Information Theoretic Games on Interference Channels

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***Abstract*—We provide a natural formulation of information theoretic games on interference channels. We analyze this game on a class of deterministic interference channels recently introduced to approximate Gaussian channels in the interference-limited regime. Our main result is a complete and simple characterization of the subset of the interference channel capacity region that can be achieved as Nash equilibria. We show that for all parameter values of the interference channel, there are always Nash equilibria which are efficient, i.e. on the boundary of the capacity region.**

I. Introduction

Interference is a central phenomenon in both wireless and wireline communication. The canonical information theoretic model for studying this phenomenon is the two-user Gaussian interference channel, where two point-to-point communication links interfere with each other through cross-talk. Each transmitter has an independent message intended only for the corresponding receiver. The capacity region of this channel is the set of all simultaneously achievable rate pairs (R_1, R_2) in the two interfering links. This characterizes the fundamental tradeoff between the performance achievable on the two links in the face of interference.

Though there has been an extensive literature on this channel, its capacity region is still unknown. Recently there has been some progress in this direction. In [1], it is shown that a very simple version of a scheme due to Han and Kobayashi [2] results in an achievable region that is within one bit of the capacity region for all values of channel parameters. This result is particularly relevant in the high SNR regime, where the noise is small and the achievable rates are high. Furthermore, it is shown in [3] that the high SNR behavior of the two-user Gaussian interference channel is in fact captured by a *deterministic* interference channel, for which the capacity region can be computed exactly. (This type of deterministic model was first proposed in [4] for Gaussian relay networks.)

Unlike the classic strategy of treating interference as Gaussian noise, information theoretic optimal or near-optimal strategies require coordination between the two users. For example, the Han-Kobayashi scheme requires the users to split their information into two streams, a common stream and a private stream. The common stream is encoded so that it can be decoded at the other user's receiver and so reduce the interference seen by that user. A natural question is: would selfish users, interested only in maximizing their own rate, have an incentive to implement such a strategy? We study such a case, where each user individually chooses an encoding/decoding scheme in order to maximize his own transmission rate. The two users can then be viewed as playing a non-cooperative game. We want to determine the set of Nash equilibria (NE) of this game and compare the performance at these equilibria to the (cooperative) capacity region. Clearly, the rates at any NE has to be in the capacity region, but the question is how many of the points in the capacity region are Nash equilibria. Our focus is on a "one-shot" game model in which each player has full information, i.e. both players know all of the channel gains, and the actions chosen by each player, as well as their pay-off function.

Other game theoretic approaches for the Gaussian interference channel have been studied before [5], [6]. However, there are two key assumptions in these works: 1) the class of encoding strategies are constrained to use random Gaussian codebooks; 2) the decoders are restricted to treat the interference as Gaussian noise and are hence sub-optimal. Because of these restrictions, the formulation in these works are not information-theoretic in nature.

In this paper, we make two contributions. First, we give an information theoretic formulation of games on general interference channels, where the users are allowed to use *any* encoding and decoding strategies. Second, we take an intermediate step toward the goal of solving this game for the Gaussian interference channel, by analyzing the corresponding problem on the two-user deterministic interference channel from [3]. Our main result is a simple characterization of the set of all rates achievable as NE inside the deterministic channel's capacity region. We also provide explicit coding schemes that achieve each rate pair as a NE. Somewhat surprisingly, we find that in all cases, there are always Nash equilibria that are *efficient*, i.e. they lie on the maximum sum-rate boundary of the capacity region. In particular, for channels with symmetrical channel gains, the symmetric rate point on the capacity region boundary is always a NE.

