{5}$$

which is the minimum power required so that the mutual information rate per channel use during the given block is equal to u/N. Provided that N is large enough, this choice will give a reasonable indication of the power needed to reliably transmit at rate u/N. One may question the reasonableness of modeling the required power using (5) when we are analyzing the performance of a system in the regime of small delays, since to communicate reliably at rates near capacity typically requires the use of long codes and subsequently long delays. The main justification for this is that we are measuring delays on the time-scale of the queue dynamics in 2; within each time-unit of this model, we

[3] We use the standard notation f' to denote $\frac{d}{dx}f$.

assume that there are still be enough degrees of freedom available in each time-slot to use sophisticated coding and approach capacity. Indeed, in many recent wireless systems data is transmitted in radio link control (RLC) blocks on a time-scale of 2-5 msec, using a bandwidth a 1-5 MHz; this results in on the order of 1000 channel uses per block.

Let $\mu : \mathcal{S} \times \mathcal{H} \mapsto \mathbb{R}^+$ denote a stationary (Markov) transmission policy that indicates U_n at any time n as a function of S_n and H_n. Under such a policy, $\{S_n\}$ will be a Markov chain. Under policy μ, we define the time-average transmission power to be

$$\bar{P}^{\mu} \equiv \limsup_{m \to \infty} \frac{1}{m} \sum_{n=1}^{m} \mathbb{E}(P(H_n, \mu(S_n, H_n))).$$

We also define the time-average delay to be,

$$\bar{D}^{\mu} \equiv \limsup_{m \to \infty} \frac{1}{m} \sum_{n=1}^{m} \frac{\mathbb{E}(S_n)}{\bar{A}}.$$

Assuming that $\{S_n\}$ is ergodic, it follows that that $\bar{P}^{\mu} = \mathbb{E}_{S,H} P(S, H)$ and $\bar{D}^{\mu} = \frac{\mathbb{E}S}{\bar{A}}$, which is equal to the average queueing delay by Little's law. Here and in the following, given an ergodic process, $\{X_n\}$, we denote by X (without an index), a random variable with the corresponding steady-state distribution. For a given channel and arrival process, the optimal power/delay trade-off, $P^*(D)$, is defined by

$$P^*(D) \equiv \inf\{\bar{P}^{\mu} : \mu \text{ such that } \bar{D}^{\mu} \leq D\}.$$

This will be a decreasing and convex function of D as shown in Fig. 2. As $D \to \infty$, $P^*(D)$ converges to an asymptotic value of $\mathcal{P}(\bar{A})$ at a rate of $\Theta\left(\frac{1}{D^2}\right)$ [15]. The asymptotic value, $\mathcal{P}(\bar{A})$ corresponds to the minimum power required to send at average rate $\bar{A}$, ignoring any delay constraints. When $P(h, u)$ is given by (5), this is the minimum power so that the channel has a *throughput capacity* of $\bar{A}/N$ bits per channel use [18].

We note that in the above definition we restricted our attention to stationary, Markov policies. More generally, one can consider transmission policies that depend on the time n, as well as the past history of buffer and channel states, i.e. $u_n = \mu(n, s_1, \ldots, s_n, h_1, \ldots, h_n)$. However, in terms of defining $P^*(D)$ such policies are not needed [15]. In other words, $P^*(D)$ also characterizes the performance that can be obtained by any such policy.

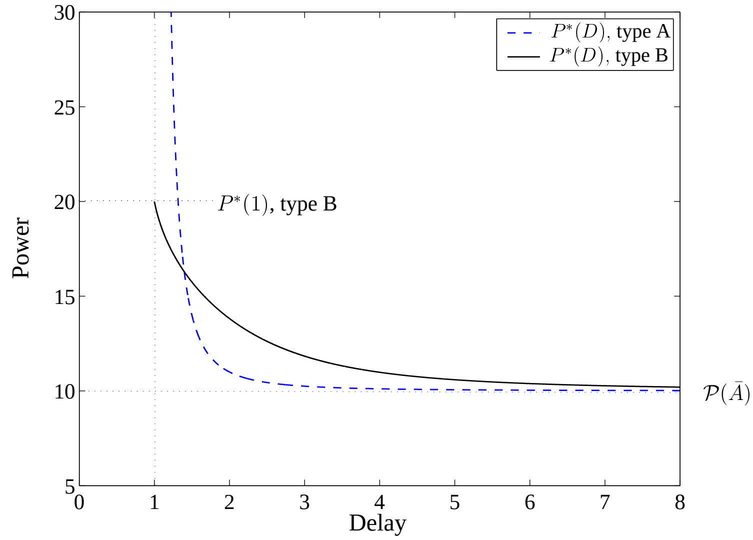

Fig. 2. Examples of the optimal power/delay trade-off $P^*(D)$ for two different channels. In the type A channel, the optimal power grows without bound as the delay approaches 1. In the type B channel the optimal power converges to the limit of $P^*(1)$.

In the following it will also be useful to define the optimal delay/power trade-off by

$$D^*(P) = \inf\{\bar{D}^\mu : \mu \text{ such that } \bar{P}^\mu \leq P\}.$$

Clearly, if $P^*(D)$ is strictly decreasing, then $D^*(P)$ will simply be its inverse. Given the above model of the buffer dynamics, all data must spend at least one time unit in the buffer, hence $D^*(P) \geq 1$ for all P. The only way that $D^*(P) = 1$ is if the transmitter used a policy such that $\mu(S_n, H_n) \geq A_n$ for all n, i.e. every bit is transmitted the time-slot after it arrives. The minimum power required by such a policy is given by

$$P^*(1) = \mathbb{E}_{A,H} P(H, A).$$

Assume that the arrival rate is constant, i.e. $A_n = \bar{A}$ for all n, and that $P(h, u)$ is given by (5). In this case, $P^*(1)$ represents the minimum power needed for the channel to have a *delay-limited capacity* of $\bar{A}/N$ bits per channel use [19].

Depending on the fading distribution, $P^*(1)$ may or may not be finite. In particular, note that since the arrival and channel processes are independent,

$$P^*(1) = \sigma^2 \mathbb{E}_H \left(\frac{1}{H} \right) \mathbb{E}_A S(A/N).$$

From this it follows that for any bounded arrival process, $P^*(1)$ is finite if and only if $\mathbb{E}_H \left(\frac{1}{H}\right) < \infty$. Therefore, every fading distribution can be classified as follows: a distribution is defined to have *positive delay-limited capacity*, if $\mathbb{E}_H \left(\frac{1}{H}\right) < \infty$; otherwise, the distribution is said to have *zero delay limited capacity*.

In the following, we will often place further restrictions on the behavior of the fading distribution near zero. In particular, we will consider the following two types of distributions:

Definition 2: A channel is defined to be of *type A* if H_n has a finite mean and $f_H(0) > 0$.

Definition 3: A channel is defined to be of *type B* if H_n has a finite mean and $f_H(h) = \Theta(h^\gamma)$ as $h \to 0$ for some $\gamma > 0$.

For example, in a Rayleigh fading channel $f_H(h)$ is an exponential distribution with $f_H(0) = \frac{1}{\mathbb{E}H}$; hence, this is a type A channel. A Ricean fading channel is also a type A channel. A Nakagami fading channel will be of type A if the Nakagami fading figure, m is less than 1. It will be type B when $m \geq 1$; in this case $\gamma = m - 1$. A Rayleigh channel with $m > 1$ independent diversity branches will also be of type B with $\gamma = m - 1$, when either selection diversity or maximal ratio combining are used. It can be seen that a type A channel will always have a zero delay-limited capacity, while a type B channel will always have a positive delay-limited capacity.

There are several properties of these channels we will use, we state these in the following two lemmas. The first lemma bounds the rate at which the channel gain's distribution function goes to zero as $h \to 0$.

Lemma 1: As $h \to 0$, for a type A channel, $F_H(h) = \Theta(h)$ and for a type B channel, $F_H(h) = \Theta(h^{\gamma+1})$.

This follows directly by writing $F_H(h) = \int_0^h f_H(\tilde{h})\,d\tilde{h}$. For a type A channel, we also use that the density $f_H(h)$ is continuous, thus $f_H(h) > 0$ for all h within some sufficiently small neighborhood of 0.

For a given fading density, $f_H(h)$, define $G(h)$ by

$$G(h) \equiv \int_h^\infty \frac{1}{\tilde{h}} f_H(\tilde{h})\,d\tilde{h}.$$

As $h \to 0$, $G(h) \to \mathbb{E}\left(\frac{1}{H}\right)$, which is infinite for any channel with zero delay-limited capacity. The next lemma characterizes how fast this quantity increases in the case of type A channels.

