Throughput Optimal Control of Cooperative Relay Networks

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Abstract

In cooperative relaying, packets are not forwarded by traditional hop-by-hop transmissions between pairs of nodes. Instead, several nodes cooperate with each other to forward a packet by, for example, forming a distributed antenna array. To date, such schemes have been primarily investigated at the physical layer with the focus on communication of a single end-to-end flow. In this paper, we consider cooperative relay networks with multiple stochastically varying end-to-end flows. The traffic from each flow is queued within the network until it can be forwarded. For such networks, we study network control policies that take into account queue dynamics to jointly optimize routing, scheduling and resource allocation. Specifically, we develop a *throughput optimal* policy, i.e., a policy that stabilizes the network for any arrival rate in its stability region. This policy is a generalization of the well-known *Maximum Differential Backlog* algorithms, which takes into account the cooperative gains in the network. Implementing this policy requires solving an optimization problem over the set of feasible transmission rates. We discuss several structural characteristics of this optimization problem for the special case of parallel relay cooperative networks.

I. Introduction

In recent years, there has been a growing body of work on "cross layer" control of wireless networks. In particular, given stochastically varying traffic demands, various *throughput optimal* control schemes have been developed that jointly address issues such as scheduling and physical-layer resource allocation (e.g. power control). This includes [1]–[8], which address various

This research was supported in part by NSF under grants CCR-0313329 and CCR-0313183, and by ARO under grant DAAD19-03-1-0229.

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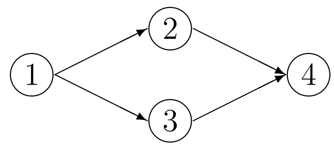

Fig. 1. A four node parallel relay network model.

models of single-hop networks, and [9], [10], which address multi-hop networks and include routing in the control strategy. Much of this work has been recently surveyed in [11]. By "throughput optimal" we mean that a control scheme stabilizes all the queues within the network whenever it is possible to do so by any (possibly non-causal) policy. In other words, such a scheme stabilizes the network for any rate in the network's *stability region*. Many of these schemes utilize some version of a *maximum differential backlog (MDB)* policy, first proposed by Tassiulas and Ephremides [9].[1] Such policies have the desirable property that they require no *a priori* knowledge of the traffic statistics and yet are throughput optimal.

A feature of all the above models is that each packet is forwarded along a single route of point-to-point links, i.e. one node transmits the packet to the next receiver; after it is received, the next node transmits the packet onward. In particular, at any time a packet resides at a single location in the network, and the resources needed for the next transmission do not depend on the previous transmissions of the packet. Recently, there has been much interest in various *cooperative relaying* techniques (e.g. [12]–[19]) that do not satisfy these assumptions. With such techniques, multiple nodes cooperate in relaying a packet. For example, consider the four node "parallel relay" network from [12], [13], shown in Figure 1. Suppose that node 1 has traffic to send to node 4. The arrows in the figure indicate the feasible links for this traffic using traditional point-to-point forwarding, i.e. node 1 could send this traffic either via node 2 or 3.[2] However, with cooperative relaying, node 1 can broadcast the same packet to both nodes 2 and 3. Nodes 2 and 3 can then cooperatively forward this packet to node 4 by, for example, forming a distributed antenna array. In certain cases, the resulting cooperative rate is greater than the

[1] In the single-hop case, this policy reduces to the so-called "max weight" policy.

[2] For simplicity, we assume that node 1 cannot directly transmit to node 4, e.g. the direct link may be of too poor a quality to be feasible.

sum of the individual rates achievable over each of the direct routes, i.e. the nodes achieve a "cooperative gain".

Cooperative communication has mainly been addressed from the physical-layer viewpoint, i.e. by studying the achievable rates or diversity gains of given cooperative schemes, assuming that all sources are backlogged and often just focusing on a single end-to-end session.[3] A goal of this paper is to develop and study models of cooperative communication that incorporate the stochastic arrival of traffic for multiple sessions (i.e. with different sources and/or destinations) and the related queueing dynamics at the various nodes in the network. For example, returning to the network in Figure 1, suppose node 2 also has its own traffic to send to node 4. In this case, in order to stabilize the network, node 1 may have to forgo any cooperative gain and use the single route through node 3. Given such a model, we are then interested in characterizing the network stability region (i.e. the set of arrival rates for which the queues in the network stay bounded), and developing an MDB-like policy which is throughput optimal without requiring *a priori* knowledge of the traffic statistics.

We focus on so-called "decode and forward" cooperative techniques, in which all of the cooperative nodes must decode a packet before forwarding it. An example of such a scheme is distributed beamforming, e.g. [18]. With such schemes, packets may now be duplicated within the network, i.e., each cooperative node must have a copy of the packet. Hence, when multiple sessions are present a new potential trade-off emerges: in order to exploit cooperative gains, the amount of congestion in the network must first increase due to this duplication. This increase in traffic can be somewhat ameliorated by exploiting the broadcast nature of the wireless medium. For example, in Figure 1, node 1 can transmit a packet to nodes 2 and 3 with a single transmission (e.g. viewing this packet as common information sent over the corresponding broadcast channel).[4]

In addition to "decode and forward," a variety of other cooperative relaying strategies have been considered, such as the "amplify and forward" technique (e.g. [16]), in which each relay simply forwards an amplified version of the received signal. We do not address such schemes here. One reason for this is that in these schemes the "commodity" at the intermediate nodes is no

[3]One paper that does take a network layer view of cooperation is [20], which addresses routing in a cooperative network for minimum energy, but does not address traffic dynamics.

[4]We note that in [21] an MDB-type of policy is given for a network that exploits such broadcasting. However, the focus in [21] is on broadcasting to improve reliability given unreliable links.

longer bits, but analog information. It is not obvious how to incorporate this into the queueing models considered here. We also note that in addition to improving throughput, cooperative relaying is often studied as a means for increasing diversity in a fading environment (e.g. [14]–[16]). Here, we focus on the case where there is no fading, and therefore do not address these diversity gains.

We begin in the next section by discussing modelling cooperative relay networks. We present a network model which can apply to a network with a general topology and multiple cooperative sets. Several examples are given to show how this model can capture different cooperative scenarios. We then move on to characterize the network stability region and give a modified version of the MDB policy that is shown to be throughput optimal. We provide a proof of this optimality and then discuss calculating this policy for some simple examples.

II. General Network Model

We study a model for a multi-hop network with an arbitrary topology and cooperative communication. For simplicity, we consider only "two-hop" cooperative communication, i.e. a node may send a packet to a group of nodes to be cooperatively forwarded to a destination node; the cooperative group then forwards this packet to the destination.[5] Of course, in an arbitrary network there can exist scenarios in which a packet could be cooperatively forwarded over several hops, e.g. one group could forward it to another group, which then forwards it to the destination. We do not consider such possibilities here, in part to simplify notation and in part because the complexity in implementing such a scheme quickly becomes intractable.

Our network model is a generalization of the model in [10] which includes cooperative communication. Specifically, the network $\mathcal{G}$ consists of a set of nodes $\mathcal{V}$, and a set $\mathcal{L}$ of feasible non-cooperative or direct links, where each non-cooperative link is simply an ordered pair of nodes (u, v) for $u, v \in \mathcal{V}$. These represent point-to-point links over which traffic can be sent.[6] Additionally, there are two other sets of "links" in the network. First, we define a set $\mathcal{S}$ of

[5]As discussed below, the cooperative group may include either the source or destination. This allows us to model several other cooperative schemes.

[6]In principle, a "link" exists between every pair of nodes in a wireless network. However, we do not require that $\mathcal{L}$ include all such links. For example, in Figure 1, the links between certain pairs of nodes may not be feasible. This may be a way of reducing routing complexity in practice.

feasible *cooperative links*. These are many-to-one links, denoted by ordered pairs (S, v), where $S \subset \mathcal{V}$ is a subset of the nodes and $v \in \mathcal{V}$ is a single destination node for the link. In this case, the nodes in S all cooperate to forward a packet to v. Second, we define a set $\mathcal{T}$ of feasible *broadcast links*, denoted by ordered pairs (u, T). These are one-to-many links originating at a node $u \in \mathcal{V}$ and terminating at a subset of of nodes $T \subset \mathcal{V}$. When a packet is sent over these "links," it is broadcast from u to all of the nodes in T. We make the assumption that each cooperative link (S, u) is matched to at least one broadcast link (u, S) whose destination set is the same as the origin set of the cooperative link. Similarly, each broadcast link is matched to at least one corresponding cooperative link. Finally, we assume that the only traffic that can be sent over a cooperative link is that which is received on one of the corresponding broadcast links. Technically, a key reason for this assumption is that it makes it easy to ensure that indeed the same packet is present at each node in the cooperative set S. Without this assumption, we would need to keep track of the specific packets within the network, not just the number of packets. From an implementation viewpoint, this is also desirable, in that it reduces the overhead needed to coordinate the cooperative transmitters.