II. Problem Formulation

Let us now formally define the communication situation for general interference channels. Communication starts at time 0. User i communicates by coding over blocks of length N_i symbols, $i = 1, 2$. Transmitter i sends on block

k information bits $b_{i1}^{(k)}, \ldots, b_{i,L_i}^{(k)}$ by transmitting a codeword denoted by $\mathbf{x}_i^{(k)} = [\mathbf{x}_i^{(k)}(1), \ldots, \mathbf{x}_i^{(k)}(N_i)]$. All the information bits are equally probable and independent of each other. Receiver i observes on each block k an output sequence $\mathbf{y}_i^{(k)} = [\mathbf{y}_i^{(k)}(1), \ldots, \mathbf{y}_i^{(k)}(N_i)]$ through the interference channel, which specifies a stochastic mapping from the input sequences of user 1 and 2 to the output sequences of user 1 and 2. Given the observed sequences up to block k, $\{\mathbf{y}_i^{(m)}\}_{m=1}^k$, receiver i generates a guess $\hat{b}_{i\ell}^{(k)}$ for each information bit. Without loss of generality, we assume that this is done via maximum-likelihood decoding on each bit.

Note that this communication scenario is more general than the one usually used in multiuser information theory, as we allow the two users to code over different block lengths. Such generality is necessary here, since even though the two users may agree *a priori* on a common block length, a selfish user may unilaterally decide to choose a different block length during the actual communication process.

A *strategy* s_i of user i is defined by its message encoding, which we assume to be the same on every block and involves:

- the number of information bits L_i and the block length N_i of the codewords,
- the codebook $\mathcal{C}_i$ employed by transmitter i,
- the encoder $f_i : \{1, \ldots, 2^{L_i}\} \times \Omega_i \to \mathcal{C}_i$, that maps on each block k the message $m_i^{(k)} := (b_{i1}^{(k)}, \ldots b_{i,L_i}^{(k)})$ to a transmitted codeword $\mathbf{x}_i^{(k)} = f_i(m_i^{(k)}, \omega_i^{(k)}) \in \mathcal{C}_i$,
- the rate of the code, $R_i(s_i) = L_i/N_i$.

A strategy s_1 of user 1 and s_2 of user 2 jointly determines the probabilities of error $p_i^{(k)} := \frac{1}{L}\sum_{\ell=1}^{L_i} \mathcal{P}(\hat{b}_{i\ell}^{(k)} \neq b_{i\ell}^{(k)})$, $i = 1, 2$. Note that if the two users use different block lengths, the error probability could vary from block to block even though each user uses the same encoding for all the blocks.

The encoder of each transmitter i may employ a stochastic mapping from the message to the transmitted codeword; $\omega_i^{(k)} \in \Omega_i$ represents the randomness in that mapping. We assume that this randomness is independent between the two transmitters and across different blocks and is only known at the respective transmitter and not at any of the receivers.

For a given error probability threshold $\epsilon > 0$, we define an ϵ-interference channel game as follows. Each user i chooses a strategy s_i, $i = 1, 2$, and receives a pay-off of $\pi_i(s_1, s_2) = R(s_i)$ if $p_i^{(k)}(s_1, s_2) \leq \epsilon$, for all k; otherwise, $\pi_i(s_1, s_2) = 0$. In other words, a user's pay-off is equal to the rate of the code provided that the probability of error is no greater than ϵ. A strategy pair (s_1, s_2) is defined to be $(1-\epsilon)$-*reliable* provided that they result in an error probability $p_i^k(s_1, s_2)$ of less than ϵ for $i = 1, 2$ and all k.

For an ϵ-game, a strategy pair (s_1^*, s_2^*) is a *Nash equilibrium* (NE) if neither user can unilaterally deviate and improve their pay-off, i.e. if for each user $i = 1, 2$, there is no other strategy s_i such that[1] $\pi_i(s_i, s_j^*) > \pi_i(s_i^*, s_j^*)$. If user i attempts to transmit at a higher rate than what he is receiving in a NE and user j does not change her strategy, then user i's error probability must be greater than ϵ. Similarly, a strategy pair (s_1^*, s_2^*) is an η-*Nash equilibrium*[2] (η-NE) of an ϵ-game if neither user can unilaterally deviate and improve their pay-off by more than η, i.e. if for each user i, there is no other strategy s_i such that $\pi_i(s_i, s_j^*) > \pi_i(s_i^*, s_j^*) + \eta$. Note that when a user deviates, it does not care about the reliability of the other user but only its own reliability. So in the above definitions (s_i, s_j^*) is not necessarily $(1-\epsilon)$-reliable.