Lemma 2: Let $f_H(h)$ be a type A fading density. Given any $h_t > 0$, then there exists a finite constant M_2 such that for all $h < h_t$,

$$G(h) \geq M_1 \ln\left(\tfrac{1}{h}\right) + M_2,$$

where $M_1 = \inf\{f_H(h) | h \leq h_t\}$. Likewise, these exists a finite constant $\tilde{M}_2$, such for all $h < h_t$,

$$G(h) \leq \tilde{M}_1 \ln\left(\tfrac{1}{h}\right) + \tilde{M}_2,$$

where $\tilde{M}_1 = \sup\{f_H(h) | h \leq h_t\}$. Furthermore, for h_t small enough, M_1 will be strictly positive.

The proof is given in Appendix I. This implies that for a type A channel, $G(h)$ grows like $\ln\left(\frac{1}{h}\right)$ as $h \to 0$.

For either type of channel, let $G^{-1}(x)$ denote the inverse of the function $G(h)$. Since, by assumption $f_H(h) > 0$ for all $h > 0$, $G(h)$ will be strictly decreasing and approach zero as $h \to \infty$. For a channel with zero delay-limited capacity, as $h \to 0$, $G(h) \to \infty$, and so $G^{-1}(x)$ is defined for all $x \in [0, \infty)$. For a channel with positive delay-limited capacity $G(0) = \mathbb{E}\left(\frac{1}{H}\right)$ is finite; in this case, $G^{-1}(x)$ is only defined for $x \in [0, G(0)]$.

III. Lower bounds on the optimal convergence rate

In this section, we lower bound the asymptotic behavior of $D^*(P)$ in the regime of small delays (high powers). We first give a lower bound on $D^*(P)$ that becomes tight as $P \to P^*(1)$. This bound holds for any channel distribution and arrival statistics that satisfy the previous assumptions. We then examine this bound for type A and type B channels and use this to bound the rate at which $D^*(P)$ approaches 1 as the average power increases to $P^*(1)$.

For any $P \leq P^*(1)$, $D^*(P)$ satisfies the following lower bound:

Proposition 1: Consider a system with a regular SNR function $S(r)$. For any $P \leq P^*(1)$,

$$D^*(P) - 1 \geq F_H \left[\left(\frac{S'(0)}{S'(a_{max})} \right) G^{-1} \left(\frac{P}{\sigma^2 \mathbb{E}_A(S(A/N))} \right) \right].$$

Note that the quantity $\sigma^2 \mathbb{E}_A S(A/N)$ is the value of $P^*(1)$ for a channel in which $H_n = 1$ for all n. In a channel with positive delay limited capacity, this satisfies $\frac{P^*(1)}{\sigma^2 \mathbb{E}_A S(A/N)} = G(0)$, and so for $P = P^*(1)$ this bound is tight, i.e. it is equal to zero. Likewise, for a channel with zero delay limited capacity, $P^*(1) = \infty$. Hence, as $P \to \infty$, the bound approaches zero and is once again tight.

To prove this proposition, we consider a "fictitious system" which is identical to the original system except that here all arriving data can be transmitted after waiting for 2 time-units without requiring any power (recall that all data must wait at least one time-unit). However, to transmit the data after one time-unit still requires the same power as in the original system. Therefore, the maximum delay in the fictitious system will be no more than 2 time-units. Let $\hat{D}(P)$ be the minimum average delay in this fictitious system under any transmission policy with average power no greater than P. Clearly, for the same arrival and channel processes, we must have $\hat{D}(P) \leq D^*(P)$ for all P. We will bound $\hat{D}(P)$ and use this relationship to derive the desired lower bound on $D^*(P)$.

Under the assumption that all arriving data leaves after 2 time-slots, the buffer dynamics in the fictitious system can be written as

$$S_{n+1} = \max(A_n - U_n + A_{n+1}, A_{n+1}).$$

Also, at each time n an optimal policy will set $U_n \leq A_n$, since any other data in the buffer will leave the system anyway without requiring any power. Therefore, an optimal policy for the fictitious system can be expressed as function of the current channel state, H_n and the number of arrivals A_n. It follows that $\hat{D}(P)$ is the solution to the following optimization problem:

$$\begin{aligned}\underset{\zeta:\mathcal{H}\times\mathcal{A}\mapsto\mathbb{R}^+}{\text{minimize}}\ & 1+\frac{1}{\bar{A}}\mathbb{E}_{H,A}\left\{(A-\zeta(H,A))^+\right\}\\ \text{subject to:}\ & \mathbb{E}_{H,A}P(H,\zeta(H,A))\leq P\\ & \zeta(h,a)\geq 0,\quad \forall h\in\mathcal{H}, a\in\mathcal{A}\end{aligned}$$

Here the objective function correspond to the expected number of packets in the system under the policy $\zeta(\cdot,\cdot)$ divided by the average arrival rate, which by Little's law is equal to the average delay. Note, we have also used the fact that the arrivals are i.i.d. This is equivalent to finding a policy $\zeta(\cdot,\cdot)$ that solves:

$$\begin{aligned}\underset{\zeta:\mathcal{H}\times\mathcal{A}\mapsto\mathbb{R}^+}{\text{maximize}}\ & \mathbb{E}_{H,A}\zeta(H,A)\\ \text{subject to:}\ & \mathbb{E}_{H,A}\frac{\sigma^2}{H}S(\zeta(H,A)/N)\leq P\\ & 0\leq\zeta(h,a)\leq a,\quad \forall h\in\mathcal{H}, a\in\mathcal{A}.\end{aligned}$$

It can be seen that the constraints are convex and that the objective in linear in $\zeta(h,a)$. From the first order optimality conditions for this problem, it follows that the optimal policy, $\zeta^*(h,a)$ is given by

$$\zeta^*(h,a)=\min\left\{N\psi\left(\tfrac{Nh}{\lambda\sigma^2}\right),a\right\}, \tag{6}$$

where $\lambda > 0$ is a Lagrange multiplier chosen to satisfy the average power constraint and $\psi(x)=\min\{r\geq 0 : S'(r)\geq x\}$. Note that for a regular SNR function, $S'(r)$ is strictly increasing and $S'(0)>0$. Thus for all $x\geq S'(0)$, $\psi(x)$ will be the inverse of $S'(r)$, and for all $x\leq S'(0)$, $\psi(x)=0$. This implies that there exists a lower channel threshold,

$$h_L\equiv\lambda S'(0)\sigma^2/N, \tag{7}$$

such that for all $h\leq h_L$, $\zeta^*(h,a)=0$. Likewise, for a regular SNR function, $S'(x)$ grows without bound as x increases, and so for each $a\in\mathcal{A}$, there exists an upper channel threshold,

$$h_U(a) \equiv \lambda S'(a/N)\sigma^2/N, \tag{8}$$

such that for all $h \geq h_U(a)$, $\zeta^*(h,a) = a$. Note that these thresholds only depend on the average power constraint through the Lagrange multipliers λ. As P is increases, λ will decrease and therefore so will h_L and $h_U(a)$.

Under this policy, the resulting power allocation can be written as:

$$P(h, \zeta^*(h,a)) = \begin{cases} 0, & \text{if } h \leq h_L, \\ \frac{\sigma^2}{h} S\left(\psi\left(\frac{Nh}{\lambda\sigma^2}\right)\right), & \text{if } h_L \leq h \leq h_U(a), \\ \frac{\sigma^2}{h} S\left(a/N\right), & \text{if } h \geq h_U(a). \end{cases} \tag{9}$$

For example, in the case where $P(h,u)$ is given by (5), then the power allocation in (9) can be written as

$$P(h, \zeta^*(h,a)) = \begin{cases} 0 & \text{if } h \leq h_L, \\ \frac{N}{\lambda} - \frac{\sigma^2}{h} & \text{if } h_L \leq h \leq h_U(a), \\ \frac{\sigma^2}{h}\left(2^{a/N} - 1\right) & \text{if } h \geq h_U(a), \end{cases} \tag{10}$$

which corresponds to the the well-known “water-filling” power allocation [20] whenever $h < h_U(a)$; for $h > h_U(a)$, the transmitter inverts the channel to transmit at the constant rate a. An example of this power allocation is shown in Figure 3

Using (9), the average power under policy $\zeta^*(\cdot,\cdot)$ satisfies

$$\begin{aligned} P &= \int_{a_{min}}^{a_{max}} \left(\int_{h_L}^{h_U(a)} \frac{\sigma^2}{h} S(\psi(\tfrac{Nh}{\lambda\sigma^2}))\, dF_H(h) + \int_{h_U(a)}^{\infty} \frac{\sigma^2}{h} S(a/N)\, dF_H(h) \right) dF_A(a) \\ &\geq \int_{a_{min}}^{a_{max}} \int_{h_U(a)}^{\infty} \frac{\sigma^2}{h} S(a/N)\, dF_H(h), dF_A(a) \\ &\geq G(h_U(a_{max}))\sigma^2 \mathbb{E}_A S(A/N). \end{aligned}$$

Equivalently,

$$h_U(a_{max}) \geq G^{-1}\left(\frac{P}{\sigma^2 \mathbb{E}_A S(A/N)}\right), \tag{11}$$

where we have used that $G(h)$ is strictly decreasing in h.

