**In situ** detection of the temporal and initial phase of the second harmonic of a microwave field via incoherent fluorescence

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Measuring the amplitude and absolute (i.e., temporal and initial) phase of a monochromatic microwave field at a specific point of space and time has many potential applications, including precise qubit rotations and wavelength quantum teleportation. Here we show how such a measurement can indeed be made using resonant atomic probes via detection of incoherent fluorescence induced by a laser beam. This measurement is possible due to self-interference effects between the positive- and negative-frequency components of the field. In effect, the small cluster of atoms here act as a highly localized pickup coil, and the fluorescence channel acts as a transmission line.

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\[ C_1(t) = i e^{-i(\omega t + \phi)} \{ \sin[g_0(t)/2] + 2 \eta \Sigma \cos[g_0(t)/2] \}, \]

where \( \Sigma = (i/2) \exp[-i(2\omega t + 2\phi)] \) and \( g_0(t) = (1/|t|) \int |g_0(t')| dt' = g_0(1 - (t/\tau_R)^2)^{-1/2} \exp[(t/\tau_R)^2]. \) If we produce this excitation using a \( \pi/2 \) pulse [i.e., \( g_0(t) = \pi/2 \)] and measure the population of state |1\> after the excitation terminates (at \( t = \tau \)), we get a signal

\[ |C_1(t, \phi)|^2 = 1/2 + \eta \sin[2(\omega \tau + \phi)]. \]

This signal contains information of both the amplitude and phase of the field \( B(t) \). The second term of Eq. (3) is related to the Bloch-Siegert shift [7,8], and we have called it the Bloch-Siegert oscillation (BSO) [2,3]. It is attributable to an interference between the so-called corotating and counter-rotating parts of the oscillating field, with the atom acting as the nonlinear mixer. For \( \eta = 0 \), we have the conventional Rabi flopping that is obtained with the RWA. For a stronger coupling field, where the RWA is not valid, the second term of Eq. (3) becomes important [2,3], and the population will depend now both on the Rabi frequency and the phase of the driving field. In recent years, this effect has also been observed indirectly using ultrashort optical pulses [9–11] under the name of carrier-wave Rabi flopping. However, to the best of our knowledge, the experiment we report here represents the first direct, real-time observation of this effect.

From the oscillation observed, one can infer the value of \( 2(\omega t + \phi) \), which represents the absolute phase of the second harmonic. This is equivalent to determine the absolute phase of the fundamental field, \( (\omega t + \phi) \), modulo \( \pi \). In principle, a simple modification of the experiment can be used to eliminate the modulus \( \pi \) uncertainty. Specifically, if one applies a dc magnetic field parallel to the rf field, it leads to a new oscillation (in the population of either level) at the fundamental frequency, with exactly the same phase as that of the driving field. In the experiment described here, we have restricted ourselves to the case of determining the absolute phase of the second harmonic only.
levels are set by the time of flight of the individual atoms in the rf field be written as by subtracting the population amplitude of each level numerically by solving the Schrödinger equation for the shown in Fig. 2, uses a thermal, effusive atomic beam. The rf field is shown in Fig. 1. The BSO oscillations for all the levels of such a system are the RWA from the population amplitude shown in the population dynamics of such a system as described below.

Consider an equally spaced, ladder-type three-level system (0), (1), and (2)). The transition frequencies for (0)-(1) and (1)-(2) are of the same magnitude ε. We also consider that a direct transition between (0) and (2) is not allowed. Now, let the system be pumped by the same field at a frequency ω. Consider also that the Rabi frequency for the (0)-(1) transition is g0 and that for (1)-(2) is also g0. Then, the Hamiltonian of the three-level system in a rotating frame can be written as

$$\hat{H} = -g_0[1 + \exp(-i2\omega t - i2\phi)]|0\rangle\langle 1| + |1\rangle\langle 2| + \text{c.c.},$$

where $\omega = \varepsilon$. The amplitudes of the three levels are calculated numerically by solving the Schrödinger equation for the above Hamiltonian. The BSO amplitudes are then calculated by subtracting the population amplitude of each level with the RWA from the population amplitude without the RWA. The BSO oscillations for all the levels of such a system are shown in Fig. 1.

The experimental configuration, illustrated schematically in Fig. 2, uses a thermal, effusive atomic beam. The rf field is applied to the atoms by a coil, and the interaction time $\tau$ is set by the time of flight of the individual atoms in the rf field before they are probed by a strongly focused and circularly polarized laser beam. The rf field couples the sublevels with $|\Delta n| = 1$, as detailed in the inset of Fig. 2. Optical pumping is employed to reduce the populations of states |1⟩ and |2⟩ compared to that of state |0⟩ prior to the interaction with the microwave field.

