

Medium Access Control via Nearest-Neighbor Interactions for Regular Wireless Networks

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Abstract—This paper studies medium access control (MAC) protocols for regular wireless networks, where only nearest-neighbor interactions are involved. Each station chooses a state in the current time slot, which determines whether it transmits or not, based on its own state and the states of all its nearest neighbors in the previous time slot. The dynamics of the network follow that of a Markov Chain of Markov Fields, which is shown to converge to a stationary distribution for certain types of interactions. It is found that this type of protocols can achieve the optimal one-hop broadcast throughput in regular wireless networks. In case each station can only distinguish between transmitting and idle neighbors, the interactions of the network can be described using the *Ising model* in statistical mechanics. For this case, a MAC protocol is designed that can achieve a throughput close to the optimum.

I. INTRODUCTION

The performance of wireless networks is limited by interference between stations, which is primarily a short-range effect among neighboring stations. Most existing medium access control (MAC) protocols such as ALOHA and carrier-sense multiple access (CSMA) exploit this phenomenon by using local interactions among stations to schedule at most one station in a neighborhood. In this paper, we study MAC protocols that only involve such nearest-neighbor interactions. Specifically, we consider time-slotted MAC protocols such that in each time slot, each station makes its transmission decision based on a local state, which is determined by its own state and that of its neighbors in the previous time slot. This model includes protocols such as ALOHA, in which a station's state is simply whether it transmits or not.

For simplicity, we consider *regular* wireless networks in which a station is placed at each point in a lattice and only neighboring stations cause significant interference to each other. Specifically, we consider the square and triangular networks shown in Fig. 1. The square, triangular and hexagonal lattices are the only two-dimensional lattices consisting of regular polygons. Note that the triangular lattice is the most representative, *i.e.*, it represents the closest possible packing of the stations in a wireless network. Hexagonal networks are not considered in this paper, although the techniques developed here also apply.

The performance of the MAC protocols we study depends on whether the interactions are symmetric or not. By symmetric interactions, we mean that the effect of state information

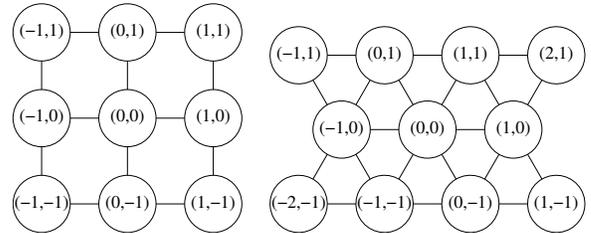


Fig. 1. Square and triangular networks.

does not depend on which neighbor it is originated from, while asymmetric interactions lack this property. If the interactions are asymmetric, stations have to distinguish the orientation of the incoming state information, therefore directional antenna or some other form of location information is required. Otherwise, omnidirectional antenna and orientation-oblivious stations suffice. This work focuses on asymmetric interactions.

We consider the *broadcast throughput* of a MAC protocol, given by the average number of successfully received packets per station per time slot. It is shown that the maximum broadcast throughput of $4/5$ can be achieved in a regular square network, if each station can choose from one of five states and inform all neighbors of its choice. Similarly, the maximum broadcast throughput of $6/7$ can be achieved in a regular triangular network, if the number of states is seven. However, in some wireless networks, it may not be easy for a station to obtain state information from its neighbors beyond whether they are transmitting or not. Therefore, it may be necessary to assume each station can only be in one of the two states: ‘transmit’ or ‘idle.’ In this aspect, it has been recognized in [1] and [2] that the resulting interactions among stations are similar to that found in the *Ising model* in statistical mechanics [3], [4]. Using the Ising model, we design a MAC protocol that can achieve a throughput of $3/4$ in square networks.

The remainder of the paper is organized as follows. Section II introduces the system model. In Section III we derive the maximum broadcast throughput for square and triangular networks; we construct an algorithm to achieve these results via nearest-neighbor interactions in Section IV. In Section V we use the Ising model to design a MAC protocol for square networks, which achieves a throughput close to the optimum. Section VI concludes the paper.