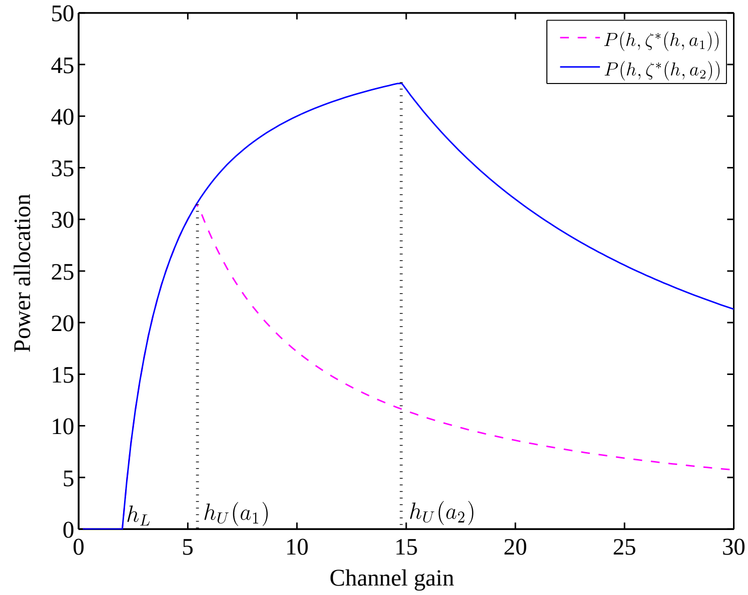

Fig. 3. Example of $P(h, \zeta^*(h, a))$ from (10); shown here are two curves corresponding the power allocation as a function of the channel gain for two different values of a with $a_1 < a_2$. The corresponding thresholds h_L and $h_U(a)$ are also indicated.

Recall that the policy in (9) achieves $\hat{D}(P) - 1$. Therefore,

$$\begin{aligned}
\hat{D}(P) - 1 &= \frac{1}{\bar{A}} \mathbb{E}_{H,A}(A - \zeta^*(H, A))^+ \\
&= \frac{1}{\bar{A}} \left(\Pr(H \le h_L)\bar{A} + \Pr(H > h_L) \mathbb{E}_{H,A} \left((A - \zeta^*(H, A))^+ \middle| H \ge h_L \right) \right) \\
&\ge \Pr(H \le h_L) \\
&= F_H(h_L) \\
&= F_H \left(\frac{S'(0)}{S'(a_{max}/N)} h_U(a_{max}) \right).
\end{aligned}$$

Here, we have used that $(A - \zeta^*(H, A))^+$ is always non-negative and is equal to A for $H < h_L$, and we also used the definitions of h_L and $h_U(a)$ in (7) and (8).

Combining (11) and (12) and recalling that $F_H(h)$ is decreasing, we get the bound in Proposition 1.

So far we have given a bound on the delay that is asymptotically tight as the $P \to P^*(1)$. In the next two corollaries we will use this bound to bound the rate at which $D^*(P)$ approaches its asymptotic limit of 1 for both type A and type B channels.

Corollary 1: For a type A channel, as $P \to \infty$, $D^*(P) - 1 = \Omega\left(e^{-\alpha P}\right)$, for any $\alpha > (\sigma^2 f_H(0)\mathbb{E}_A(S(A/N)))^{-1}$.

The proof is given in Appendix II. The result of this corollary can equivalently be expressed in terms of the power/delay trade-off, i.e. for a type A channel, $P^*(D) = \Omega\left(\ln\left(\frac{1}{D-1}\right)\right)$ as $D \to 1$. Note that in this case the constant α is not needed.

Corollary 2: For a type B channel with parameter $\gamma > 0$, as $P \to P^*(1)$ (from below), $D^*(P) - 1 = \Omega\left((P^*(1) - P)^{\frac{\gamma+1}{\gamma}}\right)$.

The proof is given in Appendix III. Notice that the exponent $\frac{\gamma+1}{\gamma}$ is decreasing in γ, and so this bound will approach 1 slower in channels with larger values of γ.

IV. Optimal and Sub-optimal Sequences of Transmission Policies

In the previous section we used the policy $\zeta^*(h, a)$ for the fictitious system to bound the optimal delay/power trade-off for the original system. Notice that for any power $P < P^*(1)$, the expected transmission rate under the policy $U^*(h, a)$ will be less than the average arrival rate, i.e. this policy will not stabilize the actual system and will result in unbounded delays. In this section, we study the performance obtained by several different types of policies that result in finite delays in the actual system. We then study the behavior of sequences of these policies as the average delays approach 1. First we consider a class of “channel threshold” polices in which the average power approaches $P^*(1)$ as the delays approach 1. For type A and type B channels, we then show that a sequence of these policies can achieve the same rate of convergence as the bounds given in Corollaries 1 and 2. It follows that these bound are tight and that these simple policies achieve the optimal convergence rate. We then consider a class of “bounded rate” policies which also exhibit optimal performance for type A channels, but not for type B channels. Finally, we consider two other sub-optimal policies which do not depend on the buffer size.

A. Channel Threshold Policies

The first type of policies we consider are *channel threshold* policies in which the transmitter only transmits when the channel gain is greater than a given threshold; when this occurs the transmitter empties the buffer. More precisely, we define a policy $\mu_k : \mathcal{H} \times \mathcal{S} \rightarrow \mathbb{R}^+$ to be a channel threshold policy with threshold, $h_k \geq 0$, if

$$\mu_{h_k}(h,s) = \begin{cases} s, & \text{if } h > h_k, \\ 0, & \text{if } h \leq h_k. \end{cases}$$

For a given channel threshold policy μ_k, let $q_k = \Pr(h \leq h_k)$ denote the probability that the channel gain is below the threshold. Also, let

$$\Sigma_n = \mathbb{E}_{A^n}\left(S\left(\frac{1}{N} \sum_{i=1}^{n} A_i \right) \right),$$

where $A^n = (A_1, \dots, A_n)$ denotes a sequence of n i.i.d. random variables, each with distribution $F_A(a)$. For $n = 1, 2, \dots,$ Σ_n represents the the expected received SNR required to transmit all the data that arrives during n time-slots. The average power and delay under such as policy is bounded in the following Proposition, whose proof is given in Appendix IV.

Proposition 2: Let μ_k be a channel threshold policy with threshold h_k, then

$$\bar{D}^{\mu_k} - 1 = \frac{q_k}{1 - q_k},$$

and for a regular SNR function,

$$\bar{P}^{\mu_k} \leq \sigma^2 G(h_k) \left(\sum_{n=1}^{\infty} q_k^{n-1}(1 - q_k)\Sigma_n \right),$$

with equality if the right-hand side is finite.

We define a *decreasing sequence* of channel threshold policies $\{\mu_k | k = 1, 2, \dots\}$ to be a sequence where the associated thresholds h_k form a decreasing sequence with

$\lim_{k\to\infty} h_k = 0$. Clearly, as k increases, the average delay will decrease with $\bar{D}^{\mu_k} \to 1$ as $k \to \infty$. Next, we characterize the rate at which this converges as a function of the average power for a type A and B channel. For this we make one additional assumption on the SNR function S.

Definition 4: A regular SNR function $S(r)$ has *exponentially bounded growth* if there exists non-negative constants M and κ such that for all $r \geq 0$, $S(r) \leq M\kappa^r$.

For example, when $S(r)$ is given by (3), it satisfies this definition with $M = 1$ and $\kappa = 2$. For such a SNR function, then for all $n = 1, 2, \ldots,$ we have

$$\Sigma_n \leq M\kappa^{na_{max}/N} \equiv M\tilde{\kappa}^n. \tag{12}$$

Corollary 3: For a type A channel, if the SNR function has exponentially bounded growth, then for any decreasing sequence of channel threshold policies $\{\mu_k\}$, as $K \to \infty$, $\bar{P}^{\mu_k} \to \infty$ and $\bar{D}^{\mu_k} - 1 = O\left(\exp(-\alpha\bar{P}^{\mu_k})\right)$, for any $\alpha < \left(\sigma^2 f_H(0)\mathbb{E}_A S(A/N)\right)^{-1}$.

The proof is given in Appendix V. Recall that in Corollary 1, we showed that $D^*(P) - 1 = \Omega\left(\exp(-\alpha P)\right)$ for any $\alpha > (\sigma^2 f_H(0)\mathbb{E}_A S(A/N))^{-1}$. This corollary implies that a sequence of decreasing channel threshold policies are nearly order optimal in the sense that we can find policies whose exponents α are arbitrarily close to the bound in Corollary 1. If instead, we consider the power/delay trade-off, then this corollary implies that for any decreasing sequence of channel threshold policies, $\bar{P}^{\mu_k} = O\left(\ln\left(\frac{1}{\bar{D}^{\mu_k}-1}\right)\right)$. In other words, in terms of the power/delay trade-off, these policies are order optimal. Therefore in the small delay regime, the optimal convergence rate of $P^*(D)$ for type A channels is $\Theta\left(\ln\left(\frac{1}{D-1}\right)\right)$. Note that this is a much faster rate of change than the $\frac{1}{D^2}$ behavior in the large delay regime.