Given any $\bar{\epsilon} > 0$, the capacity region $\mathcal{C}$ of the interference channel is the closure of the set of all rate pairs (R_1, R_2) such that for every $\epsilon \in (0, \bar{\epsilon})$, there exists a $(1-\epsilon)$-reliable strategy pair (s_1, s_2) that achieves the rate pair (R_1, R_2). The *Nash equilibrium region* $\mathcal{C}_{\text{NE}}$ of the interference channel is the closure of the set of rate pairs (R_1, R_2) such that for every $\eta > 0$, there exists a $\bar{\epsilon} > 0$ (dependent on η) so that if $\epsilon \in (0, \bar{\epsilon})$, there exists a $(1-\epsilon)$-reliable strategy pair (s_1, s_2) that achieves the rate-pair (R_1, R_2) and is a η-NE. Clearly, $\mathcal{C}_{\text{NE}} \subset \mathcal{C}$. The rest of the paper is devoted to deriving $\mathcal{C}_{\text{NE}}$.

First, we make a few comments about the definition of $\mathcal{C}_{\text{NE}}$. The parameter $\bar{\epsilon}$ is introduced so that $(1-\epsilon)$-reliable strategy pairs need only exist for "small enough" values of ϵ. In the definition of $\mathcal{C}$ this is not needed, i.e. the region is equally well defined by requiring the given conditions to hold for any $\epsilon > 0$ (since, clearly if a pair of strategies are $(1-\epsilon)$-reliable, they are also $(1-\tilde{\epsilon})$-reliable for all $\tilde{\epsilon} > \epsilon$). However, when defining $\mathcal{C}_{\text{NE}}$, this condition is important. In particular a pair of strategies can be an η-NE for an ϵ-game, but not an η-NE for an $\tilde{\epsilon}$-game for $\tilde{\epsilon} > \epsilon$, since increasing ϵ enlarges the set of possible deviations an agent may make.

Next, we turn to our use of η-NE. A more natural approach would be to define $\mathcal{C}_{\text{NE}}$ to be the closure of the rate pairs (R_1, R_2) such that for any ϵ small enough, that there exists a $(1-\epsilon)$-reliable strategy pair (s_1, s_2) which achieves the rate-pair (R_1, R_2) and is a *NE* of a ϵ-game. The difficulty with this is that to determine such a NE requires one to find a particular scheme that achieves the optimal rate for a given error probability. Finding such a scheme is extremely difficult and in general an open problem.[3] By introducing the slack η, these difficulties are removed.

Finally, we comment on the use of different block lengths. It can argued that if there is a $(1-\epsilon)$-reliable strategy pair (s_1, s_2) that achieves a rate pair (R_1, R_2) using codes of block lengths N_1, N_2, then there exists a $(1-\epsilon)$ strategy pair that achieves the same rate pair but with each user using the same block length. This follows by considering using "super-blocks" of length N, where N is the least common multiple of N_1 and N_2. Over these super-blocks the users can be viewed as using two equal-length codes. The error probabilities, being the average bit error probabilities now across longer blocks, remain less than ϵ. This means that in computing the capacity region $\mathcal{C}$, we can without loss of generality assume both users

[1]We use the convention that j always denotes the other user from i.

[2]In the game theoretic literature, this is often referred to as an ϵ-Nash equilibrium or simply an ϵ-equilibrium for a game [7, page 143].

[3]Moreover, it is not even clear if there exists such a scheme, i.e. a scheme that achieves the supremum of the rates over all $1-\epsilon$ reliable schemes.

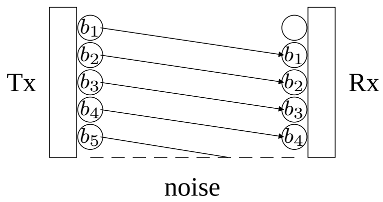

Fig. 1. The deterministic model for the point-to-point Gaussian channel. Each bit of the input occupies a signal level. Bits of lower significance are lost due to noise.

