

Comment on "The mapping of the Coulomb problem into the oscillator," by David S. Bateman, Clinton Boyd and Binayak Dutta-Roy [Am. J. Phys. 60 (9), 833–836 (1992)].

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In the above paper,¹ the authors (David S. Bateman *et al.*) have reviewed the mapping² of the 3D and 2D Schrödinger equations for the Coulomb potential problems into the Schrödinger problems for the 4D and 2D harmonic oscillator, respectively, and then claimed that the 1D Coulomb potential problem can be mapped into the radial part of the N -dimensional harmonic oscillator with proper choice of N (the dimensionality) and l (the angular momentum quantum number of the N -dimensional oscillator).

However, the authors have made serious algebraic error for the 1D oscillator which invalidates their results for the 1D oscillator as will be shown below. And also we have shown that the 1D Coulomb problem cannot be mapped into the N -dimensional harmonic oscillator.

The 1D Schrödinger equation for the Coulomb potential is given by

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} - \frac{e^2}{|x|} \psi(x) = -B\psi(x), \quad (1)$$

where B is the absolute value of the energy of the bound states.

And then under the transformations,

$$x = \rho^2 \quad \text{and} \quad \psi = \rho^{1/2} \phi. \quad (2)$$

Equation (1) transforms to

$$-\frac{\hbar^2}{2m} \left(\frac{d^2}{d\rho^2} - \frac{3}{4\rho^2} \right) \phi + 4B\rho^2\phi = 4e^2\phi. \quad (3)$$

The radial part of the Schrödinger equation for the N -dimensional harmonic oscillator³ can be written as (3),

$$-\frac{\hbar^2}{2m} \left(\frac{d^2}{dr^2} - \frac{(k-1)(k-3)}{8r^2} \right) u(r) + \frac{1}{2} m \omega^2 r^2 u(r) = E u(r), \quad (4)$$

where $k = N + 2l$.

Thus a comparison of Eqs. (3) and (4) shows that they are identical whenever

$$(k-1)(k-3) = 6, \quad (5)$$

or $k = 2 \pm 7^{1/2}$ and the identification $E = 4e^2$ and $\frac{1}{2}m\omega^2 = 4B$ is made.

The authors¹ claimed that Eqs. (3) and (4) matches for $k=4$, which is wrong and further calculation of the energy for the 1D harmonic oscillator is also incorrect.

However, for the N -dimensional harmonic oscillator $k (= N + 2l)$ is algebraic sum of the two integers and itself constrained to be an integer, a condition which is not met by the solution of Eq. (5) for $k (= 2 \pm 7^{1/2})$. In fact k is an irrational number. And any further transformations of ϕ and ρ will not make value of k as an integer.

We therefore conclude that the claim of the authors that the 1D Coulomb problem can be mapped into the radial equation for the N -dimensional harmonic oscillator is erroneous and in fact no such mapping exists.

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¹David S. Bateman, Clinton Boyd, and Binayak Dutta-Roy, "The mapping of the Coulomb problem into the oscillator," Am. J. Phys. **60**, 833–836 (1992), and references therein.

²F. J. H. Cornish, "The Hydrogen Atom and the Four-Dimensional Harmonic Oscillator," J. Phys. A **17**, 323–327 (1984).

³A. Chatterjee, "Large N Expansion in Quantum Mechanics," Phys. Rep. **186**, 249–370 (1990).

FEYNMAN'S GOD

[Feynman] believed that science and religion are natural adversaries. Einstein said, "Science without religion is lame; religion without science is blind." Feynman found this style of accommodation to be intolerable. He repudiated the conventional God: "the kind of a personal God, characteristic of Western religions, to whom you pray and who has something to do with creating the universe and guiding you in morals." Some theologians had retreated from the conception of God as a kind of superperson—Father and King—willful, white-haired, and male. Any God who might take an interest in human affairs was too anthropomorphic for Feynman—implausible in the less and less human-centered universe discovered by science.

James Gleick, *Genius—The Life and Science of Richard Feynman* (Pantheon, New York, 1992), p. 372.