For type B channels we have:

Corollary 4: For a type B channel with parameter $\gamma > 0$, if the SNR function has exponentially bounded growth, then for any decreasing sequence of channel threshold policies $\{\mu_k\}$, as $K \to \infty$, $\bar{P}^{\mu_k} \to P^*(1)$ and $\bar{D}^{\mu_k} - 1 = O\left(\left(P^*(1) - \bar{P}^{\mu_k}\right)^{\frac{\gamma+1}{\gamma}}\right)$.

The proof is given in Appendix VI. Comparing the result of this corollary to the bound in Corollary 2, it follows that for a type B channel, decreasing sequences of channel threshold policies are order optimal. Therefore, $D^*(P) - 1 = \Theta\left((P^*(1) - P)^{\frac{\gamma+1}{\gamma}}\right)$ as $P \to P^*(1)$ from below. Equivalently, $P^*(1) - P^*(D) = \Theta\left((D-1)^{\frac{\gamma}{\gamma+1}}\right)$ as $D \to 1$. As noted in Section III, $\frac{\gamma+1}{\gamma}$ is decreasing and approaches 1 as γ increases. For example, these results imply that in a Rayleigh fading channel as the number of independent diversity branches are increased (leading to larger values of γ), $D^*(P)$ will approach 1 at a slower rate. For a large number of diversity branches, the rate will be approximately linear in $P^*(1) - P$. Of course, $P^*(1)$ also decreases with additional diversity branches.

B. Policies with bounded transmission rates

Under a channel threshold policy, since the transmitter empties the buffer whenever the channel gain is greater than the threshold, the required transmission rate in a time-slot can be arbitrarily large. In this section, we look at a sequence of policies with bounded transmission rates. Such a policy may be of interest in a system with a limit on the peak transmission rate; for example, such limits may be due constraints on the available coding and modulation schemes. Next, we consider *bounded rate channel threshold policies* where the maximum available transmission rate is limited to $(a_{max} + \delta)/N$ for some small value $\delta > 0$. In such a policy, the transmitter once again only transmits when the channel gain is larger than a threshold h_k. However, given that the channel is greater than this threshold, these policies set $U_n = A_n + \delta$, i.e. they transmit at most $a_{max} + \delta$ bits, resulting in the desired maximum transmission rate. We denote such a policy by $\phi_k(h, a)$, i.e.

$$\phi_k(h, a) = \begin{cases} a + \delta, & \text{if } h > h_k, \\ 0, & \text{if } h \leq h_k. \end{cases}$$

Note that these policies do not base the transmission decision on only H_n and S_n, but can be viewed as using the past history of S_n and U_n. As noted in Section II, we permit such a dependence though it is not needed for an optimal policy. Clearly, if $s_n/N < A_n + \delta$ then there is not enough information in the buffer to transmit. In this case we can assume the transmitter sends extra "dummy" bits. This is clearly a poor choice from the view of saving power, but is sufficient for our purposes.

As in Section IV-A, we again consider a decreasing sequence of policies $\{\phi_k | k = 1, 2, \ldots\}$, where the thresholds h_k form a decreasing sequence with $\lim_{k\to\infty} h_k = 0$. We first show that for a type A channel, such a sequence is also order optimal. In other words, for type A channels, having a bounded transmission rate, does not effect the achievable rate that $D^*(P)$ converges to 1. However, for type B channels, such a sequence cannot achieve the optimal convergence rate because P^{ϕ_k} will not converge to $P^*(1)$ as $k \to \infty$. This illustrates a basic difference between channels with positive and zero delay-limited capacity.

For these results, we will use the following lemma which gives upper and lower bounds on the average buffer delay under any policy for which $U_n - A_n$ is an i.i.d. sequence. This is the clearly the case with a bounded rate channel threshold policy, since $U_n - A_n$ depends only on H_n at each time n.

Lemma 3: For any policy where $\Delta_n = U_n - A_n$ is an i.i.d. sequence, the average buffer occupancy is bounded by

$$\frac{\mathbb{E}\{([-\Delta]^+)^2\}}{2\mathbb{E}(\Delta)} \leq \mathbb{E}S - \bar{A} \leq \frac{\sigma_\Delta^2}{2(\mathbb{E}\Delta)},$$

where $[-\Delta]^+ = \max(-\Delta, 0)$ and σ_Δ^2 is variance of Δ_n.

Let $Z_n = S_n - A_n$, and $\Delta_n = U_n - A_n$. The queue dynamics in (2) can then be rewritten as

$$Z_{n+1} = (Z_n - \Delta_n)^+,$$

where by assumption $\{\Delta_n\}$ is an i.i.d. sequence. Therefore $\{Z_n\}$ is a Lindley process as is the delay in a continuous-time GI/G/1 queue. The bounds in Lemma 3 are essentially the same as Kingman's upper and lower bounds on the average delay for such a GI/G/1 system [21].

Using this lemma, we have the following result for a type A channel. Note that here we do not require the SNR function to be exponentially bounded.

Proposition 3: For a type A channel, let $\{\phi_k\}$ be a decreasing sequence of bounded rate channel threshold policies. Then as $k \to \infty$, $\bar{P}^{\phi_k} \to \infty$, and $\bar{D}^{\phi_k} - 1 = O\left(\exp(-\alpha \bar{P}^{\phi_k})\right)$, for any $\alpha < \left(\sigma^2 f_H(0) \mathbb{E}_A\{S(A/N)\}\right)^{-1}$.

The proof is given in Appendix VII. This implies that for type A channels, bounded rate channel threshold policies can achieve the same order of convergence as a channel threshold policy, which we have seen is essentially order optimal.

Next, we define a *fixed-rate, channel threshold policy* $\tilde{\phi}_k(h)$ to be a policy that transmits at a fixed rate $\tilde{a}/N$, whenever the channel gain is greater than h_k, and sends nothing otherwise. This differs from the previous bounded rate policies in that here the transmission rate does not depend on A_n. The following corollary of Proposition 3 gives a bound on the rate of convergence for such policies, when $\tilde{a} > a_{max}$

Corollary 5: For a type A channel, let $\{\tilde{\phi}_k\}$ be a decreasing sequence of fixed-rate, channel threshold policies with $\tilde{a} > a_{max}$. As $k \to \infty$, $\bar{P}^{\tilde{\phi}_k} \to \infty$ and $\bar{D}^{\tilde{\phi}_k} - 1 = O(\exp(-\alpha \bar{P}^{\tilde{\phi}_k}))$ for any $\alpha < (\sigma^2 f_H(0) S(\tilde{a}/N))^{-1}$.

The proof is given in Appendix VIII. In this corollary the constraint on the parameter α will be smaller than in Proposition 3, unless the arrival process is constant (i.e. $A_n = \bar{A}$ for all n.) Thus in general this bound does not imply than these policies are order optimal in terms of the delay/power trade-off. However, in terms of the power/delay trade-off, we can again ignore the α parameter so that these policies are order optimal in this sense. Notice that these policies do not depend on the buffer occupancy at all; this illustrates another significant difference between the small delay and large delay regimes; in the large delay regime some buffer dependence is required for any order optimal policy [15].

Next we turn to type B channels. Notice that for a type B channel or any other channel with a positive delay-limited capacity, a decreasing sequence of $\{\phi_k\}$ of bounded rate channel threshold policies, with a fixed parameter $\delta > 0$, will satisfy

$$\lim_{k \to \infty} \bar{P}^{\phi_k} = \mathbb{E}_{A,H} P(H, A + \delta) > P^*(1).$$

Therefore any such sequence is clearly not order optimal in the small delay regime. The problem here is that in the small delay limit the power wasted on transmitting extra "dummy" bits becomes significant for type B channels, while we could ignore this in type A channels.

A better approach for such channels is as k increases to reduce both the channel threshold h_k as well as the parameter $\delta = \delta_k$, with $\lim_{k\to\infty} \delta_k = 0$. In this way as $k \to \infty$, $\bar{P}^{\phi_k} \to P^*(1)$. However, as the following proposition states, such a sequence of policies still do not achieve the optimal convergence rate for type B channels. Here, for simplicity, we restrict ourselves to the case where $S(r)$ is given by (3).

Proposition 4: For a type B channel with parameter $\gamma > 0$ and $S(r)$ given by (3), let ϕ_k be a decreasing sequence of bounded rate channel threshold policies with decreasing parameters δ_k, where $\delta_k \to 0$. If as $k \to \infty$, $\bar{P}^{\phi_k} \to P^*(1)$ from below, then $\bar{D}^{\phi_k} - 1 = \Omega\left(\left(P^*(1) - \bar{P}^{\phi_k}\right)^{\frac{1}{\gamma}}\right)$.