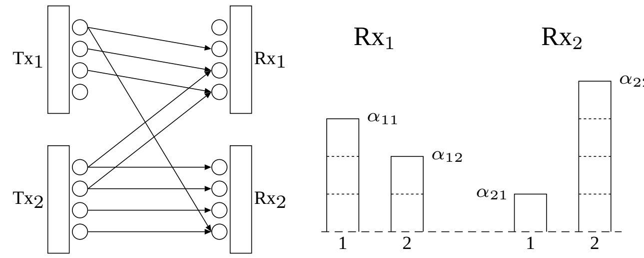

Fig. 2. At left is a deterministic interference channel. The more compact figure at right shows only the signals as observed at the receivers.

use the same block lengths. Likewise, in a η-NE, we can make this assumption (although each user is allowed to deviate using a strategy of a different block length).

III. Deterministic Channel Model

Let us now focus on a specific interference channel model: a deterministic channel model analogous to the Gaussian channel, introduced in [4]. We first describe the deterministic channel model for the point-to-point AWGN channel (see Figure 1). The real-valued channel input is written in base 2; the signal—a vector of bits—is interpreted as occupying a succession of levels: $x = 0.b_1 b_2 b_3 b_4 b_5 \ldots$. The most significant bit coincides with the highest level, the least significant bit with the lowest level. The levels attempt to capture the notion of *signal scale*; a level corresponds to a unit of power in the Gaussian channel, measured on the dB scale. Noise is modeled by truncation. Bits of smaller order than the noise are lost. Note that the number of bits above the noise floor correspond to $\log_2 \mathsf{SNR}$, where SNR is the signal-to-noise ratio of the corresponding Gaussian channel.

We proceed with the deterministic interference channel model (Fig. 2). There are two transmitter-receiver pairs (links), and as in the Gaussian case, each transmitter wants to communicate only with its corresponding receiver. The signal from transmitter i, as observed at receiver j, is scaled by a nonnegative integer gain $a_{ji} = 2^{n_{ji}}$ (equivalently, the input column vector is shifted up by n_{ji}). At each time t, the input and output, respectively, at link i are $\mathbf{x}_i(t), \mathbf{y}_i(t) \in \{0,1\}^q$, where $q = \max_{ij} n_{ij}$. Note that n_{ii} corresponds to $\log_2 \mathsf{SNR}_i$ and n_{ji} corresponds to $\log_2 \mathsf{INR}_{ji}$, where SNR_i is the signal-to-noise ratio of link i and INR_{ji} is the interference-to-noise ratio at receiver j from transmitter i in the corresponding Gaussian interference channel. To model the super-position of signals at each receiver, the bits received on each level are added *modulo two*. The channel output at receiver i is then given by

$$\mathbf{y}_i(t) = \mathbf{S}^{q-n_{i1}}\mathbf{x}_1(t) + \mathbf{S}^{q-n_{i2}}\mathbf{x}_2(t), \tag{1}$$

where summation and multiplication are in the binary field and $\mathbf{S}$ is a $q \times q$ shift matrix (e.g. see [4]).

In our analysis, it will be helpful to consult a different style of figure, as shown on the right-hand side of Fig. 2. This shows only the perspective of each receiver. Each incoming signal is shown as a column vector, with the highest element corresponding to the most significant bit and the portion below the noise floor truncated. The observed signal at each receiver is the modulo 2 sum of the elements on each level.