II. SYSTEM MODEL

Consider networks in which stations populate a finite square or triangular lattice, denoted by S . To simplify the analysis, a periodic boundary condition is assumed, *i.e.*, the network is wrapped around so that stations on the left and right boundaries are neighbors, as are stations on the top and the bottom. Consequently, there is no edge effect. In Section IV, simulation results show that the periodic boundary assumption has little effect on the performance. Let stations be labeled by their coordinates, *i.e.*, $\mathbf{r} = (i, j)$. Let $N_{\mathbf{r}}$ denote the set of neighbors of station \mathbf{r} . In square networks, $N_{(i,j)} = \{(i-1, j), (i+1, j), (i, j-1), (i, j+1)\}$, and in triangular networks,¹ $N_{(i,j)} = \{(i-1, j), (i+1, j), (i, j-1), (i, j+1), (i-1, j-1), (i+1, j+1)\}$. For boundary stations, these definitions are modified to account for the periodic boundary condition. Each station broadcasts packets to all its nearest neighbors. It is assumed that every station is saturated, so queuing is not considered.

The state of each station assumes values in some finite set E , where E may depend on the underlying network. Let $\Omega = E^S$ denote the configuration space, *i.e.*, the state space of the entire network. Let $X_{\mathbf{r}}(t) \in E$ be the state of station \mathbf{r} at time t , and $\mathbf{X}(t) = \{X_{\mathbf{r}}(t)\}_{\mathbf{r} \in S} \in \Omega$ be the configuration, or the state of the network, at time t .

We assume that at the end of each time slot, each station acquires the current states of its nearest neighbors.² Stations then choose their current states based only on the previous states of their neighbors and themselves. Precisely, we are interested in MAC protocols for which $\mathbf{X} = \{\mathbf{X}(t)\}_{t \in \mathbb{N}}$ is a *Markov Chain of Markov Fields* (MCMF) [5], *i.e.*, a process for which

- $\mathbf{X}(1), \mathbf{X}(2), \dots$ is a Markov chain on Ω , and
- for every t , $\mathbf{X}(t)$ is a Markov field³ on Ω conditioned on $\mathbf{X}(t-1)$.

Furthermore, we restrict our attention to protocols in which all stations make identically distributed decisions conditioned on the same previous states of their neighbors and themselves.⁴

III. THE MAXIMUM BROADCAST THROUGHPUT

The performance measure we use is the one-hop broadcast throughput, which can be understood as the proportion of stations that can receive a packet in each time slot. A station receives a packet in a given time slot if and only if it does not transmit and only one of its neighbors transmits. The maximum broadcast throughput for regular networks are characterized as follows.

Theorem 1. *The maximum one-hop broadcast throughput, ρ_{BC} , of a regular network is $k/(k+1)$, where k is the number*

¹In a triangular network, the geographic location of station (i, j) on \mathbb{R}^2 is $(i - j/2, \sqrt{3}j/2)$ in a rectangular coordinate system.

²A protocol would be required for exchanging such information. The design of such a protocol is beyond the scope of this paper.

³In our case, $\mathbf{X}(t)$ is simply a set of independent random variables conditioned on $\mathbf{X}(t-1)$.

⁴This assumption rules out protocols in which stations, for example, are simply assigned to transmit or not based on their location [6].

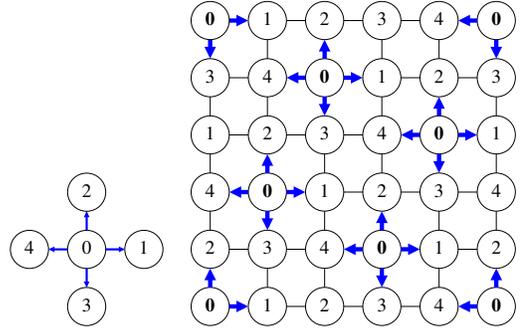


Fig. 2. Optimal pattern and its tessellation in square networks.

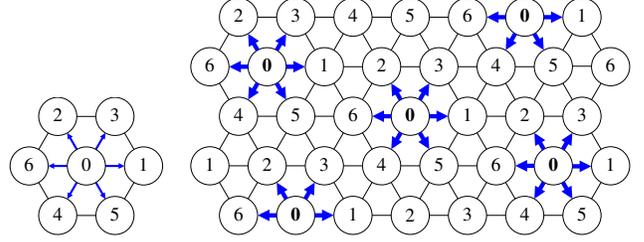


Fig. 3. Optimal pattern and its tessellation in triangular networks.

of neighbors a station has, *i.e.*, $k = 4$ and 6 for square and triangular networks, respectively.