The proof is given in Appendix IX. Note that since $\gamma > 0$, $\frac{1}{\gamma}$ will be strictly less than the optimal exponent of $\frac{\gamma+1}{\gamma}$ given by Corollary 2, and so any decreasing sequence of bounded rate channel threshold policies will not be order optimal for type B channels. This illustrates a basic difference between type A and type B channels and suggests that there is a larger class of order optimal policies for type A channels.

C. Sub-optimal policies

In this section, we consider two simple policies and show that they have suboptimal convergence rates for type A channels (these policies are also clearly sub-optimal for type B channels as they will not even converge to $P^*(1)$). Throughout this section, we will only consider the case where $S(r)$ is given by (3).

First, we consider *fixed power* policies which do not depend on the buffer state. By this we mean a policy in which the transmitter uses a fixed power, $\bar{P}_k$ in each slot and so transmits u bits, where $P(h, u) = \bar{P}_k$. Once again, if there are fewer than u bits available, we assume that the transmitter sends extra dummy bits. We denote such a policy by $\nu_k(h)$, so that for all $h \in \mathcal{H}$,

$$\nu_k(h) = N \log\left(1 + \frac{h\bar{P}_k}{\sigma^2}\right), \tag{13}$$

where we have used that $S(r)$ is given by (3). Using such a policy, the average power is clearly equal to $\bar{P}_k$.

Consider a sequence of fixed power policies, $\{\nu_k\}$, where as k increases, $\bar{P}_k$ increases, with $\lim_{k\to\infty} \bar{P}_k = \infty$. For such a sequence of policies it is also clear that the average delay will decrease with k. The next proposition shows that in the limit, the average delay approaches the minimum value of 1; however, the rate of convergence is much slower than the optimal rate of $\exp(-\alpha P)$ for a type A channel.

Proposition 5: For a type A channel, let $\{\nu_k\}$, be a sequence of fixed power policies with $\lim_{k\to\infty} \bar{P}_k = \infty$. As $k \to \infty$, $\bar{D}^{\nu_k} - 1 = O\left((\log \bar{P}_k)^{-1}\right)$ and $\bar{D}^{\nu_k} - 1 = \Omega\left((\bar{P}_k \log \bar{P}_k)^{-1}\right)$.

The proof is given in Appendix X. In this proof we again use Lemma 3 to upper and lower bound the average delay under a fixed power policy. Notice that the upper bound of $(\bar{P}_k \log \bar{P}_k)^{-1}$ is much slower than the optimal convergence rate of $\exp(-\alpha P)$ obtained by a channel threshold policy, i.e. fixed power policies are not order optimal in the small delay regime.

The second class of sub-optimal policies we consider are a sequence of "fixed water-filling" policies. By this we mean policies that use a water-filling power (and rate) allocation, once again independent of the buffer state. Let $\omega_k(h)$ denote such a policy, where

$$\omega_k(h) = \begin{cases} 0, & \text{if } h \leq \frac{\sigma^2}{\ell_k}, \\ N \log\left(\frac{h}{\sigma^2}\ell_k\right), & \text{otherwise.} \end{cases}$$

Here ℓ_k denote the "water-level" used by the policy; this is chosen to satisfy a given average power constraint, $\bar{P}^{\omega_k}$. To maximize throughput, for a backlogged system, it is well-known that this is the optimal power allocation [22]. Also in the large delay regime, the order optimal buffer threshold policies in [15] are based on a water-filling power allocation. However, as stated in the following proposition, a water-filling power allocation with no buffer dependence is not optimal in the small delay regime. Furthermore, this type of policy can not achieve a convergence rate faster than $(P \log P)^{-1}$, which is the same as the bound for a fixed power policy in Proposition 5.

Proposition 6: For a type A channel, let $\{\omega_k\}$ be a sequence of fixed water-filling policies with $\lim_{k \to \infty} \bar{P}^{\omega_k} = \infty$. As $k \to \infty$, $\bar{D}^{\omega_k} - 1 = O\left((\log \bar{P}^{\omega_k})^{-1}\right)$, and $\bar{D}^{\omega_k} - 1 = \Omega\left((\bar{P}^{\omega_k} \log \bar{P}^{\omega_k})^{-1}\right)$.

A sketch of the proof is given in Appendix XI.

V. Conclusions

In this paper we have analyzed the optimal power/delay trade-off for a single user fading channel in the regime of small delays and large power. In this regime, the optimal trade-off was shown to strongly depend on the behavior of the fading distribution near zero. We focused on two broad classes of fading channels. For "type A" channels where the fading density is strictly positive at zero, the average delay was shown to decrease at a rate of $\Theta(e^{-\alpha P})$ as the average power increases, where α is a parameter that depends on the arrival statistics and the fading density at zero. For "type B" channels, where the fading density approaches zero like $\Theta(h^\gamma)$, the average delay was shown to decrease at a rate $\Theta((P^*(1) - P)^{\frac{\gamma+1}{\gamma}})$ as the average power approaches $P^*(1)$, the minimum power required to achieve the minimum delay. In both cases, a simple channel threshold policy was shown to be order optimal. For type A channels we also showed that a "bounded rate" policy is also essentially order optimal; however, such policies are not optimal for type B channels. Finally, we showed that a "fixed power" policy and a "fixed water-filling" policy are not order optimal for either channel.

Here we have focused on a single user communicating over a memoryless fading channel with only a long-term average power constraint. Potential directions for future work include relaxing these modeling assumptions, for example considering multi-user systems or channels with memory. Another possible direction is to consider models with imperfect channel knowledge, in which case outages may occur requiring data to be retransmitted.

APPENDIX I

Proof of Lemma 1: We give a proof for the lower bound; the upper bound can be derived in the same manner. For $h < h_t$, we have

$$\begin{aligned} G(h) &\geq \int_h^{h_t} \frac{1}{h} M_1 \, dh + \int_{h_t}^{\infty} \frac{1}{h} f_H(h) \, dh \\ &\geq M_1(\ln(h_t) - \ln(h)) + \int_{h_t}^{\infty} \frac{1}{h} f_H(h) \, dh \\ &= M_1 \ln\left(\frac{1}{h}\right) + M_2, \end{aligned}$$

where,

$$\begin{aligned} M_2 &= M_1 \ln(h_t) + \int_{h_t}^{\infty} \frac{1}{h} f_H(h) \, dh \\ &\leq M_1 \ln(h_t) + \int_{h_t}^{\infty} \frac{1}{h_t} f_H(h) \, dh \\ &\leq M_1 \ln(h_t) + \frac{1}{h_t}. \end{aligned}$$

Here we used that for $h \geq h_t$, $\frac{1}{h} \leq \frac{1}{h_t}$. From this it follows that $|M_2| < \infty$. Also, from the continuity of $f_H(h)$, it follows that for h_t small enough, M_1 must be greater than 0. ■

APPENDIX II

Proof of Corollary 1: From Lemma 1, for a type A channel, $F_H(h) = \Theta(h)$ as $h \to 0$. Using this in the bound from Proposition 1, we have

$$D^*(P) - 1 = \Omega\left(G^{-1}\left(\frac{P}{\sigma^2 \mathbb{E}_A(S(A/N))}\right)\right).$$

To complete the proof we will bound the rate at which $G^{-1}(x)$ approaches zero as $x \to \infty$ for a type A channel. Let $h = G^{-1}(x)$, so that $x = G(h)$. Pick some constant $h_t > 0$ such that $M_1 > 0$ in Lemma 2. As x increases, h decreases to zero. Thus there exists some x', such that for all $x > x'$, $h < h_t$. And so, from Lemma 2, for all $x > x'$,

$$x = G(h) \geq M_1 \ln\left(\frac{1}{h}\right) + M_2.$$

Equivalently, for $x > x'$,

$$h \geq \exp\left(-\tfrac{x}{M_1} + \tfrac{M_2}{M_1}\right).$$

This implies that $G^{-1}(x) = \Omega\left(e^{-x/M_1}\right)$ as $x \to \infty$. Combining this with the above we have that as $P \to \infty$, $D^*(P) - 1 = \Omega\left(e^{-\alpha P}\right)$, where $\alpha = \left(M_1 \sigma^2 \mathbb{E}_A(S(A/N))\right)^{-1}$. Finally, we note that in the above bound, $M_1 = \inf\{f_H(h) | h \leq h_t\} \leq f_H(0)$ and can be made arbitrarily close to this value by choosing h_t small enough. This gives the desired lower bound on α. Notice that if $f_H(h)$ is increasing at $h = 0$, then we can choose $\alpha = \left(\sigma^2 f_H(0) \mathbb{E}_A(S(A/N))\right)^{-1}$; otherwise, α can be made arbitrarily close to this value, but not equal to it. ■