Returning to the example four node network in Figure 1. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{S}, \mathcal{T})$ be a model for this network. Here $\mathcal{V} = \{1, 2, 3, 4\}$, and $\mathcal{E}$ consists of the four direct links, shown by the arrows in the figure. Assume that 2 and 3 can cooperate to relay a message to 4. We model this by setting $\mathcal{S} = \{(\{2, 3\}, 4)\}$ and $\mathcal{T} = \{(1, \{2, 3\})\}$. Under the above assumption, to send a packet over the cooperative link, $(\{2, 3\}, 4)$, it must first be sent over the broadcast link $(1, \{2, 3\})$.

Next, we turn to the feasible rates over each link in the network. We assume the network operates in slotted time, where the length of each time-slot is normalized to 1. For simplicity, we assume that there is no fading or changes in the topology over the time-scale of interest.[7] Within time-slot t, let $\boldsymbol{R}(t) = (R_l(t))$ denote the vector of realized transmission rates for all $l \in \mathcal{L} \cup \mathcal{S} \cup \mathcal{T}$, i.e., this indicates the transmission rate on each link of all three types in the network. For all t, we assume that $\boldsymbol{R}(t) \in \mathcal{C}$, where $\mathcal{C}$ denotes the *instantaneous link capacity region*, which we assume is a bounded subset of $\mathbb{R}_+^{|\mathcal{L} \cup \mathcal{S} \cup \mathcal{T}|}$. In other words, $\mathcal{C}$ denotes the set of feasible link rates in any time-slot t. Any constraints on the set of links that may be active are included in this set. Note that $\mathcal{C}$ includes the feasible rates on all cooperative and broadcast

[7]Such effects can be incorporated in our analysis at the expense of more complicated notation.

links.

Next, we illustrate several examples of $\mathcal{C}$. For these examples, we assume that the channel between each pair of nodes i, j is given by an additive Gaussian noise channel with gain $\sqrt{h_{ij}}$, unit variance noise, and bandwidth $W = 1$ Hz. We further assume that each transmitter has a power constraint of P during each time-slot. If link (i, j) is the only link activated, then we model the feasible transmission rate by

$$R_{ij} = \log(1 + h_{ij}P),$$

i.e., the Shannon capacity of this point-to-point channel. This is reasonable provided that each time-slot has sufficiently many degrees of freedom to allow for sophisticated coding. We emphasize that the results in Section III are not restricted to this case, but apply to any model for $\mathcal{C}$ that gives a bounded subset of $\mathbb{R}_+^{|\mathcal{L}\cup\mathcal{S}\cup\mathcal{T}|}$.

Example 1: Consider the four-node parallel relay network in Figure 1. Let $\boldsymbol{R} = (R_{1S}, R_{12}, R_{13}, R_{S4}, R_{24}, R_{34})$ be the vector of transmission rates for the 6 links in this model, where $S = \{2, 3\}$, e.g. R_{1S} is the rate of the broadcast link $(1, \{2, 3\})$ and, R_{S4} is the rate on the cooperative link $(\{2, 3\}, 4)$. Suppose that in any time-slot, only one of the following two sets of transmitters may be active: $\mathcal{A}_1 = \{1\}$ or $\mathcal{A}_2 = \{2, 3\}$. Note that this enforces a *half-duplexing constraint* (see e.g. [16], [17]), at nodes 2 and 3, so that these nodes can not be both transmitting and receiving within a time-slot.[8] With this assumption, the link capacity region, $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2$, where $\mathcal{C}_i$ is the set of feasible rates corresponding to activation set $\mathcal{A}_i$ (if $\boldsymbol{R} \in \mathcal{C}_1$, then the last three components of $\boldsymbol{R}$ must be zero).

Suppose additionally that during a time-slot, multiple feasible links must be served using time-division multiplexing (TDM) (i.e. at any time only one link is active.). Let $\overline{R}_{ij} = \log(1 + h_{ij}P)$ be the maximum feasible bit rate for direct link (i, j) when it is active. It follows that

$$\begin{aligned} \mathcal{C}_1 = \Big\{ & (R_{1S}, R_{12}, R_{13}, 0, 0, 0) \in \mathbb{R}_+^6 : R_{12} \le \tau_1 \overline{R}_{12}, R_{13} \le \tau_2 \overline{R}_{13}, \\ & R_{1S} \le \tau_3 \min(\overline{R}_{12}, \overline{R}_{13}), \sum_{i=1}^{3} \tau_i = 1, \tau_i \ge 0 \ \forall i \Big\}. \end{aligned} \tag{1}$$

[8] In this example, we do not allow some schedules that do not violate the half-duplexing constraint, such as a scenario where node 1 transmits to node 2, while node 3 transmits to node 4. Such schedules can easily be accommodated in the general model; here we omit them to simplify the discussion.

Similarly, if $\overline{R}_{S4}$ is the maximum feasible rate on link $(\{2,3\},4)$, then

$$\mathcal{C}_2 = \Big\{(0,0,0,R_{S4},R_{24},R_{34}) \in \mathbb{R}_+^6 : R_{24} \leq \tau_1 \overline{R}_{24}, R_{34} \leq \tau_2 \overline{R}_{34}, \\ R_{S4} \leq \tau_3 \overline{R}_{S4}, \sum_{i=1}^{3} \tau_i = 1, \tau_i \geq 0 \, \forall i \Big\}. \tag{2}$$

As an example of a cooperative gain, assume nodes 2 and 3 cooperate by beamforming, then

$$\overline{R}_{S4} = \log(1 + (\sqrt{h_{24}} + \sqrt{h_{34}})^2 P). \tag{3}$$

This is greater than the rate achieved by either direct link $(2,4)$ or $(3,4)$.[9]

For the network in Figure 1, we can instead define other link capacity regions by changing our assumptions on the allowable physical-layer techniques. For example, when node 1 is transmitting, the network can be viewed as a Gaussian broadcast channel. Note that the traffic sent over the broadcast link $(1,\{2,3\})$ represents the *common information* in the broadcast channel. Without loss of generality, assume that $h_{12} \leq h_{13}$. Let $\mathcal{C}_{BC}$ be the capacity region of the corresponding two-user Gaussian broadcast channel. It follows that the rates (R_{1S}, R_{12}, R_{13}) must satisfy $(R_{12}+R_{1S}, R_{13}) \in \mathcal{C}_{BC}$. Therefore, we can define $\mathcal{C}_1$ as the set of all $(R_{1S}, R_{12}, R_{13}, 0, 0, 0) \in \mathbb{R}_+^6$ such that $(R_{12} + R_{1S}, R_{13}) \in \mathcal{C}_{BC}$. For a symmetric network ($h_{12} = h_{13}$), $\mathcal{C}_1$ reduces to the set of $(R_{1S}, R_{12}, R_{13}, 0, 0, 0) \in \mathbb{R}_+^6$ that lie in the simplex defined by

$$R_{1S} + R_{12} + R_{13} \leq \log\left(1 + h_{12} P\right). \tag{4}$$

Similarly, when set $\mathcal{A}_2$ is active, nodes 2 and 3 transmit to node 4 over a Gaussian multiaccess channel. When these nodes send only direct traffic ($R_{S4} = 0$), the transmission rates (R_{24}, R_{34}) must lie in the corresponding multiaccess capacity region $\mathcal{C}_{MAC}$. This is the set of $(0,0,0,0,R_{24},R_{34}) \in \mathbb{R}_+^6$ satisfying

$$\sum_{i \in \mathcal{V}} R_{i4} \leq \log\left(1 + \sum_{i \in \mathcal{V}} h_{i4} P\right) \; \forall \mathcal{V} \subseteq \{2,3\}. \tag{5}$$

When both nodes send only cooperative traffic ($R_{24} = R_{34} = 0$), the transmission rate R_{S4} is again given by (3). In addition, we can allow the nodes to transmit both cooperative and

[9] Of course achieving this rate requires that the two transmitters have perfect synchronization and therefore can coherently combine their signals at the receiver. Other models for distributed beamforming that relax this assumption can also be found, e.g. [18].These can be incorporated into the model by simply re-defining $\overline{R}_{S4}$.

direct traffic simultaneously. One way to model this is to allow time-sharing between the above two modes. More generally, we can view this as a type of three-user multiaccess channel, with two users corresponding to the direct traffic for nodes 2 and 3, respectively, and a third user corresponding to the cooperative traffic.[10] The difference here is that the power constraints of the "users" are coupled. We assume that if both users 2 and 3 devote a fraction $\alpha \in [0,1]$ of their power to cooperative traffic, then they can achieve any rates $(0,0,0,R_{S4},R_{24},R_{34}) \equiv (0,0,0,R_4,R_5,R_6) \in \mathbb{R}^6_+$ satisfying

$$\sum_{i \in \mathcal{V}} R_i \leq \log\left(1 + \sum_{i \in \mathcal{V}} P_i(\alpha)\right) \quad \forall \mathcal{V} \subseteq \{4,5,6\}, \tag{6}$$

where $P_4(\alpha) = (\sqrt{h_{24}} + \sqrt{h_{34}})^2 \alpha P$, $P_5(\alpha) = h_{24}(1-\alpha)P$, and $P_6(\alpha) = h_{34}(1-\alpha)P$. Let $\mathcal{C}_{CMAC}(\alpha)$ be the set of rates $(0,0,0,R_4,R_5,R_6)$ which satisfy (6) for a particular power splitting parameter α. We can then set

$$\mathcal{C}_2 = \bigcup_{\alpha \in [0,1]} \mathcal{C}_{CMAC}(\alpha).$$

It can be verified that the resulting region is convex.[11]

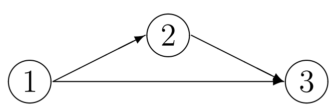

Fig. 2. A three node simple relay network model.