Proof: The converse part of the theorem is first established. Let p be the fraction of stations transmitting over any given slot. Since a station cannot transmit and receive simultaneously, $\rho_{\text{BC}} \leq 1 - p$. Also, a transmission results in at most k successful receptions, therefore, $\rho_{\text{BC}} \leq kp$. Hence,

$$\rho_{\text{BC}} \leq \min\{1 - p, kp\}.$$

The maximum of the right-hand-side over all p satisfies $1 - p = kp$, which implies that the instantaneous throughput of any slot is upper bounded by $k/(k+1)$, and so is ρ_{BC} .

We next describe a transmission schedule which achieves the maximum throughput. For square networks, consider the pattern shown in Fig. 2, where 0 means transmitting, 1, 2, 3 and 4 mean receiving a packet from the left, down, up and right transmitters respectively. This pattern tessellates the plane as shown in Fig. 2 and achieves a throughput of $4/5$. For triangular networks, the required pattern is shown in Fig. 3, where 0 means transmitting, 1, 2, 3, 4, 5 and 6 mean receiving a packet from the left, lower-right, lower-left, upper-right, upper-left and right transmitters respectively. This pattern tessellates the plane as shown in Fig. 3, and a throughput of $6/7$ can be achieved. ■

IV. ACHIEVING THE MAXIMUM BROADCAST THROUGHPUT VIA NEAREST-NEIGHBOR INTERACTIONS

In this section, a distributed algorithm is developed, which allows the patterns shown in Figs. 2 and 3 to emerge and shift periodically, *starting from arbitrary initial configuration, using only nearest-neighbor interactions.*

A. Square Networks

Choose $E = \{0, 1, \dots, l-1\}$ with $l = 5$, and consider the following dynamics or protocol. Let \oplus and \ominus denote addition and subtraction modulo l respectively. At time t , station \mathbf{r} collects state information $\{X_{\mathbf{r}'}(t)\}_{\mathbf{r}' \in N_{\mathbf{r}}}$ and selects its state at time $t+1$ from the multi-set

$$\mathcal{V}_{(i,j)}(t) = [X_{(i-a,j-b)}(t) \oplus h \oplus ad_1 \oplus bd_2 : (a,b) \in N_{(0,0)} \cup \{(0,0)\}] \quad (1)$$

where $N_{(0,0)}$ is the set of neighbors of station $(0,0)$. Each state $s \in \mathcal{V}_{\mathbf{r}}(t)$ is selected with a probability proportional to $f(n_s)$, where n_s is the number of occurrences of s in $\mathcal{V}_{\mathbf{r}}(t)$ and $f: \mathbb{N} \mapsto \mathbb{R}$ is some increasing function with $f(0) = 0$. The parameters l, h, d_1, d_2 represent the size of the smallest repeating unit, the direction of pattern shift, and the difference in the states of neighboring stations along the horizontal and vertical directions, respectively. The multi-set $\mathcal{V}_{\mathbf{r}}(t)$ can be interpreted as a collection of votes on a station's next state from all of its neighbors and itself. The more votes on a given state, the higher probability the state will be chosen. No transition is made to any state that receives zero vote because $f(0) = 0$. The interactions are asymmetric, since $\mathcal{V}_{\mathbf{r}}(t)$ depends on the orientation of the state information. An example of f is $f(n_s) = e^{n_s} \mathbf{1}_{\{n_s > 0\}}$, where $\mathbf{1}_{\{n_s > 0\}}$ is 1 if $n_s > 0$ and 0 otherwise, which results in the Boltzmann distribution on $\mathcal{V}_{\mathbf{r}}(t)$. We next show that the throughput of the system converges to the maximum under these dynamics.