APPENDIX III

Proof of Corollary 2: This proof follows a similar argument as in Corollary 1. In this case, since the channel is of type A, then from Lemma 1, we have that $F_H(h) = \Theta\left(h^{\gamma+1}\right)$. Therefore, combining this with Proposition 1 gives us,

$$D^*(P) - 1 = \Omega\left(\left(G^{-1}\left(\frac{P}{\sigma^2 \mathbb{E}_A(S(A/N))}\right)\right)^{\gamma+1}\right). \tag{14}$$

Let $h = G^{-1}(x)$, so that $x = G(h)$. Define, $\bar{G}(h) = G(0) - G(h)$, so that

$$\bar{G}(h) = \int_0^h \frac{1}{\tilde{h}} f_H(\tilde{h})\, d\tilde{h},$$

and note that for a type B channel this integral will be finite. By assumption, $f_H(h) = \Theta(h^\gamma)$ as $h \to 0$. From this it follows that, $\bar{G}(h) = \Theta(h^\gamma)$ as $h \to 0$. Therefore,

$$G^{-1}(x) = \Theta\left((G(0) - x)^{\frac{1}{\gamma}}\right),$$

as $x \to G(0)$. Combining this with (14) yields

$$D^*(P) - 1 = \Omega\left(\left(G(0) - \frac{P}{\sigma^2 \mathbb{E}_A(S(A/N))}\right)^{\frac{\gamma+1}{\gamma}}\right). \tag{15}$$

Finally, using that $P^*(1) = G(0)\sigma^2 \mathbb{E}_A(S(A/N))$, we have

$$G(0) - \frac{P}{\sigma^2 \mathbb{E}_A(S(A/N))} = \left(\frac{P^*(1) - P}{P^*(1)}\right) G(0).$$

Substituting this into (15), the desired bound follows. ■

APPENDIX IV

Proof of Proposition 2: Consider a given channel threshold policy μ_k. Under this policy, each time-slot can be classified as either a feasible time-slot (if $H > h_k$) or a non-feasible time-slot (if $H \leq h_k$).[4] Let T_m denote the number of time-slots between the $(m-1)$th and mth feasible slot, i.e. if the $(m-1)$th feasible time-slot occurred at time n, then the mth feasible time-slot is at time $n + T_m$. Let for $n = 1, 2, \ldots,$ let M_n denote the number of feasible time-slots that have occurred up to and including time n. Then $\{M_n\}$ is a renewal process and $\{T_m\}$ is the sequence of inter-renewal times. For a channel threshold policy the inter-renewal times will be geometrically distributed with $\mathbb{E}(T_n) = \frac{1}{1-q_k}$, which is finite for all $q_k > 0$. We next calculate the average delay and average power for such a policy using renewal-reward theory [23].

To calculate the average delay, define a reward R_m to be the sum of the buffer occupancy between the $(m-1)$th and mth renewals, i.e. if the $(m-1)$th renewal occurred at time n, then

$$\begin{aligned} R_m &= \sum_{l=n+1}^{n+T_m} S_l \\ &= (T_m)A_{n+1} + (T_m - 1)A_{n+2} + \cdots A_{n+T_m}. \end{aligned}$$

From renewal-reward theory it follows that the average buffer occupancy under policy μ_k is given by $\bar{S}^{\mu_k} = \frac{\mathbb{E}R_m}{\mathbb{E}T_m}$, where

$$\begin{aligned} \mathbb{E}R_k &= \mathbb{E}_{T_m}(\mathbb{E}_{A^{T_m}}(T_m(A_1) + +(T_m - 1)A_2 + \cdots A_{T_m}|T_m) \\ &= \mathbb{E}_{T_m}\left(\frac{\bar{A}}{2}(T_m)(T_m+1)\right) \\ &= \frac{\bar{A}}{(1-q_k)^2}. \end{aligned}$$

Therefore, $\bar{S}^{\mu_k} = \frac{\bar{A}}{(1-q_k)}$ and so by Little's law, $\bar{D}^{\mu_k} = \frac{1}{1-q_k}$ or equivalently $\bar{D}^{\mu_k} - 1 = \frac{q_k}{1-q_k}$, as desired.

[4]Note a slot may be feasible and still result in no transmissions if the queue is empty.

Next we calculate the average power. First note that if $\sum_{n=1}^{\infty} q_k^{n-1}(1-q_k)\Sigma_n = \infty$, the desired bound is trivially true. Therefore, we assume that $\sum_{n=1}^{\infty} q_k^{n-1}(1-q_k)\Sigma_n < \infty$ in the following. In this case, we show that the given bound is met with equality. Now define a reward P_m to be the power used at the end of the mth renewal period, so that $\bar{P}^{\mu_k} = \frac{\mathbb{E}P_m}{\mathbb{E}T_m}$. Using that the arrival process and channel gain are independent we have

$$\begin{aligned}
\mathbb{E}P_m &= \mathbb{E}_{T_m}(\mathbb{E}(P_m|T_m)) \\
&= \mathbb{E}_{T_m}\left(\mathbb{E}_{H,A^{T_m}}\left(P(H, A_1 + \cdots + A_{T_m})\middle| H > h_k, T_m\right)\right) \\
&= \mathbb{E}_{T_m}\left(\mathbb{E}_{H,A^{T_m}}\left(\frac{\sigma^2}{H} S\left(\frac{1}{N} A_1 + \cdots + A_{T_m}\right)\middle| H > h_k, T_m\right)\right) \\
&= \sigma^2 \frac{G(h_k)}{1-q_k} \mathbb{E}_{T_m}(\Sigma_{T_m}) \\
&= \sigma^2 G(h_k) \sum_{n=1}^{\infty} q_k^{n-1} \Sigma_n.
\end{aligned}$$

By assumption this is finite, so we can apply the renewal-reward theorem. Dividing by $\mathbb{E}\, T_m = \frac{1}{1-q_k}$ the desired expression follows. ■

Appendix V

Proof of Corollary 3: Let $\{\mu_k\}$ be a decreasing sequence of channel threshold policies, and let $S(r)$ be exponentially bounded by $M\kappa^r$. From Proposition 2, the average delay for each policy μ_k satisfies $\bar{D}^{\mu_k} - 1 = \frac{F_H(h_k)}{1-q_k}$, and from lemma 1, for a type A channel, $F_H(h_k) = \Theta(h_K)$ as $h_k \to 0$. Therefore, there exists constants $M_d > 0$ and $K_1 > 0$ such that for all $k > K_1$,

$$\bar{D}^{\mu_k} - 1 \leq M_d h_k. \tag{16}$$

From Proposition 2, the average power of policy μ_k can be bounded by

$$\bar{P}^{\mu_k} \leq \sigma^2 G(h_k) \left(\sum_{n=1}^{\infty} q_k^{n-1}(1-q_k)\Sigma_n \right). \tag{17}$$

The final term in this bound satisfies

$$\sum_{n=1}^{\infty} q_k^{n-1}(1-q_k)\Sigma_n = (1-q_k)\left(\Sigma_1 + \frac{1}{q_k}\sum_{n=2}^{\infty} q_k^n \Sigma_n\right)$$
$$\leq (1-q_k)\left(\Sigma_1 + \frac{M}{q_k}\sum_{n=2}^{\infty} (q_k\tilde{\kappa})^n\right),$$

where $\tilde{\kappa} = \kappa^{a_{max}/N}$. Here we have used the bound in (12) for each Σ_n, $n \geq 2$. As $k \to \infty$, it can be shown that the right-hand side of this bound converges to Σ_1. It follows that for any $\tilde{\Sigma} > \Sigma_1 = \mathbb{E}_A S(A/N)$, there exists a K_2 such that for all $k > K_2$,

$$\sum_{n=1}^{\infty} q_k^{n-1}(1-q_k)\Sigma_n \leq \tilde{\Sigma}.$$

Also, from Lemma 2, for all $k > K_2$,

$$G(h_k) \leq \tilde{M}_1 \ln\left(\frac{1}{h_k}\right) + \tilde{M}_2,$$

where $\tilde{M}_1 = \sup\{f_H(h) | h \leq h_k\} \geq f_H(0)$. Using these in (17), we have that for $k > K_2$,

$$\bar{P}^{\mu_k} \leq \sigma^2 \tilde{S}\left(\tilde{M}_1 \ln\left(\tfrac{1}{h_k}\right) + \tilde{M}_2\right),$$

or equivalently,

$$h_k \leq M_h \exp\left(-\alpha \bar{P}^{\mu_k}\right),$$

where $M_h = \exp\left(\frac{\tilde{M}_2}{\tilde{M}_1}\right)$ and $\alpha = (\sigma^2 \tilde{M}_1 \tilde{S}_1)^{-1}$. Combining this with (16), we have that for $k \geq \max\{K_1, K_2\}$,

$$\bar{D}^{\mu_k} - 1 \leq M_d M_h \exp\left(-\alpha \bar{P}^{\mu_k}\right).$$

Hence, $\bar{D}^{\mu_k} - 1 = O\left(\exp\left(-\alpha \bar{P}^{\mu_k}\right)\right)$, where $\alpha < (\sigma^2 f_H(0)\mathbb{E}_A S(A/N))^{-1}$ and can be made arbitrarily close by choosing K_2 large enough. ■