Example 2: The next example we consider is the three node relay network shown in Figure 2, which is based on the classical relay channel [22]. For this example, we again focus on the case where all information is intended for a single destination (node 3), and all feasible direct links are indicated via an arrow. We assume that both nodes 1 and 2 can generate traffic for node 3.

[10]A key assumption here is that the encoding of the traffic by these three "users" depends only on their own message and that the messages are independent.

[11]Note that here we require both nodes 2 and 3 to devote the same fraction of their power to the cooperative traffic. More generally, one can consider a model where each may devote a different fraction.

Furthermore, we assume that $h_{12} > h_{13}$. For this network, we discuss two ways in which packets from node 1 can be cooperatively relayed to node 3. First, we can consider a cooperative link $(\{1,2\},3)$, in which nodes 1 and 2 cooperatively forward a packet to node 3, e.g. using distributed beamforming. To utilize this cooperative link, node 1 must first send a packet to node 2 and save a copy of the packet for itself. Then in the next time slot, both nodes can transmit the packet to node 3. To incorporate such a scheme into our model, we view the first transmission as occurring over a broadcast link $(1,\{1,2\})$, i.e. a link in which the source is also one of the destination nodes. Of course, node 1 need not actually transmit a packet over this link and thus the maximum transmission rate on the link is simply $\overline{R}_{1\{1,2\}} = \log(1+h_{12}P)$, i.e. the direct rate from node 1 to node 2. In this case, $\mathcal{C}$ contains vectors of the form $(R_{1S}, R_{12}, R_{13}, R_{S3}, R_{23})$. Given a duplexing constraint at node 2, $\mathcal{C}$ can again be decomposed into two sets $\mathcal{C}_1$ and $\mathcal{C}_2$, where $\mathcal{C}_1$ ($\mathcal{C}_2$) is the set of feasible rates given that node 2 is receiving (transmitting), i.e., $\mathcal{C}_1$ contains vectors of the form $(R_{1S}, R_{12}, R_{13}, 0, 0)$, while $\mathcal{C}_2$ contains vectors of the form $(0, 0, R_{13}, R_{S3}, R_{23})$. Note that here link $(1,3)$ can be active in either case. As in Example 1, we can model $\mathcal{C}_1$ and $\mathcal{C}_2$ as in (1) by assuming TDM transmissions or assuming a more general rate region as in (4) (e.g. allowing node 1 to simultaneously broadcast direct traffic to node 3 and cooperative traffic to node 2 over the underlying Gaussian broadcast channel.)

A second possible cooperative scenario for this model is for node 1 to first transmit a packet to node 2, but for node 3 to also store the received signal from this transmission (even though it can not decode it). Then in the next time-slot, node 2 forwards the packet to node 3, which uses the information from both transmissions to decode the packet. We model this case by including a broadcast link $(1,\{2,3\})$ and a cooperative link $(\{2,3\},3)$. The maximum rate for the broadcast link $(1,\{2,3\})$ is again the rate at which node 1 can transmit to node 2 (since node 3 is not decoding), i.e., $\overline{R}_{1\{2,3\}} = \log(1+h_{12}P)$. The corresponding rate on the cooperative link $(\{2,3\},3)$ is

$$\overline{R}_{\{2,3\}3} = \log(1+h_{23}P) + \log(1+h_{13}P). \tag{7}$$

Here, the first term reflects the mutual information received from node 2's transmission and the second term is the mutual information received from node 1's original transmission to node 2. In this case, one can again define $\mathcal{C}$ for given duplexing and multiplexing constraints. For example, the cooperative link can still achieve rate $\overline{R}_{\{2,3\}3}$ in (7), while node 1 can simultaneously send at rate

$$\overline{R}_{1,3} = \log\left(1 + \frac{h_{13}P}{1 + h_{23}P}\right). \tag{8}$$

This can be accomplished by having node 3 successively decode both transmissions starting with node 1's direct transmission.[12] Of course, we can also define an instantaneous link capacity region which includes both types of cooperative links. Such a region would contain vectors of dimension 7 corresponding to the three direct rates, two cooperative rates, and two broadcast rates.

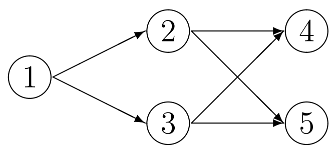

Fig. 3. A cooperative network with multiple commodities.

Example 3: The next example we consider is the network in Figure 3, which is similar to that in Figure 1, except now there are two destinations to which traffic can be cooperatively relayed. That is, nodes 2 and 3 can cooperative forward traffic to either node 4 or node 5 in a given time-slot. Note that in this case there are two cooperative links which are matched to the same broadcast link. The feasible link capacity region $\mathcal{C}$ is nine-dimensional, including the rates for the six direct rates, the one broadcast link, and the two cooperative links.

Example 4: The final example we give is shown in Figure 4. This can be viewed as a generalization of Example 1 to the case where there are n relay nodes between a node a and node b. In this case all n nodes may form a cooperative link, i.e. a link of the form $(\{1, \dots, n\}, b)$. More generally, any subset of these n nodes can form a cooperative link. Allowing all such possibilities, there are potentially $2^n - 1 - n$ different cooperative links between a and b in this network. Each such link would also have its own corresponding broadcast link. In this case, the instantaneous link capacity region would have a dimension of $2(2^n - 1 - n) + 2n$. Of course, to

[12]For this model, we require that node 1 transmit on link $(1, \{2, 3\})$ will full power P. Otherwise, the corresponding rate on the cooperative link would depend on the power used in the previous time-slots.

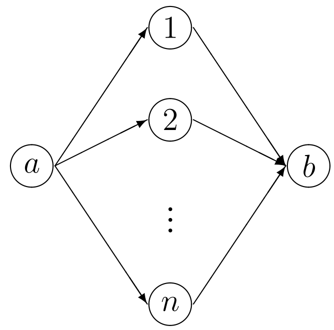

Fig. 4. A $n+2$ node parallel relay network model.

reduce the implementation complexity, one might limit the number of cooperative links in such a setting.

In this section, we have focused on relatively simple network topologies to illustrate some possibilities for cooperation. In a general network, several of these scenarios, as well as others, could exist at different locations in the network. Moreover, we emphasize that while we restricted our attention to two-hop cooperative transmissions, we do not restrict the overall network to have a two-hop topology. For example, a session could route a packet using more than two hops, where the route consists of one or more cooperative links.

A. *Traffic and queueing dynamics*

Next we turn to describing our model of the traffic and queueing dynamics within the network. Following [10], all traffic that enters the network is classified according to a particular "commodity," which specifies its desired destination.[13] Let $\mathcal{K} \subset \mathcal{V}$ denote the set of commodities in the network, where commodity k has destination node k. Exogenous traffic corresponding to each commodity $k \in \mathcal{K}$ is assumed to arrive into the network at node $i \in \mathcal{V} \setminus k$, according to an ergodic process $B_i^k(t)$, where $B_i^k(t)$ is the number of exogenous bit arrivals to node i in time-slot t. Each node buffers all arriving packets for each commodity until they are transmitted.

Let $U_i^k(t)$ be the number of untransmitted bits (unfinished work) of commodity k at node i, which is to be sent over a direct or broadcast link (we refer to this as the direct traffic).

[13] More generally a commodity could have any node from a given subset as a destination; however, we do not consider this here.

Additionally, for each cooperative link $(S,u) \in \mathcal{S}$, let $U_S^k(t)$ be the unfinished work of commodity k traffic, which is to be forwarded cooperatively by the nodes in S. We view each of these quantities as the backlog for a separate queue at the corresponding nodes. In other words, each node keeps separate queues for each commodity of the direct traffic as well as each commodity of traffic for each cooperative set S to which it belongs. Thus, potentially a node would need $(M+1)|\mathcal{K}|$ queues, where M is the number of cooperative sets a node is involved in. Note if a node is part of several cooperative links involving the same cooperative set S (c.f. Example 3), all the traffic of a given commodity for each of these cooperative links can be stored in one queue.