Theorem 2. *For a finite square network with periodic boundary condition, under the preceding Markov dynamics with $\gcd(h, l) = 1$, \mathbf{X} has the unique stationary distribution*

$$\pi(\mathbf{x}) = \begin{cases} l^{-1}, & \text{if } \mathbf{x} = \mathbf{x}^{(k)} \text{ for } k = 0, \dots, l-1 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

where the l recurrent configurations $\mathbf{x}^{(k)} = \{x_{(i,j)}^{(k)}\}, k = 0, \dots, l-1$, have the following properties:

- i) $x_{(i,j)}^{(k)} = x_{(i-1,j)}^{(k)} \oplus d_1$,
- ii) $x_{(i,j)}^{(k)} = x_{(i,j-1)}^{(k)} \oplus d_2$, and
- iii) $x_{(i,j)}^{(k)} = x_{(i,j)}^{(k \oplus 1)} \oplus h$,

and all other configurations are transient.

Proof: First note that there will be exactly l configurations that satisfy properties i) and ii) in the theorem. Moreover, these configurations can always be ordered so that property iii) is satisfied. Given such an ordered list of configurations, we next show that this set must be recurrent. Suppose the previous configuration is $\mathbf{x}^{(k)}$. Observe that $x_{(i-1,j)}^{(k)} \oplus h \oplus d_1, x_{(i,j-1)}^{(k)} \oplus h \oplus d_2, x_{(i,j)}^{(k)} \oplus h, x_{(i,j+1)}^{(k)} \oplus h \ominus d_2, x_{(i+1,j)}^{(k)} \oplus h \ominus d_1$ are all equal by properties i) and ii). Hence, the current state of station (i,j) will be $x_{(i,j)}^{(k)} \oplus h$ with probability 1, and from property iii) the current configuration is $\mathbf{x}^{(k \oplus 1)}$. Therefore, $\mathbf{x}^{(0)}, \dots, \mathbf{x}^{(l-1)}$ form a cycle in the configuration space, and so are absorbing.

To see that the remaining configurations are transient, select any of them to be the initial configuration. Fix a station \mathbf{r}

and let its state be s . Under the given Markov dynamics, station \mathbf{r} will vote for itself to transition to state $s \oplus h$ and for each of its neighbors to transition to the state that is locally consistent with $s \oplus h$ under one of the recurrent configurations. Hence, in one step of the Markov dynamics there is a positive probability that all stations in $N_{\mathbf{r}} \cup \{\mathbf{r}\}$ transition to such a local configuration. Likewise, in the next transition, there will be a positive probability that all stations in $N_{\mathbf{r}} \cup \{\mathbf{r}\}$ remains locally consistent with a recurrent configuration and the neighborhood of each station in $N_{\mathbf{r}}$ is also consistent with the same configuration. Continuing it follows that with sufficiently many steps there will be a positive probability that the entire network transitions to one of the recurrent configurations. Since the l recurrent configurations are absorbing, there is a nonzero probability that the initial configuration will never be visited again and so it must be transient.

From symmetry it then follows that the stationary distribution for these dynamics is given by (2). ■

To prevent starvation and avoid collisions, we need $\gcd(h, l) = \gcd(d_1, l) = \gcd(d_2, l) = \gcd(d_1 \pm d_2, l) = 1$. For the pattern in Fig. 2, we have $d_1 = 1, d_2 = 2$. The protocol allows the pattern to emerge via nearest-neighbor interactions, and the pattern can be shifted in any direction by choosing h , where $h = 1$ shifts the pattern to the left, $h = 2$ shifts it down, $h = 3$ shifts it up and $h = 4$ shifts it to the right.

B. Triangular Networks

For a triangular lattice, similar dynamics can be defined by using a state space with $l = 7$ states. At time t , station \mathbf{r} selects its state at time $t+1$ from the multi-set $\mathcal{V}_{\mathbf{r}}(t)$ given by (1), where $N_{(0,0)}$ is now the set of neighbors of station $(0,0)$ in a triangular network. The $l = 7$ recurrent configurations also satisfy

$$\text{iv) } x_{(i,j)}^{(k)} = x_{(i-1,j-1)}^{(k)} \oplus d_1 \oplus d_2$$

in addition to properties i), ii) and iii) in Theorem 2. Following similar arguments as in the proof of Theorem 2, these are the only recurrent configurations, and the stationary distribution resulting from these dynamics is also given by (2).