IV. Main Results

To begin, we give the capacity region, $\mathcal{C}$, of our two-user deterministic interference channel. This region is given by Theorem 1 in [8], which applies to a larger class of deterministic interference channels. For our model, the resulting region becomes the set of non-negative rates satisfying:[4]

$$R_i \le n_{ii}, \ i = 1,2 \tag{2}$$
$$R_1 + R_2 \le (n_{11} - n_{21})^+ + \max(n_{22}, n_{21}) \tag{3}$$
$$R_1 + R_2 \le (n_{22} - n_{12})^+ + \max(n_{11}, n_{12}) \tag{4}$$
$$R_1 + R_2 \le \max(n_{12}, (n_{11} - n_{21})^+) + \max(n_{21}, (n_{22} - n_{12})^+) \tag{5}$$
$$2R_1 + R_2 \le \max(n_{11}, n_{12}) + (n_{11} - n_{21})^+ + \max(n_{21}, (n_{22} - n_{12})^+) \tag{6}$$
$$R_1 + 2R_2 \le \max(n_{22}, n_{21}) + (n_{22} - n_{12})^+ + \max(n_{12}, (n_{11} - n_{21})^+). \tag{7}$$

Our main result, stated in Theorem 1 below is to completely characterize $\mathcal{C}_{\mathrm{NE}}$ for the two-user deterministic interference channel model. This characterization is in terms of $\mathcal{C}$ and a "box" $\mathcal{B}$ in $\mathbb{R}_+^2$ given by

$$\mathcal{B} = \{(R_1, R_2) : L_i \le R_i \le U_i, \forall i = 1,2\},$$

where for each user $i = 1,2$, $L_i = (n_{ii} - n_{ij})^+$, and

$$U_i = \begin{cases} n_{ii} - \min(L_j, n_{ij}), & \text{if } n_{ij} \le n_{ii}, \\ \min((n_{ij} - L_j)^+, n_{ii}), & \text{if } n_{ij} > n_{ii}. \end{cases} \tag{8}$$

Theorem 1: $\mathcal{C}_{NE} = \mathcal{C} \cap \mathcal{B}$. Moreover, $\mathcal{C}_{NE}$ always contains at least one efficient point and is either equal to $\mathcal{B}$ or is the intersection of $\mathcal{B}$ with the simplex corresponding to the sum-rate constraint for $\mathcal{C}$.

First let us interpret the bounds L_1, L_2, U_1, U_2. The number L_i is the number of levels at receiver i that never see any interference from user j. These are always the most significant bits of user i's signal. In the example in Fig. 2,

[4]The boundaries of the region in [8] is given in terms of conditional entropies that must be maximized over any product distribution on the channel inputs. For our model the optimizing input distribution for each bound is always uniform over the input alphabet. The given bounds follow.

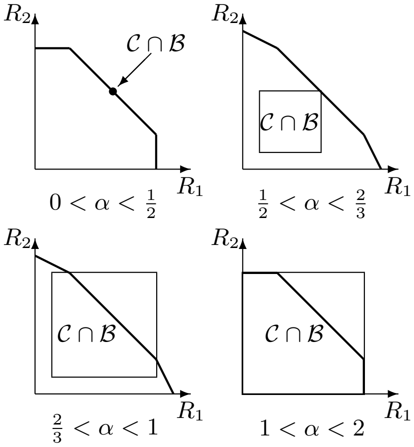

Fig. 3. Examples of $\mathcal{C}_{\text{NE}} = \mathcal{C} \cap \mathcal{B}$ for a symmetric interference channel with normalized cross gain α.

these correspond to the top level for transmitter 1 ($L_1 = 1$) and the top 3 levels for transmitter 2 ($L_2 = 3$). The number U_i is the number of levels at receiver i that receive signals from transmitter i but are free of interference from the top L_j levels from transmitter j. In Fig. 2, these correspond to the top level at receiver 1 ($U_1 = 1$) and the top three levels at receiver 2 ($U_2 = 3$).

Intuitively, it is clear that at any η-NE, user i should have rate at least L_i: these levels are interference-free and user i can always send information at the maximum rate on these levels. This will create interference of maximum entropy at a certain subset of levels at receiver j and render them un-usable for user j. The rate for user j is bounded by the number of remaining levels that it can use. This is precisely the upper bound U_j. What Theorem 1 says is that any rate pair in the capacity region $\mathcal{C}$ subject to these natural constraints is in $\mathcal{C}_{\text{NE}}$.