Let $\boldsymbol{U}(t) = ((U_i^k(t))_{i\in\mathcal{V}}, (U_S^k(t))_{S\in\mathcal{U}})_{k\in\mathcal{K}}$ denote the joint queue state at time t, i.e. the unfinished work of each commodity in every direct or cooperative queue in the network. We consider the case where given $\boldsymbol{U}(t)$ at time t, a network controller specifies a joint rate allocation/routing assignment denoted by $(R_l^k(t))_{l\in\mathcal{L}\cup\mathcal{S}\cup\mathcal{T},k\in\mathcal{K}}$, where $R_l^k(t)$ denotes the rate allocated to commodity k over link l at time t. For feasibility, $(R_l^k(t))_{l\in\mathcal{L}\cup\mathcal{S}\cup\mathcal{T},k\in\mathcal{K}}$ must satisfy

$$\sum_{k\in\mathcal{K}} R_l^k(t) \le R_l(t) \text{ for all } l, \quad \text{and} \quad \boldsymbol{R}(t) \equiv (R_l(t))_{l\in\mathcal{L}\cup\mathcal{S}\cup\mathcal{T}} \in \mathcal{C} \tag{9}$$

where $R_l(t)$ is the aggregate rate allocated over link l at time t.

Given a feasible routing decision, the dynamics of the direct queue backlogs $U_i^k(t)$, for all i, k, satisfy:

$$U_i^k(t+1) \le \left[U_i^k(t) - \sum_{T\in\mathcal{T}_i} R_{iT}^k(t) - \sum_{j\in\mathcal{O}_i} R_{ij}^k(t) + \sum_{S\in\mathcal{S}_i} R_{Si}^k(t) + \sum_{m\in\mathcal{I}_i} R_{mi}^k(t) + B_i^k(t)\right]^+. \tag{10}$$

Here, $\mathcal{O}_i \equiv \{j \in \mathcal{V}|(i,j) \in \mathcal{L}\}$, $\mathcal{T}_i \equiv \{T \subseteq \mathcal{V}|(i,T) \in \mathcal{T}\}$, $\mathcal{I}_i \equiv \{m \in \mathcal{V}|(m,i) \in \mathcal{L}\}$, $\mathcal{S}_i \equiv \{S \subseteq \mathcal{V}|(S,i) \in \mathcal{S}\}$, and $[x]^+$ denotes $\max(x,0)$.

Similarly, the dynamics of the backlog for each cooperative queue $U_S^k(t)$ for all S, k, satisfy:

$$U_S^k(t+1) \le \left[U_S^k(t) - \sum_{j\in\mathcal{O}_S} R_{Sj}^k(t) + \sum_{m\in\mathcal{I}_S} R_{mS}^k(t)\right]^+. \tag{11}$$

Here, $\mathcal{O}_S \equiv \{j \in \mathcal{V}|(S,j) \in \mathcal{S}\}$ and $\mathcal{I}_S \equiv \{m \in \mathcal{V}|(m,S) \in \mathcal{T}\}$. Note that all arrivals to cooperative queues arrive via broadcast links. In particular, there are no exogenous arrivals. This means that at any time, all the source nodes involved in a cooperative set has the same queue backlog in the corresponding cooperative queues. We briefly highlight one important caveat to this statement. This concerns the second cooperative model in Example 2. In that case, the cooperative link given by $(\{2,3\},3)$ corresponds to the case where node 3 cannot decode node 1's transmission to node 2, but stores some information about the received signal to aid it in decoding node 2's transmission. Thus, in this case, the cooperative queue backlog does not correspond to the actual amount of information stored at node 3 (since it is not decoding the packet but rather storing it). If we assume that the amount of data stored by node 3 is no greater than some bounded multiple of the actual number of bits transmitted, then the stability of $U_S^k(t)$ still implies the stability of the node 3 cooperative queue at node 3.

III. Network Stability Region and Throughput Optimal Rate Allocation

Given the model in Section II, we proceed to characterize the network stability region and the throughput optimal joint rate allocation/routing policy. Although the results we obtain here may be reminiscent of results for conventional networks [9], [10], we shall find that the cooperative nature of the relay network introduces some significantly new elements.

A. Stability Region

Let $\rho_i^k = \lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t} B_i^k(\tau)$ be the exogenous bit arrival rate of traffic to the direct queue at node i for commodity k. Denote the size of the direct queue at node i for commodity k at time t by $U_i^k(t)$. We say that the queue is ***stable*** if $\limsup_{t\to\infty} \frac{1}{t}\sum_{\tau=1}^{t} 1_{[U_i^k(\tau)>\xi]} d\tau \to 0$ as $\xi \to \infty$, where $1_{\{\cdot\}}$ is the indicator function. As noted earlier, for any cooperative set S and commodity k, the lengths of the cooperative queues for S and k are the same for all nodes i in the set S and given by $U_S^k(t)$. The notion of stability for the cooperative queues is defined in the same manner as for the direct queues.

The ***network stability region*** Λ is defined as the closure of the set of all $(\rho_i^k)_{i\in\mathcal{V},k\in\mathcal{K}} \in \mathbb{R}_+^{|\mathcal{K}||\mathcal{V}|}$ for which there exists some feasible joint rate allocation and routing policy $\mathcal{R}(\boldsymbol{u})$ which can guarantee that all queues are stable. This includes all policies which dynamically make rate allocation and routing decisions given (possibly non-causal) knowledge of the joint queue backlogs, $\boldsymbol{u}(t) = ((u_i^k(t))_{i\in\mathcal{V}}, (u_S^k)_{S\in\mathcal{U}})_{k\in\mathcal{K}}$. By feasible, we mean that at each time t, the policy specifies a rate vector $(R_l^k(t))_{l\in\mathcal{L}\cup\mathcal{S}\cup\mathcal{T},k\in\mathcal{K}}$ satisfying (9). The following result characterizes the stability region for a cooperative relay network. The proof is a direct generalization of the arguments in [9], [10], and so is omitted.

Theorem 1: The stability region Λ of a network $\mathcal{G} = (\mathcal{V}, \mathcal{L}, \mathcal{S}, \mathcal{T})$ with two-hop cooperative forwarding is the set of all $(\rho_i^k)_{i\in\mathcal{V},k\in\mathcal{K}} \in \mathbb{R}_+^{|\mathcal{K}||\mathcal{V}|}$ for which there exist non-negative flow variables $((f_{ij}^k)_{(i,j)\in\mathcal{L}},\ (f_{iT}^k)_{T\in\mathcal{T}},\ (f_{Si}^k)_{S\in\mathcal{S}})_{k\in\mathcal{K}}$ which support $(\rho_i^k)_{i\in\mathcal{V},k\in\mathcal{K}}$ relative to a weighted graph defined by the long-term rates in the convex hull of $\mathcal{C}$, $\mathrm{conv}(\mathcal{C})$. That is, the following flow conservation relations must be satisfied:

$$\rho_i^k = \sum_{j\in\mathcal{O}_i} f_{ij}^k + \sum_{T\in\mathcal{T}_i} f_{iT}^k - \sum_{m\in\mathcal{I}_i} f_{mi}^k - \sum_{S\in\mathcal{S}_i} f_{Si}^k,$$

for all $k \in \mathcal{K}$ and all $i \in \mathcal{V} \setminus k$;

$$0 = \sum_{j\in\mathcal{O}_S} f_{Sj}^k - \sum_{m\in\mathcal{I}_S} f_{mS}^k,$$

for all $k \in \mathcal{K}$ and all cooperative sets S; and

$$\sum_{i\in\mathcal{V}} \rho_i^k = \sum_{i\in\mathcal{I}_k} f_{ik}^k + \sum_{(S,k)\in\mathcal{S}} f_{Sk}^k,$$

for all $k \in \mathcal{K}$. In addition, $((f_{ij}^k)_{(i,j)\in\mathcal{L}}, (f_{iT}^k)_{T\in\mathcal{T}}, (f_{Si}^k)_{S\in\mathcal{S}})_{k\in\mathcal{K}} \in \mathrm{conv}(\mathcal{C})$.

The first flow conservation relation requires that the flow of direct traffic for each commodity into and out of each node which is not the destination of the commodity, must be the same. The second relation is a similar constraint for each cooperative set. The third constraint ensures that the arrival rate of a commodity is equal to its departure rate. Note, we can write the flow conservation equations in this way because we have implicitly assumed that once data is sent over a broadcast link to a cooperative set, it must be routed over a corresponding cooperative link. The careful reader may have noted in principle this is not always required. For example, in Figure 1, node 1 could broadcast a packet to nodes 2 and 3. Then, at a later time, node 2 could forward this packet directly, while node 3 could simply drop the packet. However, it can be shown that allowing for such strategies does not increase the stability region. In particular, note that under a non-causal policy one would never need to broadcast a packet to a node which will not forward it. Since the stability region includes the rates achievable under all non-causal policies, we do not reduce it by restricting ourselves to the above assumption.