To prevent starvation and avoid collisions, we need $\gcd(h, l) = \gcd(d_1, l) = \gcd(d_2, l) = \gcd(d_1 \pm d_2, l) = \gcd(d_1 + 2d_2, l) = \gcd(2d_1 + d_2, l) = 1$. For the pattern in Fig. 3, we have $d_1 = 1, d_2 = 2$. The pattern will emerge from the local interaction protocol and can be shifted in any direction by choosing h , e.g., the pattern shifts to the left if $h = 1$, whereas it shifts toward the lower-right corner if $h = 2$.

C. Simulations

The upper part of Fig. 4 shows a simulation result of the protocol for square networks of size 20×20 and 100×100 . The function f defined in the protocol is $f(n_s) = e^{n_s} \mathbf{1}_{\{n_s > 0\}}$. The throughput increases at the beginning, and then it stays very close to the optimum of $4/5$ for the remainder of the simulation. The throughput stays at the optimum once the network reaches one of the recurrent configurations and begins to cycle through all recurrent configurations. The time it takes

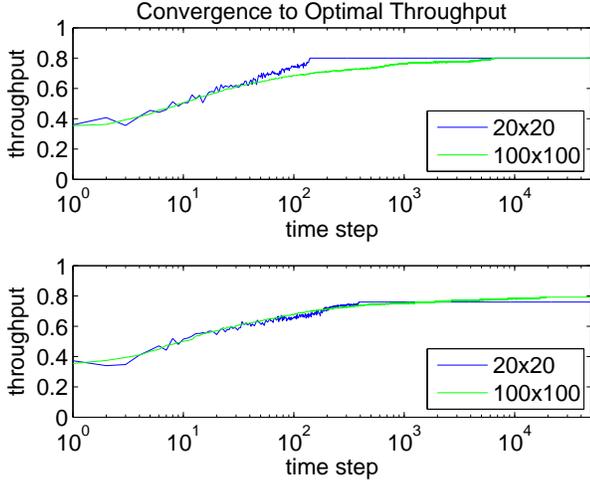


Fig. 4. Simulation of the 5-state protocol for square networks with (upper) and without (lower) periodic boundary condition.

for the protocol to reach the optimal throughput increases with the network size.

Note that without the periodic boundary condition, boundary stations have fewer neighbors. The protocol achieves a throughput slightly smaller than that in Theorem 1 [7], as is shown in the lower part of Fig. 4. This is due to the fewer transmissions by the boundary stations, *e.g.*, a station on the left boundary cannot transmit a packet to its ‘neighbor’ on the right boundary.

V. A MAC PROTOCOL BASED ON THE ISING MODEL

In the above protocol, we assume there are multiple idle states. It would be difficult for stations to distinguish the states of their idle neighbors unless we allow idle stations to explicitly notify their neighbors. In this section we discuss a protocol for square networks based on the Ising model, in which a station only needs to know whether its neighbors transmitted or not in the previous time slot.

The Ising model [3], [4] was originally proposed to model a spin system in stationarity. Each spin can be in ‘up’ or ‘down’ state, which is affected by the external magnetic field acting on it and its interactions with neighboring spins. We can use the Ising model to model regular wireless networks, such that the spins represent the stations, and $E = \{+1, -1\}$, where $+1$ means transmitting or ‘up’, and -1 means idle or ‘down’ [2].

Let $\mathbf{x} = \{x_{\mathbf{r}}\}_{\mathbf{r} \in S}$ and $\mathbf{y} = \{y_{\mathbf{r}}\}_{\mathbf{r} \in S}$ be the previous and current configurations, respectively. Consider the dynamics in which each station \mathbf{r} observes the previous states of stations in $N_{\mathbf{r}}$ and chooses its current state according to

$$p_{\mathbf{r}}(y_{\mathbf{r}}|\mathbf{x}) \propto \exp\left(\sum_{\mathbf{r}' \in N_{\mathbf{r}}} J_{\mathbf{r}'-\mathbf{r}} x_{\mathbf{r}'} y_{\mathbf{r}}\right) \quad (3)$$

where $J_{\mathbf{r}'-\mathbf{r}}$ denotes the interaction from station \mathbf{r}' to station \mathbf{r} . To simplify the notation, we will use J_l, J_r, J_d, J_u to denote

the interactions from the left, right, down and up neighbors, respectively.