A given atom interacts with the rf field for a duration $\tau$ prior to excitation by the probe beam that couples state |0⟩ to an excited sublevel in $5^2S_{1/2}$. The rf field was tuned to 0.5 MHz, with a power of about 10 W, corresponding to a Rabi frequency of about 4 MHz for the |0⟩ → |1⟩ as well as the |1⟩ → |2⟩ transition. The probe power was 0.5 mW focused to a spot of about 30 µm diameter, giving a Rabi frequency of about 60 Γ, where Γ(6.06 MHz) is the lifetime of the optical transition. The average atomic speed is 500 m/s, so that the effective pulse width of the probe, $\tau_{LP}$, is about 60 ns, which satisfies the constraint that $\tau_{LP} \approx 1/\omega$. Note that the resolution of the phase measurement is essentially given by the ratio of $\min[\tau_{LP}, 1/\Gamma]$ and $1/\omega$, and can be increased further by making the probe zone shorter. The fluorescence observed under this condition is essentially proportional to the population of level |0⟩, integrated over a duration of $\tau_{LP}$, which corresponds to less than 0.3 Rabi period of the rf driving field [for $g_{0\text{ff}}/(2\pi) = 4$ MHz]. Within a Rabi oscillation cycle, the BSO signal is maximum for $g_0(\tau + n\pi)/2 = (2n+1)\pi/2$, where $n = 0, 1, 2, \ldots$, so that there is at least one maximum of the BSO signal within the region of the probe.
When the rf intensity is increased a component of the BSO at the probe beam is blocked, there is no signal magnetic field applied in the the BSO signal amplitude varies as a function of an external each of the three ground Zeeman sublevels. We observed that of level information is not clearly present, since the total population probed. When the probe is tuned to \( F \leftrightarrow \) is locked to the \( F \) is a constant. The observed residual phase in-

Note that atoms with different velocities have different interaction times with the rf field and produce a spread in the BSO signal amplitude within the probe region. However, the phase of the BSO signal is the same for all the atoms, since it corresponds to the value of \( \langle \omega r + \phi \rangle \) at the time and location of interaction. Thus, there is no washout of the BSO signal due to the velocity distribution in the atomic beam.

Figure 3 shows the spectrum of the observed BSO signal. In Fig. 3(a), we show that the BSO stays mainly at 2\( \omega \) when the probe beam is blocked, there is no signal [Fig. 3(b)]. When the rf intensity is increased a component of the BSO at 4\( \omega \) begins to develop, as predicted. For the data in Fig. 4, the second harmonic of the driving field is used to trigger an 100-MHz digital oscilloscope and the fluorescence signal is averaged 256 times. When the probe beam is tuned to the \( F=1 \leftrightarrow F'=0 \) transition, the population at \( m=-1 \) state is probed. When the probe is tuned to \( F=1 \leftrightarrow F'=1 \), the combined populations of \( m=-1 \) and \( m=0 \) states are probed. That results in an effective detection of the complement of the population of \( m=1 \). On the other hand, when the probe beam is locked to the \( F=1 \leftrightarrow F'=2 \) transition, all three Zeeman sublevels of \( F=1 \) are simultaneously probed and the phase information is not clearly present, since the total population of level \( F=1 \) is a constant. The observed residual phase information is a result of different coupling efficiencies for each of the three ground Zeeman sublevels. We observed that the BSO signal amplitude varies as a function of an external magnetic field applied in the \( \hat{z} \) direction, with a peak corresponding to a Zeeman splitting matching the applied frequency of 0.5 MHz.

In Fig. 5, we show that the fluorescence signal is phase locked to the second harmonic of the driving field. First, we placed a delay line of 0.4 \( \mu s \) on the cable of the reference field used to trigger the oscilloscope and recorded the fluorescence [Fig. 5(a)]. Then, we put the 0.4-\( \mu s \) delay line on the BSO signal cable and recorded the fluorescence [Fig. 5(b)]. The phase difference between the signals recorded in Figs. 5(a) and 5(b) is approximately 0.8 \( \mu s \), as expected for a phase locked fluorescence signal. The data presented were for the probe resonant with the transition \( F=1 \leftrightarrow F'=1 \), but the same results were observed for \( F=1 \leftrightarrow F'=0 \).

To summarize, we report the first direct observation of the absolute phase of the second harmonic of an oscillating elec-

FIG. 5. Demonstration of phase-locked fluorescence. \( T \) is the period of the Bloch-Siegert oscillation. (a) Population vs time when a 0.47 delay line was inserted in the reference field cable. (b) Population vs time when the same 0.47 delay line was placed in the fluorescence signal cable. The figure shows that signal (b) is about 0.87 ahead of signal (a), confirming that the atomic fluorescence carries phase information which is locked to the absolute rf field phase. The solid and dashed sinusoidal smooth curves are fittings to the experimental data and were used for period and delay determination.
tromagnetic field using self-interference in an atomic resonance. This process is important in the precision of quantum bit rotations at a high speed. The knowledge of the absolute phase of a rf field at a particular point of space may also be useful for single-atom quantum optics experiments. For example, an extension of this concept may possibly be used to teleport the wavelength of an oscillator, given the presence of degenerate distant entanglement, even in the presence of unknown fluctuations in the intervening medium [4–6,12]. Finally, this localized absolute phase detector may prove useful in mapping of radio-frequency fields in microcircuits. Although a particular alkali-metal atom was used in the present experiment, the mechanism is robust and could be observed in virtually any atomic or molecular species.

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