B. Throughput Optimal Rate Allocation and Routing

Theorem 1 states that if $\boldsymbol{\rho} = (\rho_i^k)_{i\in\mathcal{V},k\in\mathcal{K}} \in \mathrm{int}(\Lambda)$, then the queues can be stabilized. In general, however, this may require knowing the value of $\boldsymbol{\rho}$. In reality, $\boldsymbol{\rho}$ can be learned only over time, and may be variable. One would prefer to find *adaptive* rate allocation/routing policies which can stabilize the network *without* knowing $\boldsymbol{\rho}$, as long as $\boldsymbol{\rho} \in \mathrm{int}(\Lambda)$. As pointed out previously in [10], a throughput optimal resource allocation policy for stochastic networks with physical-layer capacity regions turns out to be a generalization of the *maximum differential backlog* (MDB) policy first proposed by Tassiulas [9]. Due to cooperative transmissions, however, the general relay networks considered here is somewhat different from the networks considered in [10]. Nevertheless, we show that the MDB policy can be adapted to produce a throughput optimal rate allocation/routing policy for a cooperative relay network.

Let $\boldsymbol{B}(t) = (B_i^k(t))_{i\in\mathcal{V},k\in\mathcal{K}}$ be the vector of bit arrivals in the tth time slot. In this section to simplify our arguments, we restrict attention to the case where $\{\boldsymbol{B}(t) : t \in \mathbb{Z}_+\}$ are i.i.d. according to distribution $\pi_{\boldsymbol{B}}$ with finite mean $\mathsf{E}[\boldsymbol{B}] = \boldsymbol{\rho}$, where $\boldsymbol{\rho} = (\rho_i^k)_{i\in\mathcal{V},k=\in\mathcal{K}}$ is the vector of exogenous bit arrival rates. Furthermore, assume that $\mathsf{E}[(B_i^k)^2] < \infty$ for each i and each k, and $\Pr\left(\cap_{i\in\mathcal{V}} \cap_{k\in\mathcal{K}} \{B_i^k = 0\}\right) > 0$. These assumptions on the arrival process clearly hold, for example, for independent homogeneous Poisson arrival processes. Following similar arguments as in [11], the above assumptions can be relaxed to the Markov modulated case.

Theorem 2: A throughput optimal rate allocation/routing policy $\mathcal{R}^*(\boldsymbol{u})$ for a network with two-hop cooperative forwarding is given by first finding a rate allocation $\boldsymbol{R}^*$ which is a solution to the following optimization:

$$\max_{\boldsymbol{R}\in\mathcal{C}} \sum_{(i,j)\in\mathcal{L}} b_{ij}^* R_{ij} + \sum_{(i,T)\in\mathcal{T}} b_{iT}^* R_{iT} + \sum_{(S,i)\in\mathcal{S}} b_{Si}^* R_{Si} \tag{12}$$

where

$$b_{ij}^* \equiv \max_{k\in\mathcal{K}} u_i^k - u_j^k, \tag{13}$$

$$b_{iT}^* \equiv \max_{k\in\mathcal{K}} u_i^k - |T| u_T^k, \tag{14}$$

$$b_{Si}^* \equiv \max_{k\in\mathcal{K}} |S| u_S^k - u_i^k. \tag{15}$$

The corresponding routing policy is implemented by sending only bits from traffic class k^* which attains the maximum in (13) ((14) and (15), respectively) at rate R_{ij}^* (R_{iT}^* and R_{Si}^*, respectively) for all $(i,j) \in \mathcal{L}$ ($(i,T) \in \mathcal{T}$ and $(S,i) \in \mathcal{S}$, respectively). That is, over link $l \in \mathcal{L} \cup \mathcal{S} \cup \mathcal{T}$, $R_l^k = R_l^*$ for $k = k^*$ and $R_l^k = 0$ otherwise.

Note that the policy in (12) is the not the same as the conventional MDB policy of [9], [10]. In particular, the terms $u_i^k - |T|u_T^k$ and $|S|u_S^k - u_i^k$ reflect the *queue coupling* effect induced by the cooperative transmission structure.[14] We refer to the policy of (12) as the *Cooperative Maximum Differential Backlog* (CMDB) policy.

Proof of Theorem 2: To show that the CMDB policy stabilizes this network for any $\boldsymbol{\rho} = (\rho_i^k)_{i\in\mathcal{V},k\in\mathcal{K}} \in \text{int}(\Lambda)$, it is convenient to consider a "fictitious network" $\mathcal{G}_f$ that is the same as the network $\mathcal{G}$, except that arrivals are allowed to enter the cooperative queues. Let $\mathcal{U}$ be the set of all cooperation sets. In the fictitious network, for each $S \in \mathcal{U}$, $i \in S$, and $k \in \mathcal{K}$ let ρ_{iS}^k denote the exogenous bit arrival rate to the queue at node i for cooperative set S and commodity k; where we assume that the same arrivals occur simultaneously at each $i \in S$, so that $\rho_{iS}^k = \rho_S^k$ for all $i \in S$. Let Λ_f be the stability region of $\mathcal{G}_f$; this can be characterized as in Theorem 1, except the second flow conservation equation will now have ρ_S on the left-hand side. It is clear that if the CMDB policy stabilizes $\mathcal{G}_f$ for all $((\rho_i^k), (\rho_{iS}^k)_{S\in\mathcal{U}})_{i\in\mathcal{V},k\in\mathcal{K}} \in \text{int}(\Lambda_f)$ such that $\rho_{iS}^k = 0$ for $S \ni i$, all $i \in \mathcal{V}$ and all $k \in \mathcal{K}$, then CMDB also stabilizes $\mathcal{G}$ for all $\boldsymbol{\rho} = (\rho_i^k)_{i\in\mathcal{V},k\in\mathcal{K}} \in \text{int}(\Lambda)$. Therefore, from now on, we concentrate on the artificial network $\mathcal{G}_f$.

To show that the CMDB policy stabilizes $\mathcal{G}_f$ for all $((\rho_i^k), (\rho_{iS}^k)_{S\in\mathcal{U}})_{i\in\mathcal{V},k\in\mathcal{K}}$ such that $\rho_{iS}^k = 0$ for $S \ni i$, all $i \in \mathcal{V}$ and all $k \in \mathcal{K}$, we use an extension of Foster's Criterion for the convergence of Markov chains [5], [6], [10]. Consider the Lyapunov function

$$V(\boldsymbol{u}) \equiv \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{V}} \left[(u_i^k)^2 + \sum_{S\ni i} (u_S^k)^2 \right] = \sum_{k\in\mathcal{K}} \left[\sum_{i\in\mathcal{V}} (u_i^k)^2 + \sum_{S\in\mathcal{U}} |S| (u_S^k)^2 \right]. \qquad (16)$$

We wish to show that there exists a compact subset $\Gamma \subset \mathbb{R}_+^{|\mathcal{K}|(|\mathcal{V}|+|\mathcal{U}|)}$ such that under the CMDB policy,

$$\mathsf{E}[V(\boldsymbol{U}(t+1)) - V(\boldsymbol{U}(t)) | \boldsymbol{U}(t) = \boldsymbol{u}] < -\epsilon$$

for all $\boldsymbol{u} \notin \Gamma$, where $\epsilon > 0$. This, along with some other technical conditions [10], implies the existence of a steady state distribution for $\boldsymbol{U}$.

[14]The exact value of these terms is due to our choice of the Lyapunov function used in the proof of Theorem 2, which is a natural generalization of the Lyapunov function used in [9], [10]. Other choices of Lyapunov functions can be used to derive other throughput optimal policies.

We have from (10),

$$\begin{aligned}(U_i^k(t+1))^2 &\le \left(\left[U_i^k(t) - \sum_{T\in\mathcal{T}_i} R_{iT}^k(t) - \sum_{j\in\mathcal{O}_i} R_{ij}^k(t) + \sum_{S\in\mathcal{S}_i} R_{Si}^k(t) + \sum_{m\in\mathcal{I}_i} R_{mi}^k(t) + B_i^k(t)\right]^+\right)^2 \\ &\le \left(U_i^k(t) - \sum_{T\in\mathcal{T}_i} R_{iT}^k(t) - \sum_{j\in\mathcal{O}_i} R_{ij}^k(t) + \sum_{S\in\mathcal{S}_i} R_{Si}^k(t) + \sum_{m\in\mathcal{I}_i} R_{mi}^k(t) + B_i^k(t)\right)^2 \\ &\le (U_i^k(t))^2 - 2U_i^k(t)\left(\sum_{T\in\mathcal{T}_i} R_{iT}^k(t) + \sum_{j\in\mathcal{O}_i} R_{ij}^k(t) - \sum_{S\in\mathcal{S}_i} R_{Si}^k(t) + \sum_{m\in\mathcal{I}_i} R_{mi}^k(t) - B_i^k(t)\right) \\ &\quad +(B_i^k(t))^2 + 2B_i^k(t)\left(\sum_{S\in\mathcal{S}_i} R_{Si}^k(t) + \sum_{m\in\mathcal{I}_i} R_{mi}^k(t)\right) \\ &\quad + \left(\sum_{T\in\mathcal{T}_i} R_{iT}^k(t) - \sum_{j\in\mathcal{O}_i} R_{ij}^k(t)\right)^2 + \left(\sum_{S\in\mathcal{S}_i} R_{Si}^k(t) + \sum_{m\in\mathcal{I}_i} R_{mi}^k(t)\right)^2 \end{aligned} \tag{17}$$