To illustrate this result, consider a symmetric interference channel in which $n_{11} = n_{22}$ and $n_{12} = n_{21}$. Let $\alpha = n_{ji}/n_{ii}$ be the normalized cross gain. Four examples of $\mathcal{C}$ and $\mathcal{B}$ corresponding to different ranges of α are shown in Fig. 3. For $0 < \alpha < \frac{1}{2}$, $\mathcal{C}_{\text{NE}} = \mathcal{B}$ is a single point, which lies at the symmetric sum-rate point of $\mathcal{C}$. For $\frac{1}{2} < \alpha < \frac{2}{3}$, again $\mathcal{C}_{\text{NE}} = \mathcal{B}$. $\mathcal{C}_{\text{NE}}$ contains a single efficient point (the symmetric sum-rate point in $\mathcal{C}$), but now there are additional interior points of $\mathcal{C}$ which may be achieved as a Nash equilibrium.[5] For $\frac{2}{3} < \alpha < 1$, $\mathcal{C}_{\text{NE}}$ is a the intersection of the simplex formed by the sum-rate constraint of $\mathcal{C}$ and $\mathcal{B}$. In this case, there are multiple efficient points; in fact, the entire sum-rate face of $\mathcal{C}$ is included in $\mathcal{C}_{\text{NE}}$. For $1 < \alpha < 2$, $\mathcal{C} \subset \mathcal{B}$ and so $\mathcal{C}_{\text{NE}} = \mathcal{C}$. For $2 \leq \alpha$ (not shown) $\mathcal{C} = \mathcal{B}$ and so again $\mathcal{C}_{\text{NE}} = \mathcal{C}$. Note that in all cases, the symmetric rate point is in $\mathcal{C}_{\text{NE}}$.

V. Analysis

To prove Theorem 1, we first show that points outside of $\mathcal{B}$ cannot be achievable as a NE. The intuition behind this result was discussed in the previous section. We then show that all points inside $\mathcal{C} \cap \mathcal{B}$ can be achieved as NE. The proof of this is based on first noting that since $\mathcal{C}$ and $\mathcal{B}$ are convex polytopes, their intersection must also be a convex polytope. We first explicitly construct strategies which show that the corner points of $\mathcal{C} \cap \mathcal{B}$ are in $\mathcal{C}_{\text{NE}}$. In these strategies each user either transmits only un-coded bits or uses a repetition code across levels (but no coding over time) and in fact achieves perfect reliability. Since $\mathcal{C} \cap \mathcal{B}$ is a convex polytope, any point can be expressed as a convex combination of the corner points. Using this and a time-sharing argument, we can then show that the remaining points in $\mathcal{C} \cap \mathcal{B}$ are also in $\mathcal{C}_{\text{NE}}$. The additional properties of $\mathcal{C}_{\text{NE}}$ given in Theorem 1 then directly follow. In the following, we focus on the first step in this argument, namely showing that the corner points of $\mathcal{C} \cap \mathcal{B}$ are in $\mathcal{C}_{\text{NE}}$.

A pair of strategies (s_1, s_2) are defined to be a *partial Bernoulli* pair if: (*i*) For $i = 1, 2$, user i's transmitted signal at each channel use contains $k_j \leq \min(n_{jj}, n_{ji})$ i.i.d. Bernoulli-1/2 bits which create interference for user j's received signal (the remaining bits of user i's signal can be arbitrary values possibly dependent on these k_j levels); and (*ii*) each user i transmits at rate $R_i = n_{ii} - k_i$ with zero probability of error. The first property in this definition states that each user j sees interference of maximum entropy on k_j levels. Intuitively, on these levels, each user can not reliably convey any information. This leaves the user with $n_{jj} - k_j$ levels. The second property states that each user is transmitting at maximum rate over these remaining levels; this requires the user to essentially transmit uncoded bits on these levels. The next lemma shows why these strategies are useful in characterizing $\mathcal{C}_{\text{NE}}$.

Lemma 1: If there exists a partial Bernoulli pair of strategies that achieves the rate pair (R_1, R_2) then $(R_1, R_2) \in \mathcal{C}_{\text{NE}}$.