If we choose $J_l \gg 0, J_r \ll 0, J_u \gg 0, J_d \ll 0$, each station tends to choose the same state as its left and up neighbors, and the opposite state as its right and down neighbors. It can be shown that with these interactions, the pattern in the upper-left of Table I is stationary, except that it shifts to the lower-right corner. More generally, the pattern in the upper-left of Table I is formed (up to shifts) by choosing

- $J_l \gg 0, J_r \ll 0, J_u \gg 0, J_d \ll 0$, in which the pattern shifts toward the lower-right corner, or
- $J_l \ll 0, J_r \gg 0, J_u \ll 0, J_d \gg 0$, in which the pattern shifts toward the upper-left corner,

while the pattern in the upper-middle of Table I is formed (up to shifts) by choosing

- $J_l \ll 0, J_r \gg 0, J_u \gg 0, J_d \ll 0$, in which the pattern shifts toward the lower-left corner, or
- $J_l \gg 0, J_r \ll 0, J_u \ll 0, J_d \gg 0$, in which the pattern shifts toward the upper-right corner.

The MAC protocol we introduce here is based on *two* Ising models. The state of station \mathbf{r} is a two-dimensional vector $(x_{\mathbf{r},1}, x_{\mathbf{r},2})$ where $\{x_{\mathbf{r},1}\}_{\mathbf{r} \in S}$ form the first Ising model, while $\{x_{\mathbf{r},2}\}_{\mathbf{r} \in S}$ form the second Ising model. That is, the protocol consists of two independent copies of the dynamics described by (3). In the first Ising model, we choose $J_{l,1} \gg 0, J_{r,1} \ll 0, J_{u,1} \gg 0, J_{d,1} \ll 0$ so that it forms the stationary pattern in the upper-left of Table I, which shifts toward the lower-right corner, and in the second Ising model, we choose $J_{l,2} \ll 0, J_{r,2} \gg 0, J_{u,2} \gg 0, J_{d,2} \ll 0$ so that it forms the stationary pattern in the upper-middle of Table I, which shifts toward the lower-left corner. In each time slot, station \mathbf{r} transmits when $x_{\mathbf{r},1} = x_{\mathbf{r},2} = +1$ and remains idle otherwise. One possible resulting activity pattern is shown in the upper-right of Table I. Each station either is a transmitter or receives one packet from a neighboring transmitter without collision, and a throughput of $3/4$ is achieved. This activity pattern shifts downward, which is the vector addition of the direction of shifts of the patterns formed in the two Ising models. More generally, the activity pattern shifts

- downward if $J_{l,1}, J_{u,1} \gg 0, J_{r,2}, J_{u,2} \gg 0, J_{r,1}, J_{d,1} \ll 0$, and $J_{l,2}, J_{d,2} \ll 0$,
- upward if $J_{r,1}, J_{d,1} \gg 0, J_{l,2}, J_{d,2} \gg 0, J_{l,1}, J_{u,1} \ll 0$, and $J_{r,2}, J_{u,2} \ll 0$,
- to the right if $J_{l,1}, J_{u,1} \gg 0, J_{l,2}, J_{d,2} \gg 0, J_{r,1}, J_{d,1} \ll 0$ and $J_{r,2}, J_{u,2} \ll 0$,
- to the left if $J_{r,1}, J_{d,1} \gg 0, J_{r,2}, J_{u,2} \gg 0, J_{l,1}, J_{u,1} \ll 0$ and $J_{l,2}, J_{d,2} \ll 0$.

If we shift the pattern formed in the second Ising model in Table I one position to the left, the resultant activity pattern would be almost the same as the one in the upper-right of Table I, except that it is rotated by 90 degrees. The activity pattern still shifts downward, but in this case the stations in every other column are starved. In Table I, every station can be a transmitter, so the problem of starvation does not occur. But since each station always transmits at the same time with

TABLE I
ACTIVITY PATTERN IN SQUARE NETWORKS RESULTING FROM USING TWO ISING MODELS.