Similarly, from (11), since $\rho_{iS}^k = 0$ for all $i \in S$, $S \in \mathcal{U}$ and $k \in \mathcal{K}$,

$$\begin{aligned}(U_S^k(t+1))^2 &\le \left(\left[U_S^k(t) - \sum_{j\in\mathcal{O}_S} R_{Sj}^k(t) + \sum_{m\in\mathcal{I}_S} R_{mS}^k(t)\right]^+\right)^2 \\ &\le \left(U_S^k(t) - \sum_{j\in\mathcal{O}_S} R_{Sj}^k(t) + \sum_{m\in\mathcal{I}_S} R_{mS}^k(t)\right)^2 \\ &\le (U_S^k(t))^2 - 2U_S^k(t)\left(\sum_{j\in\mathcal{O}_S} R_{Sj}^k(t) - \sum_{m\in\mathcal{I}_S} R_{mS}^k(t)\right) \\ &\quad + \left(\sum_{j\in\mathcal{O}_S} R_{Sj}^k(t)\right)^2 + \left(\sum_{m\in\mathcal{I}_S} R_{mS}^k(t)\right)^2. \end{aligned} \tag{18}$$

Taking conditional expected value of both sides of inequalities (17)-(18) given the event $\boldsymbol{U}(t) = \boldsymbol{u}$, and re-arranging, we have

$$\begin{aligned}&\mathsf{E}[V(\boldsymbol{U}(t+1)) - V(\boldsymbol{U}(t))|\boldsymbol{U}(t) = \boldsymbol{u}] \\ &\le \sum_{k\in\mathcal{K}}\left\{\sum_{i\in\mathcal{V}} -2u_i^k \mathsf{E}\left[\sum_{T\in\mathcal{T}_i} R_{iT}^k(t) + \sum_{j\in\mathcal{O}_i} R_{ij}^k(t) - \sum_{S\in\mathcal{S}_i} R_{Si}^k(t) - \sum_{m\in\mathcal{I}_i} R_{mi}^k(t) - B_i^k(t)|\boldsymbol{U}(t) = \boldsymbol{u}\right]\right. \\ &\quad \left. + \sum_{S\in\mathcal{U}} -2u_S^k|S|\mathsf{E}\left[\sum_{j\in\mathcal{O}_S} R_{Sj}^k(t) - \sum_{m\in\mathcal{I}_S} R_{mS}^k(t)|\boldsymbol{U}(t) = \boldsymbol{u}\right]\right\} + \beta \end{aligned} \tag{19}$$

where $\beta > 0$ is an upper bound on a sum of terms involving the second moments of the bit arrivals in the tth slot (which are bounded since the second moments of the packet arrivals and the packet sizes are bounded), and powers of transmission rates (which are bounded since $\mathcal{C}$ is bounded).

Let $\mathsf{E}_{\boldsymbol{u}}[X]$ denote $\mathsf{E}[X|\boldsymbol{U}(t) = \boldsymbol{u}]$. Note that

$$\sum_{k\in\mathcal{K}}\left\{\sum_{i\in\mathcal{V}} u_i^k \mathsf{E}_{\boldsymbol{u}}\left[\sum_{T\in\mathcal{T}_i} R_{iT}^k(t) + \sum_{j\in\mathcal{O}_i} R_{ij}^k(t) - \sum_{S\in\mathcal{S}_i} R_{Si}^k(t) - \sum_{m\in\mathcal{I}_i} R_{mi}^k(t)\right]\right.$$
$$\left. + \sum_{S\in\mathcal{U}} u_S^k |S| \mathsf{E}_{\boldsymbol{u}}\left[\sum_{j\in\mathcal{O}_S} R_{Sj}^k(t) - \sum_{m\in\mathcal{I}_S} R_{mS}^k(t)\right]\right\}$$
$$= \sum_{k\in\mathcal{K}}\left\{\sum_{(i,j)\in\mathcal{L}} \mathsf{E}_{\boldsymbol{u}}[R_{ij}^k(t)](u_i^k - u_j^k) + \sum_{(i,T)\in\mathcal{T}} \mathsf{E}_{\boldsymbol{u}}[R_{iT}^k(t)](u_i^k - |T|u_T^k) + \sum_{(S,i)\in\mathcal{S}} \mathsf{E}_{\boldsymbol{u}}[R_{Si}^k(t)](|S|u_S^k - u_i^k)\right\} \tag{20}$$

For any $((\rho_i^k), (\rho_{iS}^k)_{S\in\mathcal{U}})_{i\in\mathcal{V},k\in\mathcal{K}} \in \text{int}(\Lambda_f)$ such that $\rho_{iS}^k = 0$ for $S \ni i$, all $i \in \mathcal{V}$ and all $k \in \mathcal{K}$, there exists $\delta > 0$ such that $((\rho_i^k + \delta), (\rho_{iS}^k)_{S\in\mathcal{U}})_{i\in\mathcal{V},k\in\mathcal{K}} \in \Lambda_f$ such that $\rho_{iS}^k = \delta$ for $S \ni i$, all $i \in \mathcal{V}$ and all $k \in \mathcal{K}$. Therefore, an application of Theorem 1 to $\mathcal{G}_f$ shows that there exist non-negative flow variables $((f_{ij}^k)_{(i,j)\in\mathcal{L}}, (f_{iT}^k)_{T\in\mathcal{T}}, (f_{Si}^k)_{S\in\mathcal{S}})_{k\in\mathcal{K}} \in \text{conv}(\mathcal{C})$ such that

$$\begin{aligned} \rho_i^k + \delta &= \sum_{j\in\mathcal{O}_i} f_{ij}^k + \sum_{T\in\mathcal{T}_i} f_{iT}^k - \sum_{m\in\mathcal{I}_i} f_{mj}^k - \sum_{S\in\mathcal{S}_i} f_{Si}^k, \quad i \in \mathcal{V}, k \in \mathcal{K}. \\ \delta &= \sum_{j\in\mathcal{O}_S} f_{Sj}^k - \sum_{m\in\mathcal{I}_S} f_{mS}^k, \quad S \in \mathcal{U} \end{aligned}$$

We therefore have

$$\sum_{k\in\mathcal{K}}\left\{\sum_{i\in\mathcal{V}} u_i^k(\rho_i^k + \delta) + \sum_{S\in\mathcal{U}} |S|u_S^k\delta\right\}$$
$$= \sum_{k\in\mathcal{K}}\left\{\sum_{(i,j)\in\mathcal{L}} f_{ij}^k(u_i^k - u_j^k) + \sum_{(i,T)\in\mathcal{T}} f_{iT}^k(u_i^k - |T|u_T^k) + \sum_{(S,i)\in\mathcal{S}} f_{Si}^k(|S|u_S^k - u_i^k)\right\}.$$

Let $((R_{ij}^k)_{(i,j)\in\mathcal{L}}, (R_{iT}^k)_{T\in\mathcal{T}}, (R_{Si}^k)_{S\in\mathcal{S}})_{k\in\mathcal{K}}$ be chosen according to the CMDB rule described in (12). Then, since $((f_{ij}^k)_{(i,j)\in\mathcal{L}}, (f_{iT}^k)_{T\in\mathcal{T}}, (f_{Si}^k)_{S\in\mathcal{S}})_{k\in\mathcal{K}} \in \text{conv}(\mathcal{C})$, $\sum_{k\in\mathcal{K}}\{\sum_{i\in\mathcal{V}} u_i^k(\rho_i^k + \delta) + \sum_{S\in\mathcal{U}} |S|u_S^k\delta\}$ is less than or equal to the RHS of (20). Combining this fact with a rearrangement of the RHS of (19), and noting $\mathsf{E}[B_i^k(t)] = \rho_i^k$, we have

$$\mathsf{E}[V(\boldsymbol{U}(t+1)) - V(\boldsymbol{U}(t))|\boldsymbol{U}(t) = \boldsymbol{u}] \leq \beta - 2\delta\left(\sum_{k\in\mathcal{K}}\left\{\sum_{i\in\mathcal{V}} u_i^k + \sum_{S\in\mathcal{U}} |S|u_S^k\right\}\right)$$

Let $\Gamma = \{\boldsymbol{u} : \sum_{k\in\mathcal{K}}[\sum_{i\in\mathcal{V}} u_i^k + \sum_{S\in\mathcal{U}} |S|u_S^k] \leq \frac{\beta+\epsilon}{2\delta}\}$. Then, for any $\epsilon > 0$, and any $\boldsymbol{u} \notin \Gamma$, $\mathsf{E}[V(\boldsymbol{U}(t+1)) - V(\boldsymbol{U}(t))|\boldsymbol{U}(t) = \boldsymbol{u}] < -\epsilon$.