APPENDIX VI

Proof of Corollary 4: Let $\{\mu_k\}$ be a decreasing sequence of channel threshold policies, and let $S(r)$ be exponentially bounded by $M\kappa^r$. Once again, from Proposition 2 and Lemma 1, $\bar{D}^{\mu_k} - 1 = \frac{F_H(h_k)}{1-q_k} = \Theta(h_k^{\gamma+1})$ as $k \to \infty$. Therefore, there exists positive constants M_d and K_1 such that for all $k > K_1$,

$$\bar{D}^{\mu_k} - 1 \leq M_d h_k^{\gamma+1}. \tag{18}$$

Also, using Proposition 2,

$$\begin{aligned} P^*(1) - \bar{P}^{\mu_k} &\geq \sigma^2\Sigma_1 G(0) - \sigma^2 G(h_k)\left(\sum_{n=1}^{\infty} q_k^{n-1}(1-q_k)\Sigma_n\right) \\ &= \sigma^2 S_1(G(0) - G(h_k)) + q_k\sigma^2\Sigma_1 G(h_k) - \sigma^2 G(h_k)\left(\sum_{n=2}^{\infty} q_k^{n-1}(1-q_k)\Sigma_n\right). \end{aligned}$$

As in the proof of Corollary 2, let $\bar{G}(h) = G(0) - G(h)$ so that

$$\frac{P^*(1) - \bar{P}^{\mu_k}}{\bar{G}(h_k)} \geq \sigma^2\Sigma_1 + \frac{q_k\sigma^2 S_1 G(h_k)}{\bar{G}(h_k)} - \frac{\sigma^2 G(h_k)\left(\sum_{n=2}^{\infty} q_k^{n-1}(1-q_k)\Sigma_n\right)}{\bar{G}(h_k)}. \tag{19}$$

As shown in the proof of Corollary 2, for a type B channel $\bar{G}(h_k) = \Theta(h_k^{\gamma})$ as $h_k \to 0$. Using this and that $q_k = \Theta(h_k^{\gamma+1})$, it follows that as $k \to \infty$, the right-hand side of (19) converges to $\sigma^2\Sigma_1$. Here, as in the proof of Corollary 3, we have used that the SNR function is exponentially bounded to bound the last term in (19). Therefore, there exists positive constants, M_p and K_2 such that for all $k > K_2$, $P^*(1) - \bar{P}^{\mu_k} \geq M_2(h_k)^{\gamma}$, or equivalently,

$$h_k \leq \left(M_p P^*(1) - \bar{P}^{\mu_k}\right)^{\frac{1}{\gamma}}.$$

Combining this with (18), we have that for all $k \geq \max\{K_1, K_2\}$,

$$\bar{D}^{\mu_k} - 1 \leq M_d M_p^{\frac{1}{\gamma}} \left(P^*(1) - \bar{P}^{\mu_k}\right)^{\frac{\gamma+1}{\gamma}},$$

which implies that $\bar{D}^{\mu_k} - 1 = O\left(\left(P^*(1) - \bar{P}^{\mu_k}\right)^{\frac{\gamma+1}{\gamma}}\right)$, as desired. ■

Appendix VII

Proof of Proposition 3: Let $\{\phi_k\}$ be a decreasing sequence of bounded rate channel threshold policies with a fixed parameter $\delta > 0$.

The average power of such a policy is given by

$$\bar{P}^{\phi_k} = \mathbb{E}_A \left\{ \mathbb{E}_H \left(\sigma^2 \frac{1}{H} S\left(\frac{A+\delta}{N}\right) \middle| H > h_k, A \right) \right\}$$
$$= \sigma^2 \frac{G(h_k)}{1-q_k} \mathbb{E}_A \left\{ S\left(\frac{A+\delta}{N}\right) \right\},$$

where $q_k = \Pr(H \leq h_k)$. By a similar argument as in the proof of Proposition 1, it can be shown that as $\bar{P}^{\phi_k}$ increases,

$$h_k = O\left(e^{-\alpha \bar{P}^{\phi_k}}\right), \tag{20}$$

where $\alpha = \left(\sigma^2 \tilde{K}_1 \mathbb{E}_A \left\{ S\left(\frac{A+\delta}{N}\right)\right\}\right)^{-1}$ and $\tilde{K}_1$ is the constant from Lemma 2. This satisfies $\alpha < \left(\sigma^2 f_H(0) \mathbb{E}_A \left\{ S\left(A/N\right)\right\}\right)^{-1}$ and can be made arbitrarily close by choosing δ small enough.

Next we bound the average delay using Lemma 3. Applying Little's law to the upper bound from this lemma, we have

$$\bar{D}^{\phi_k} - 1 \leq \frac{\sigma_\Delta^2}{2(\mathbb{E}\Delta)\bar{A}}. \tag{21}$$

Evaluating (21) for policy ϕ_k yields,

$$\bar{D}^{\phi_k} - 1 \leq \frac{(1-q_k)q_k(\delta^2 + 2\delta\bar{A}) + q_k\bar{A}^2 - q_k^2(\bar{A})^2}{2((1-q_k)\delta - (q_k)\bar{A})(\bar{A})}.$$

From this it can be seen that $\bar{D}^{\phi_k} - 1 = O(q_k)$ as $q_k \to 0$. Also, from Lemma 1, $q_k = \Theta(h_k)$. Hence, combining these with (20), we have $D^{\phi_k} - 1 = O\left(\exp(-\alpha \bar{P}^{\phi_k})\right)$. ■

Appendix VIII

Proof of Corollary 5: Let $\{\tilde{\phi}_k\}$ be a decreasing sequence of fixed rate, channel threshold policies with parameter $\tilde{a} > a_{max}$.

Following the same argument as in the proof of Proposition 3, we have

$$\bar{P}^{\tilde{\phi}_k} = \sigma^2 \frac{G(h_k)}{1-q_k} S\left(\tilde{a}/N\right),$$

and so,

$$h_k = O\left(e^{-\alpha \bar{P}^{\tilde{\phi}_k}}\right), \tag{22}$$

where $\alpha < \left(\sigma^2 f_H(0) S(\tilde{a}/N)\right)^{-1}$ and can be made arbitrarily close.

To bound the average delay under policy $\tilde{\phi}_k$, let ϕ_k be a related bounded rate channel threshold policy with the same threshold h_k and with parameter $\delta = \tilde{a} - a_{max}$. Consider a fixed sample path of arrivals and channel states $\{a_n, h_h\}$. Then for all n, $\tilde{\phi}_k(h_n) \geq \phi_k(h_n, a_n)$. From this it follows that the buffer occupancy under the fixed rate policy is always less than or equal to the buffer occupancy under the related bounded rate policy. Therefore, we have

$$\bar{D}^{\tilde{\phi}_k} - 1 \leq \bar{D}^{\phi_k} - 1 = O(q_k),$$

and so $\bar{D}^{\tilde{\phi}_k} - 1 = O(\exp(-\alpha \bar{P}^{\tilde{\phi}_k}))$ as desired. ■

APPENDIX IX

Proof of Proposition 4: Let $\{\phi_k\}$ be a decreasing sequence of bounded rate channel threshold policies with decreasing parameters δ_k, and $\lim_{k\to\infty} \delta_k = 0$. Then,

$$\begin{aligned}
P^*(1) - \bar{P}^{\phi_k} &= \sigma^2 \mathbb{E}_H\left(\frac{1}{H}\right) \mathbb{E}_A S(A/N) - \sigma^2 G(h_k) \mathbb{E}_A S\left(\frac{A+\delta_k}{N}\right) \\
&= \sigma^2 \bar{G}(h_k) \mathbb{E}_A S\left(\frac{A+\delta_k}{N}\right) - \sigma^2 \mathbb{E}_H\left(\frac{1}{H}\right)\left[\mathbb{E}_A S\left(\frac{A+\delta_k}{N}\right) - \mathbb{E}_A S\left(AN\right)\right] \\
&= \sigma^2 \bar{G}(h_k) \mathbb{E}_A S\left(\frac{A+\delta_k}{N}\right) - \sigma^2 \mathbb{E}\left(\frac{1}{H}\right) \mathbb{E}_A S(A/N)\left(2^{\delta_k/N} - 1\right).
\end{aligned}$$

In the last step we have used that by assumption $S(r) = 2^r - 1$.