The proof of this is based on using Fano's inequality to bound the pay-off a user can receive from deviating from a partial Bernouli pair of strategies. The next two lemmas show that three corners of $\mathcal{B}$ are always in $\mathcal{C}_{\text{NE}}$.

Lemma 2: The rate pairs (U_1, L_2), and (L_1, U_2) are always in $\mathcal{C}_{\text{NE}}$.

Proof: Without loss of generality, consider the pair (U_1, L_2). This can be achieved by the following strategy pair (see Fig. 4): (*i*) User 2 transmits uncoded information on its L_2 most significant levels and nothing on the remaining levels. (*ii*) User 1 transmits uncoded information on every level that is not interfered with by user 2's signal and transmits i.i.d. Bernouli-1/2 noise on all remaining levels that are received above the noise floor at *either* receiver. It can be seen that this is a partial Bernoulli pair and so $(U_1, L_2) \in \mathcal{C}_{\text{NE}}$. ∎

By a similar argument we have:

Lemma 3: The rate pair (L_1, L_2) is in $\mathcal{C}_{\text{NE}}$.

In general, the equilibrium strategies in Lemma 2 are not efficient, e.g. for both examples in Fig. 4, the maximum sum-rate obtainable in $\mathcal{C}$ is 6, which is not obtained by these strategies. This efficiency loss is due to the random noise or "junk" that user 1 is transmitting on the levels marked with a "J." The overall system performance can be improved by better utilizing these "junk levels." Since user 1 is already transmitting at the upper bound, it cannot improve its own

[5]In a slight abuse of terminology, we say that points in $\mathcal{C}_{\text{NE}}$ can be "achieved as a NE."

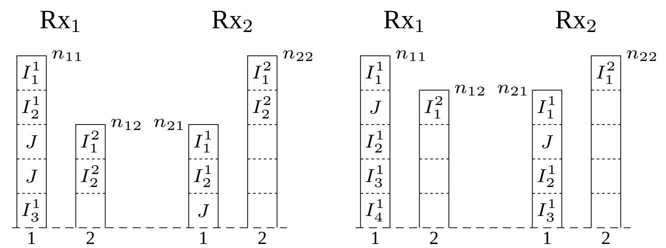

Fig. 4. Two examples of the equilibrium strategies used in Lemma 2. On the left is the pair of strategies achieving $(U_1, L_2) = (3, 2)$; on the right is the pair of strategies achieving $(U_1, L_2) = (4, 1)$. Here a level labeled with I_k^j indicates that the kth uncoded information bit of user j is sent on that level; a level labeled with a J indicates that the user sends noise (junk) on that level. On other levels a user does not transmit.

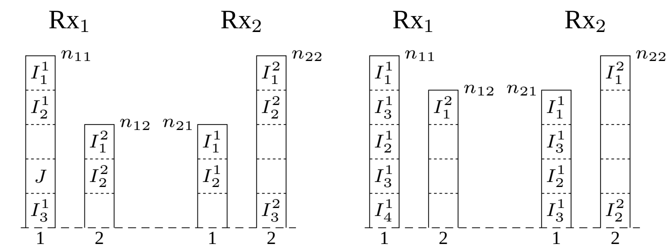

Fig. 5. Examples of the efficient strategies for the two interference channels from Fig. 4. Note that on the right user 1 transmits two copies of I_3^1.

rate (in a NE) by changing its strategy. However, if the rate pair (U_1, L_2) is not efficient, user 1 can change its strategy on only the junk levels so that there is a new NE in which user 2's rate is increased without decreasing user 1's rate. Moreover, the resulting rate pair will meet the sum-rate bound of $\mathcal{C}$ and so is efficient. This provides the remaining corners of $\mathcal{C} \cup \mathcal{B}$.