We have shown that the CMDB policy stabilizes $\mathcal{G}_f$ for all $((\rho_i^k), (\rho_{iS}^k)_{S\in\mathcal{U}})_{i\in\mathcal{V},k\in\mathcal{K}} \in \text{int}(\Lambda_f)$ such that $\rho_{iS}^k = 0$ for $S \ni i$, all $i \in \mathcal{V}$ and all $k \in \mathcal{K}$. Thus, we have also shown that the CMDB policy stabilizes $\mathcal{G}$ for all $\boldsymbol{\rho} = (\rho_i^k)_{i\in\mathcal{V},k\in\mathcal{K}} \in \text{int}(\Lambda)$. □

As noted above, the CMDB policy defined in Theorem 2 differs from the usual MDB policy in the definition of the differential backlog weights b_{iT}^* and b_{Si}^* defined in (14) and (15). From the above proof, it can be seen that this is coupled to our choice of Lyapunov function in (16); namely, the fact that we include the backlog at each node in a cooperative set.

IV. Calculating the CMBD policy

In this section, we focus on the calculation of the CMBD policy in several simple network examples first presented in Section II.

A. Four Node Parallel Relay Network with TDM

Consider first the four-node network in Example 1, using the TDM rate regions given by (1) and (2). We focus on the case where all traffic is destined for node 4, i.e. there is a single commodity. In this network, there are three direct queues (at nodes 1, 2, and 3) and one cooperative queue (representing traffic stored at both nodes 2 and 3). At a give time t, let u_1, u_{2d}, u_{3d}, and u_c represent the respective backlogs for the queues. For simplicity of notation, let $(w_1, w_2, w_3, w_4, w_5, w_6)$ denote $(u_1 - 2u_c,\ u_1 - u_{2d},\ u_1 - u_{3d},\ 2u_c,\ u_{2d},\ u_{3d})$, and let $(R_1$, R_2, R_3, R_4, R_5, $R_6)$ denote $(R_{1\{2,3\}}$,R_{12},R_{13},$R_{\{2,3\}4}$,R_{24}, $R_{34})$. The CMDB policy can now be expressed as

$$\max_{\boldsymbol{R}\in\mathcal{C}} \sum_{i=1}^{6} w_i R_i. \tag{21}$$

Recall that in this example, $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2$, where $\mathcal{C}_1$ and $\mathcal{C}_2$ are orthogonal sets. Therefore, it follows that the solution to (21) is either $\boldsymbol{R}_1^*$ or $\boldsymbol{R}_2^*$, where for $j = 1, 2$, $\boldsymbol{R}_j^*$ solves

$$\max_{\boldsymbol{R}\in\mathcal{C}_j} \sum_{i=1}^{6} w_i R_i. \tag{22}$$

For $j = 1$ or 2, note that the corresponding regions $\mathcal{C}_j$ in (1) and (2) both constrain the non-zero rates to lie within a simplex in $\mathbb{R}^3$. It follows that the solution to (22) is simply a "max-weight" type of rule, i.e. $\boldsymbol{R}_j^*$ can always be chosen so that only one of its components is positive and equal to its maximum value $\overline{R}_i$. The index of the positive component satisfies

$$i^*(j) = \arg\max_i w_i \overline{R}_i(j).$$

where $\overline{R}_i(j)$ denote the maximum possible value of R_i in the set $\mathcal{C}_j$. The corresponding solution to (22) is $w_{i^*(j)}\overline{R}_{i^*(j)}$ for each j. Comparing the solutions for $j = 1$ and $j = 2$, it follows that the overall solution to (21) is given by the same type of rule.

Lemma 1: Consider the four node network in Example 1, where the feasible link capacity region is given by $\mathcal{C}_1 \cup \mathcal{C}_2$, with the regions $\mathcal{C}_1$ and $\mathcal{C}_2$ given by (1) and (2). A solution to the MDB problem in (21) is given by a rate vector $\boldsymbol{R}^*$ for which every component but one is zero. The non-zero component i^* is allocated its maximum value in $\mathcal{C}$, $\overline{R}_{i^*}$, and i^* satisfies

$$i^* = \arg\max_i w_i \overline{R}_i.$$

In other words, under the MDB policy, there is no need to time-share among multiple feasible links in different active sets within a time-slot, even though this is allowed.

B. Four Node Parallel Relay Network with Multiaccess/Broadcast

Next, suppose that instead, within a time-slot, we allow the network controller to choose any rates in the *convex hull* of $\mathcal{C}$, conv$(\mathcal{C})$, by solving the optimization problem

$$\max_{\boldsymbol{R} \in \text{conv}(\mathcal{C})} \sum_{i=1}^{6} w_i R_i. \tag{23}$$

This would correspond to time-sharing among different active sets within a time-slot. It can be seen that since $\mathcal{C}_1$ and $\mathcal{C}_2$ are both convex polytopes in $\mathbb{R}_+^6$, conv$(\mathcal{C})$ is also a convex polytope. Furthermore the objective in (23) is linear, i.e. (23) is a linear programming problem. It follows that (23) must have a solution $\boldsymbol{R}^*$ which is an extreme point of conv$(\mathcal{C})$. Furthermore, the extreme points of conv$(\mathcal{C})$ must be either points in $\mathcal{C}_1$ or $\mathcal{C}_2$, i.e. $\boldsymbol{R}^*$ is also the solution to (21). Again, even though we are allowed to time-share among active sets, the MDB policy does not do this.

For the next scenario, we consider the network in Example 1 using the broadcast and multi-access rate regions described in (4)-(6). We focus on a symmetric scenario where $h_{12} = h_{13} =$

$h_{24} = h_{34} = 1$. In this case, the power variables used in the definition of $\mathcal{C}_{CMAC}(\alpha)$ become $P_4(\alpha) = 4\alpha P$, $P_5(\alpha) = P_6(\alpha) = (1-\alpha)P$.

Suppose that we allow for time-sharing in any given time slot. Thus, within a time slot, the network controller can choose any rate in $\text{conv}(\mathcal{C}_1, \mathcal{C}_2)$, where $\mathcal{C}_2 = \cup_{\alpha\in[0,1]}\mathcal{C}_{CMAC}(\alpha)$. Note that the solution $\boldsymbol{R}^*$ to (21) lies in $\text{conv}(\mathcal{C}_1,\, \mathcal{C}_{CMAC}(\alpha^*))$ for some $\alpha^* \in [0,1]$. Since $\mathcal{C}_1$ and $\mathcal{C}_{CMAC}(\alpha^*)$ are both convex polytopes, $\text{conv}(\mathcal{C}_1, \mathcal{C}_{CMAC}(\alpha^*))$ is also a convex polytope. Following the above discussion, $\boldsymbol{R}^*$ is an extreme point of $\text{conv}(\mathcal{C}_1, \mathcal{C}_{CMAC}(\alpha^*))$, and therefore is either an extreme point of $\mathcal{C}_1$ or an extreme point of $\mathcal{C}_{CMAC}(\alpha^*)$. We now consider these cases separately.

Case 1: $\boldsymbol{R}^*$ is an extreme point of $\mathcal{C}_1$. Using the symmetry of the regions and results from earlier discussion, $\boldsymbol{R}^*$ takes the form $R^*_{i^*} = \log(1+P)$ and $R^*_i = 0$ otherwise, where $i^* = \arg\max_{i=1,2,3} w_i$. Thus, whenever the CMDB policy operates in the broadcast mode, it allocates maximum rate $\log(1+P)$ to the traffic type with the largest weight w_i.

Case 2: $\boldsymbol{R}^*$ is an extreme point of $\mathcal{C}_{CMAC}(\alpha^*)$. In this case, $\boldsymbol{R}^*$ has the form $(0,0,0,R^*_4,R^*_5,R^*_6)$, and (R^*_4,R^*_5,R^*_6) solves

$$\max_{\boldsymbol{R}\in\mathcal{C}_{CMAC}(\alpha^*)} \sum_{i=4}^{6} w_i R_i \tag{24}$$

Since $\mathcal{C}_{CMAC}(\alpha^*)$ is a *polymatroid* [23], (R^*_4,R^*_5,R^*_6) can be explicitly given as follows. Let $w_{[4]}, w_{[5]}, w_{[6]}$ be the largest, second largest, and smallest element of $\{w_4,w_5,w_6\}$, respectively. Then

$$R^*_{[i]} = \log\left(1 + \frac{P_{[i]}(\alpha^*)}{1+\sum_{j<i} P_{[j]}(\alpha^*)}\right), \quad i = 4,5,6. \tag{25}$$

For instance, if $w_4 \geq w_5 \geq w_6$, then $R_4 = \log(1+4\alpha^* P)$, $R_5 = \log\left(1+\frac{(1-\alpha^*)P}{1+4\alpha^* P}\right)$, $R_6 = \log\left(1+\frac{(1-\alpha^*)P}{1+(3\alpha^*+1)P}\right)$.

Next, to find α^*, we solve

$$\max_{\alpha\in[0,1]} \sum_{i=4}^{6} w_{[i]} \log\left(1+\frac{P_{[i]}(\alpha)}{1+\sum_{j<i} P_{[j]}(\alpha)}\right). \tag{26}$$

Let $L(\alpha)$ be the objective in (26). For the case of $w_4 \geq w_5 \geq w_6$, it can be verified that $L(\alpha)$ is concave in α over $[0,1]$, and that $L'(\alpha) \geq 0$ for all $\alpha \in [0,1]$. Thus, $\alpha^* = 1$, i.e. *all power is allocated to cooperative transmission* over the MAC. In other cases, $L(\alpha)$ may not be concave, and one needs to solve for the stationary points of $L(\alpha)$ and compare the value of $L(\alpha)$ at the stationary points with its values on the boundaries.