From this it follows that a necessary condition for $\bar{P}^{\phi_k}$ to approach $P^*(1)$ from below is that as $k \to \infty$,

$$\left(2^{\delta_k/N} - 1\right) = O(\bar{G}(h_k)). \tag{23}$$

In other words δ_k must decrease fast enough, or the resulting sequence of policies will require too much power. Note that $\left(2^{\delta_k/N}-1\right)=\Theta(\delta_k)$ as $k\to\infty$, and for a type B channel, $\bar{G}(h_k)=\Theta\left((q_k)^{\frac{\gamma}{\gamma+1}}\right)$. Therefore, (23) is equivalent to

$$\delta_k = O\left((q_k)^{\frac{\gamma}{\gamma+1}}\right). \tag{24}$$

For any sequence of policies, that satisfy (23), as $k\to\infty$,

$$\begin{aligned} P^*(1)-\bar{P}^{\phi_k} &= O\left(\bar{G}(h_k)\right) \\ &= O\left(\bar{G}\left(F_H^{-1}(q_k)\right)\right) \\ &= O\left((q_k)^{\frac{\gamma}{1+\gamma}}\right), \end{aligned}$$

where in the last step we used Lemma 1 for a type B channel. Equivalently,

$$q_k = \Omega\left(\left(P^*(1)-\bar{P}^{\phi_k}\right)^{\frac{1+\gamma}{\gamma}}\right). \tag{25}$$

Next we lower bound the average buffer delay under such a policy. For this we use the lower bound from Lemma 3 and Little's law, which yields

$$\bar{D}^{\mu_k}-1 \geq \frac{\mathbb{E}(A_n^2)q_k}{2\left[(1-q_k)\delta_k - q_k\bar{A}\right]\bar{A}}.$$

If q_k and δ_k satisfy (24), then it follows that as $k\to\infty$,

$$\bar{D}^{\mu_k}-1 = \Omega\left((q_k)^{\frac{1}{\gamma+1}}\right).$$

Finally, combining this with (25), we have

$$\bar{D}^{\mu_k}-1 = \Omega\left(\left(P^*(1)-\bar{P}^{\phi_k}\right)^{\frac{1}{\gamma}}\right),$$

as desired. ■

APPENDIX X

Proof of Proposition 5: First we prove the upper bound $\bar{D}^{\nu_k}-1=O\left(\frac{1}{\log\bar{P}_k}\right)$. Under a fixed power policy, U_n is a function only of H_n. Hence, $\{U_n-A_n\}$ is an i.i.d. sequence and so Lemma 3 applies. Using the upper bound from this lemma, along with Little's law gives us that

$$\bar{D}^{\nu_k} - 1 \leq \frac{\sigma_A^2 + \sigma_{\nu_k(H)}^2}{2\bar{A}\left(\mathbb{E}_H\{\nu_k(H)\} - \bar{A}\right)},$$

where σ_A^2 and $\sigma_{\nu_k(H)}^2$ are respectively the variances of A and $\nu_k(H)$. Here we have used that U_n and A_n are independent under a fixed power policy.

Under a fixed power policy, using (13), the expected transmission rate is given by

$$\mathbb{E}_H\{\nu_k(H)\} = \mathbb{E}_H\left\{N \log\left(1 + \frac{H\bar{P}_k}{\sigma^2}\right)\right\},$$

which is increasing with $\bar{P}_k$ at a rate of $\Theta(\log(\bar{P}_k))$.

Next we consider the variance, $\sigma_{\nu_k(H)}^2$. It can be shown that as $\bar{P}_k$ increases, the variance is increasing. However, asymptotically the variance is bounded, as stated in the following lemma.

Lemma 4: Under any fixed power policy, ν_k,

$$\sigma_{\nu_k(H)}^2 \leq \frac{N}{2}\sigma_{\log(H)}^2,$$

where $\sigma_{\log(H)}^2$ is the variance of the random variable $\log(H)$.

To see this note that

$$\sigma_{\nu_k(H)}^2 = \frac{1}{2}\mathbb{E}_{H,\tilde{H}}\left(\nu_k(H) - \nu_k(\tilde{H})\right),$$

where $\tilde{H}$ is another random variable, independent of H and identically distributed. Substituting the expression for $\nu_k(h)$ from (13) yields

$$\sigma_{\nu_k(H)}^2 = \frac{1}{2}\mathbb{E}_{H,\tilde{H}}\left\{N \log\left(\frac{1 + \frac{H\bar{P}_k}{\sigma^2}}{1 + \frac{\tilde{H}\bar{P}_k}{\sigma^2}}\right)\right\}.$$

As $\bar{P}_k \to \infty$ this converges to

$$\frac{N}{2}\mathbb{E}_{H,\tilde{H}}\left\{\log\left(\frac{H}{\tilde{H}}\right)\right\} = \tfrac{N}{2}\sigma_{\log(H)}^2,$$

as desired. Also, it can be shown that for a type A channel $\sigma_{\log(H)}^2 < \infty$.

Continuing with the proof of Proposition 5, it follows that

$$\bar{D}^{\nu_k} - 1 \leq \frac{M_f}{\mathbb{E}_H\left\{\nu_k(H)\right\} - \bar{A}},$$

where $M_f = \frac{1}{2\bar{A}}(\sigma_a^2 + \frac{N}{2}\sigma_{\log(H)}^2)$ is a constant depending on the arrival and channel statistics but not on k.

Finally, since $\mathbb{E}_H\left\{\nu_k(H)\right\}$ increases at rate $\Theta(\log \bar{P}_k)$, the desired upper bound of $O\left((\log \bar{P}_k)^{-1}\right)$ follows.

Next we consider the lower bound for the convergence rate of a fixed power policy. This follows from using the lower bound from Lemma 3. Using this lemma, the average delay under the policy ν_k is lower bounded by

$$D^{\nu_k} - 1 \geq \frac{\mathbb{E}_{H,A}\left\{((A - \nu_k(H))^+)^2\right\}}{2\bar{A}\left(\mathbb{E}_H\{\nu_k(H)\} - \bar{A}\right)}.$$

Once again the denominator will increase at rate $\Theta(\log(\bar{P}_k))$. To bound the numerator, we use Markov's inequality which states that for any $\epsilon > 0$,

$$\mathbb{E}_{H,A}\left\{((A - \nu_k(H))^+)^2\right\} \geq \Pr(A - \nu_k(H) > \epsilon)\epsilon^2.$$

Using (13),

$$\begin{aligned}\Pr(A - \nu_k(H) \geq \epsilon) &= \Pr\left(H \leq \frac{\sigma^2(2^{(A-\epsilon)/N} - 1)}{\bar{P}_k}\right) \\ &= \int_{a_{min}}^{a_{max}} \Pr\left(H \leq \frac{\sigma^2(2^{(a-\epsilon)/N} - 1)}{\bar{P}_k}\right) dF_A(a).\end{aligned}$$

Choosing some $\epsilon < a_{max}/2$, this quantity can be bounded by

$$\begin{aligned}\Pr(A - \nu_k(H) \geq \epsilon) &\geq \int_{2\epsilon}^{a_{max}} \Pr\left(H \leq \frac{\sigma^2(2^{(a-\epsilon)/N} - 1)}{\bar{P}_k}\right) dF_A(a) \\ &\geq Pr\left(H \leq \frac{\sigma^2(2^{\epsilon/N} - 1)}{\bar{P}_k}\right) \Pr(A \geq 2\epsilon) \\ &= F_H\left(\frac{\sigma^2(2^{\epsilon/N} - 1)}{\bar{P}_k}\right) \Pr(A \geq 2\epsilon).\end{aligned}$$

From Lemma 1, it follows that for a type A channel,

$$\Pr(A - \nu_k(H) \geq \epsilon) = \Theta\left(\frac{1}{\bar{P}_k}\right)$$

as $k \to \infty$. Finally, combining these results we have that $D^{\mu_k} - 1 = \Omega\left((\bar{P}_k \log \bar{P}_k)^{-1}\right)$ as desired. ■

APPENDIX XI

Proof (sketch) of Proposition 6: This proof follows a similar argument as the proof of Proposition 5, and so we only give a sketch of the main argument. Let $\{\omega_k\}$ be a sequence of fixed water-filing policies, such that as $k \to \infty$, the water-level ℓ_k (and hence the average power) increases to infinity. First, using similar arguments as in the proof of Lemma 2, it can be shown that under a fixed water-filling policy, the average power $\bar{P}^{\omega_k}$, increases like $\Theta(\ell_k)$ as $k \to \infty$.

As in the proof of Proposition 5, we use the bounds from Lemma 3 to bound the average power. In this case, it can be shown that the average transmission rate, $\mathbb{E}_H(\omega_k(H))$ increases like $\Theta(\ell_k)$ as $k \to \infty$. As in the proof of Proposition 5, it can again be shown that the the variance, $\sigma^2_{\omega_k(H)}$ is bounded. Finally to derive the lower bound, we again bound $\mathbb{E}_{A,H}((A - \omega_k(H))^+)^2$ using Markov's inequality and then show that for ϵ small enough $\Pr(A - \omega_k(H) \geq \epsilon) = \Theta\left(\frac{1}{\ell_k}\right)$. Combining these the desired bounds follow. ■

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