Let us see how this improvement can be done for the examples in Fig. 4. In the left example, user 1 has two junk levels that it can release to improve user 2's rate. The lower junk level appears below the noise floor at receiver 2 so what user 1 does there is immaterial. The upper junk level appears at the bottom level at receiver 2. If user 1 sends nothing instead of junk at that level, then user 2 can send 1 more bit of information by using the bottom level. This additional signal from user 2 is appearing below noise floor at receiver 1 and therefore will not deteriorate user 1's rate (see Fig. 5, left). Thus, we can now achieve $(3, 3)$, which is efficient. Moreover, the strategies remain a partial Bernoulli pair and therefore $(3, 3) \in \mathcal{C}_{\text{NE}}$

Now, consider the right example in Fig. 4, where user 1 has a single junk level. If user 1 turns off the junk level, user 2 can transmit on the third level and gain one extra bit. But unlike the previous example, this extra signal is harmful to user 1 since it causes interference at the receiver of user 1 at the second level from the bottom. The problem here is that the bottom level of user 2 is actually harmless to user 1, but it is currently being interfered with by an information bit from user 1 and so is not usable. But we can get around this problem by having user 1, instead of transmitting nothing on the junk level, send a copy of this interfering information bit (see Fig. 5, right). This way, user 2 can see that bit on level 3, subtract it from the received signal at the bottom level and free that level for transmitting its own bit. Again this will be a partial Bernoulli pair of strategies, and the resulting rates are efficient.

We can generalize this construction as follows. The L_i most significant levels of each user i are defined to be that user's *guaranteed levels*. Each user can always transmit uncoded bits on each of these levels regardless of the other user's strategy. Let user i's *open levels* be that user's non-guaranteed levels that (i) appear above the noise floor at user j's receiver *and* (ii) at user i's own receiver are either interfered with by user j's guaranteed levels or appear below the noise floor. These are levels where user i cannot send information for itself but may be useful for helping user j. In Fig. 4, the "junk" levels of user 1 correspond to his open levels. Finally, the *harmless* levels of user i are the non-guaranteed levels that do not create interference at receiver j. User i's signal on his harmless levels does not effect user j's performance. On the left-hand side of Fig. 4, each user's two least significant levels are harmless. For $i = 1, 2$, let O_i denote user i's open levels and H_i his harmless levels.

Lemma 4: The rate pairs $(U_1, L_2 + \min(O_1, H_2))$ and $(L_1 + \min(O_2, H_1), U_2)$ are always in $\mathcal{C}_{\text{NE}}$. Furthermore, these rate pairs always satisfy the sum-rate bound for $\mathcal{C}$ with equality.

ACKNOWLEDGMENT

This work has benefited from convergence on a common game-theoretic formulation for interference channels that also appears in [10].

REFERENCES

[1] R. Etkin, D. Tse, and H. Wang, "Gaussian Interference Channel Capacity to within One Bit," submitted to *IEEE Trans. on Info. Theory*, 2007.

[2] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. on Info. Theory*, vol. 27, pp. 49-60, Jan. 1981.

[3] G. Bresler and D. Tse, "The Two-User Gaussian Interference Channel: A Deterministic View," to appear.

[4] S. Avestimehr, S. Diggavi, and D. Tse, "Wireless Network Information Flow," in *Allerton Conference on Communication, Control, and Computing*, (Monticello, IL), September 2007.

[5] R. Etkin, A. P. Parekh and D.Tse, "Spectrum Sharing in Unlicensed Bands", *IEEE Journal on Selected Areas of Communication*, vol. 25, no. 3, pp. 517-528, April 2007.

[6] S. T. Chung, S. J. Kim, J. Lee, and J.M. Cioffi, "A game-theoretic approach to power allocation in frequency-selective Gaussian interference channels," *Proc. of IEEE ISIT*, pp. 316-316, June 2003.

[7] R. Myserson, *Game Theory: Analysis of conflict*, Harvard University Press, Cambridge MA, 1991.

[8] A. El Gamal and M. Costa, "The Capacity Region of a Class of Deterministic Interference Channels," *IEEE Transactions on Information Theory*, Vol. IT-28, No. 2, pp. 343-346, March 1982.

[9] R.G. Gallager, *Information Theory and Reliable Communication*, Wiley, 1968.

[10] R. Yates, D. Tse and Z. Li, "Secret Communication on Interference Channels", submitted to ISIT 2008.