Let $L^*_{CMAC}(w_4, w_5, w_6)$ be the optimal objective of (26). Using the above arguments, it is easy to see that the following is true:

Lemma 2: Consider the symmetric four node network in Example 1 where the feasible link capacity region is given by $\mathrm{conv}(\mathcal{C}_1 \cup \mathcal{C}_2)$, with the regions $\mathcal{C}_1$ and $\mathcal{C}_2$ given by (4)-(6). Let the differential queue backlogs be $w_1, w_2, \ldots, w_6$. If $(\max_{i=1,2,3} w_i) \log(1+P) \geq L^*_{CMAC}(w_4, w_5, w_6)$, then the optimal solution $\boldsymbol{R}^*$ to (21) is such that $R^*_{i^*} = \log(1+P)$ and $R^*_i = 0$ otherwise, where $i^* = \arg\max_{i=1,2,3} w_i$. Otherwise, $\boldsymbol{R}^* = (0, 0, 0,\ R^*_4, R^*_5, R^*_6)$, where $R^*_{[i]}$ is given by (25) and α^* is given by (26).

C. $n+2$ Node Parallel Relay Network with Multiaccess/Broadcast

Finally, we consider the generalization of the four node network considered in Section IV-B to the n relay case. The resulting $n+2$ node parallel relay network was first introduced in Example 4 in Section II. Assuming half-duplexing constraints at the n relay nodes, and allowing for time-sharing within a slot, the link capacity region is again given by $\mathcal{C} = \mathrm{conv}(\mathcal{C}_1 \cup \mathcal{C}_2)$, where $\mathcal{C}_1$ corresponds to activation set $\mathcal{A}_1 = \{1\}$ and $\mathcal{C}_2$ corresponds to activation set $\{1, \ldots, n\}$. As mentioned in Example 4, in this case, any subset of the n relays can potentially form a cooperative link. Let $\mathcal{U} \subseteq \{1, \ldots, n\}$ be the set of all cooperation subsets. For simplicity of notation, we include all direct links, i.e. singleton subsets, in $\mathcal{U}$.

Without loss of generality, assume $h_{a1} \leq \cdots \leq h_{an}$. Let $\mathcal{C}_{BC}$ be the capacity region of the n-user Gaussian broadcast channel corresponding to the model. It follows that the rate vector $((R_{aS})_{S\in\mathcal{U}}, \mathbf{0}) \in \mathbb{R}_+^{2|\mathcal{U}|}$ lies in $\mathcal{C}_1$ if and only if $(R_{aS})_{S\in\mathcal{U}}$ satisfies

$$R_i = \sum_{S\in\mathcal{U}_i} R_{aS}, i = 1, \ldots, n \quad \text{and} \quad (R_1, \ldots, R_n) \in \mathcal{C}_{BC} \tag{27}$$

where $\mathcal{U}_i = \{S \in \mathcal{U} : i = \min S\}$. For a symmetric network ($h_{a1} = \cdots = h_{an}$), $\mathcal{C}_1$ reduces to the set of all $((R_{aS})_{S\in\mathcal{U}}, \mathbf{0}) \in \mathbb{R}_+^{2|\mathcal{U}|}$ satisfying the simplex constraint

$$\sum_{S\in\mathcal{U}} R_{aS} \leq \log(1 + h_{a1}P). \tag{28}$$

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For the multiaccess side of the parallel relay network, let $\boldsymbol{\alpha} = (\alpha_S^i)_{S \ni i, i=1,\ldots,n}$ be the vector of power splitting parameters, where $\alpha_S^i P$ is the power allocated by node i to cooperation set $S \ni i$. The multiaccess capacity region $\mathcal{C}_{CMAC}(\boldsymbol{\alpha})$ for a given $\boldsymbol{\alpha}$ is the set of all $(\mathbf{0}, (R_{Sb}))_{S \in \mathcal{U}} \in \mathbb{R}_+^{2|\mathcal{U}|}$ such that

$$\sum_{S \in \mathcal{V}} R_{Sb} \leq \log\left(1 + \sum_{S \in \mathcal{V}} P_S(\boldsymbol{\alpha})\right) \quad \forall \mathcal{V} \subseteq \mathcal{U}. \tag{29}$$

Note that $\mathcal{C}_{CMAC}(\boldsymbol{\alpha})$ is defined by as many as $2^{2^n-1} - 1$ constraints! Finally, the overall multiaccess region $\mathcal{C}_2 = \cup_{\boldsymbol{\alpha}} \mathcal{C}_{CMAC}(\boldsymbol{\alpha})$.

Let u_a be the queue backlog at node a and u_S be the queue backlog corresponding to cooperative set S. The CMDB policy can now be expressed as

$$\max_{\boldsymbol{R} \in \mathcal{C}} \sum_{S \in \mathcal{U}} (u_a - |S| u_S) R_{aS} + |S| u_S R_{Sb}. \tag{30}$$

As before, the solution $\boldsymbol{R}^*$ to (30) lies in $\mathrm{conv}(\mathcal{C}_1, \mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*))$ for some $\boldsymbol{\alpha}^*$. Since $\mathcal{C}_1$ and $\mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*)$ are orthogonal and the objective is linear, $\boldsymbol{R}^*$ lies either in $\mathcal{C}_1$ or in $\mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*)$. We again consider two cases.

Case 1: $\boldsymbol{R}^*$ lies in $\mathcal{C}_1$. In the symmetric case, $\boldsymbol{R}^*$ takes the form $R_{aS^*}^* = \log(1 + h_{a1} P)$ and $R_{aS}^* = 0$ otherwise, where $S^* = \arg\max_{S \in \mathcal{U}} [u_a - |S| u_S]$. Thus, at any time, only one cooperative set (or direct link) is active. In the asymmetric case, due to the linear constraint in (27), (30) reduces to

$$\max_{(R_1,\ldots,R_n) \in \mathcal{C}_{BC}} \sum_{i=1}^{n} \left(\max_{S \in \mathcal{U}_i} [u_a - |S| u_S] \right) \cdot R_i. \tag{31}$$

Let $(R_1^*, \ldots, R_n^*)$ be the solution to (31). Then the optimal solution to (30) has the form $R_{aS^*}^* = R_i^*$ for $S^* = \arg\max_{S \in \mathcal{U}_i} [u_a - |S| u_S]$ and $R_{aS}^* = 0$ for all other $S \in \mathcal{U}_i$, $i = 1, \ldots, n$. That is, at any time, multiple cooperative sets (or direct links) can be active, but *each relay node i participates in only one cooperative set (or direct link)*, namely the cooperative set (or direct link) in $\mathcal{U}_i$ with the largest differential backlog $u_a - |S| u_S$. For a general broadcast region $\mathcal{C}_{BC}$, the optimization in (31) can be solved using the greedy technique from [24], [25]. Note that even though the number of variables $(R_{aS})_{S \in \mathcal{U}}$ in the original optimization (30) can be exponentially large in n, the actual resulting optimization problem in (31) is only n-dimensional.

Case 1: $\boldsymbol{R}^*$ lies in $\mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*)$. In this case, the optimization in (30) reduces to

$$\max_{(\mathbf{0}, R_{Sb})_{S \in \mathcal{U}} \in \mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*)} \sum_{S \in \mathcal{U}} |S| u_S R_{Sb} \tag{32}$$

As mentioned above, $\mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*)$ is potentially defined by a doubly exponential number of constraints. However, since $\mathcal{C}_{CMAC}(\boldsymbol{\alpha}^*)$ is a *polymatroid* [23], the maximization in (32) merely involves a *sorting* of the coefficients $|S|u_S$. The solution to (32) is then given by successively decoding the cooperative sets (or direct links) in increasing order of the coefficients $|S|u_S$ [23]. Since there are at most $2^n - 1$ coefficients $|S|u_S$, the maximization in (32) can be solved in $O(n)$ (linear) time.

V. Conclusions

In this paper, we considered throughput optimal control for a model of wireless networks with cooperative relaying and stochastically varying traffic. This model applies to a general network topology and several different types of cooperative scenarios. We established the network stability region and gave a variation of the Maximum Differential Backlog policy, which we proved to be throughput optimal. This policy is modified to incorporate the potential gains of cooperative communication. Specifically, under this policy the size of the cooperative set is included when determining the differential backlog metric. In this paper, we focused on a centralized implementation. In practice, a distributed solution is more desirable, particularly for managing the complexity of a cooperative network. Moreover, in a large network, there may be many potential cooperative sets. Allowing all of these would likely result in prohibitively high complexity. A useful direction for future work would be to develop a means for determining the most "fruitful" of these sets which are to be used. Finally, as we noted above, we considered only decode and forward models for cooperation. Incorporating other cooperative models into this framework is also of interest.

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