| time | first Ising model | | | | | second Ising model | | | | | activity pattern | | | | | | | |
|---------|-------------------|---|---|---|---|--------------------|---|---|---|---|------------------|---|---|---|---|---|---|---|
| t | - | - | + | + | - | - | - | - | + | + | - | - | - | - | + | + | - | - |
| | - | + | + | - | - | + | + | + | - | - | + | + | - | - | - | - | - | |
| | + | + | - | - | + | + | + | + | + | - | - | + | + | + | - | - | + | + |
| | + | - | - | + | + | - | - | - | + | + | - | - | + | - | - | - | - | - |
| | - | - | + | + | - | - | - | - | - | + | + | - | - | - | + | + | - | - |
| | - | + | + | - | - | + | + | + | - | - | + | + | - | - | - | - | - | - |
| $t + 1$ | + | - | - | + | + | - | - | - | + | + | - | - | + | - | - | - | - | - |
| | - | - | + | + | - | - | - | - | - | + | + | - | - | - | + | + | - | - |
| | - | + | + | - | - | + | + | + | - | - | + | + | - | - | - | - | - | - |
| | + | + | - | - | + | + | + | + | + | - | - | + | + | + | - | - | + | + |
| | + | - | - | + | + | - | - | - | + | + | - | - | + | - | - | - | - | - |
| | - | - | + | + | - | - | - | - | - | + | + | - | - | - | + | + | - | - |

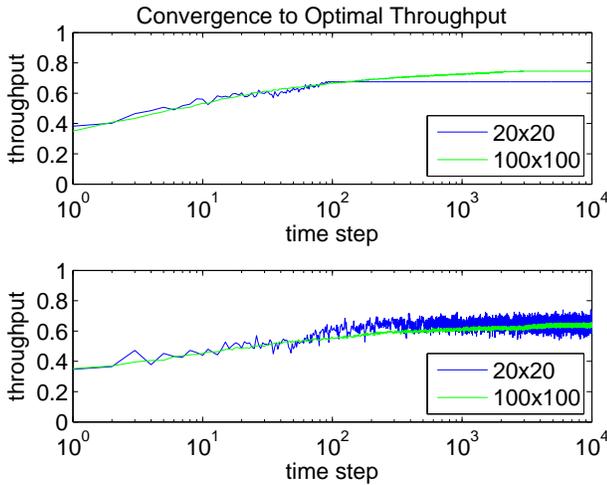


Fig. 5. Simulation of the protocol using two Ising models for square networks with strong (upper) and weak (lower) interactions.

either its left or right neighbor, each station has exactly one neighbor that it cannot deliver a packet to. Also, there is no control on which activity pattern to occur.

Finally, for state information exchange between neighbors, we can let each time slot be preceded by two mini-slots. Station r transmits in the k -th mini-slot if $x_{r,k} = +1$, for $k = 1, 2$.

The upper part of Fig. 5 illustrates a simulation result of the evolution of the throughput of this protocol. In this example, the interaction potentials are chosen such that $J_{l,1} = J_{u,1} = J_{r,2} = J_{u,2} = 5$ and $J_{r,1} = J_{d,1} = J_{l,2} = J_{d,2} = -5$. Similar to the protocol in Section III, the throughput increases at the beginning. But in this case, the throughput stays close to $3/4$ for an extremely long time. The reason is that at the time the throughput is close to $3/4$, the network can be partitioned into large clusters such that the activity pattern in Table I appears within each cluster but not on the boundary between the clusters. When this happens, it is difficult to ‘merge’ the clusters so that the pattern appears in the entire network. Comparing with the size of the entire network, the boundary of the clusters only occupies a small area, hence the throughput

is close to $3/4$.

The lower part of Fig. 5 shows the same simulation, but with weak interactions. All interaction potentials are reduced to one-tenth of those in the upper part of Fig. 5. In this case, the throughput fluctuates below $3/4$ instead of staying constant. The range of fluctuation is larger if the interactions are weaker.

VI. CONCLUSION

In this paper, we have studied MAC protocols in regular wireless networks in which stations choose to transmit or not based on their own previous states and those of their neighbors. We have shown that, if neighboring stations are able to exchange a few bits of state information, then a simple protocol can be used to achieve the optimal one-hop broadcast throughput. In case a station can only discern transmitting or idle states of its neighbors, it is still possible to obtain a throughput close to the optimum.

The interactions considered in this paper are asymmetric, which requires directional antennas or additional information exchange so that each station has orientation of the state information it receives. If the interactions are required to be symmetric, whether it is possible to achieve the maximum throughput via nearest-neighbor interactions is open and left as future work. The protocol developed here can also be extended to unicast traffic [7].

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