NORTHWESTERN UNIVERSITY Department of Electrical and Computer Engineering

ECE-221 Fundamentals of Circuits

LAB 7: FILTERS

Much of the material in this lab is discussed in chapter 13 in the textbook – *Engineering Circuit Analysis*, by W. H. Hayt, Jr. and J. E. Kemmerly (McGraw-Hill, 1993).

Any periodic signal can be represented as the superposition of sinusoids at appropriate frequencies. As a result, the frequency response of a given circuit tells us much about the response of that circuit to arbitrary periodic signals. A filter is a **two-port network** which is designed to have a specific frequency response between input and output, known as the transfer function (see figure below), and denoted by $\mathbf{H}(\mathbf{s})$ NOTE: $(\mathbf{s} = j \mathbf{w})$.



In this lab, we will measure the response of two different filters to a sinusoidal input of varying frequency, and compare our results with what is predicted by theory. More specifically, we will produce theoretical and experimental plots of the frequency response $H_{dB}(\mathbf{w})$ (dB denotes decibel scale), where

$$H_{dB}(\mathbf{w}) = 20 \log_{10}(|\mathbf{H}(j\mathbf{w})|).$$

1: Theory

For each of the two filters we will obtain $H_{dB}(\mathbf{w})$ using SPICE.

(a) The first filter to be considered in this lab is the familiar series RC circuit.



- 1.a.1 Construct the above diagram in the PSPICE Schematic grid. Replace Vin with an AC source (VAC). Set the magnitude to 5V. From the top menu, select <u>Markers and choose Mark Advanced</u>. Select part vdb and place it at Vout (+), then the output voltage is in the unit of dB instead of voltages. Connect a GND_EARTH to Vin(-) / Vout (-). From the Analysis Setup, click on AC Sweep. In AC sweep type, select 'Decade' to make the frequency axis in the logarithm scale. Make sure that <u>Start Freq</u>: and <u>End Freq</u>: encompass the range 100 < f < 10,000,000 Hz. Exit Analysis Setup, and run the sweep. Print out the resulting plot for your report.
- (b) The second filter for this experiment has a complicated geometry, but its transfer function has only two poles.



Note that this filter contains two T structures, each of which is connected to the same three nodes. If we perform a T- Π transformation on both, and add the resulting impedances in parallel, we obtain the following equivalent circuit:



wherein

$$\mathbf{Z}_{a} = 500 + \frac{1.25 \times 10^{7}}{\mathrm{s}}$$
$$\mathbf{Z}_{b} = \frac{6 \times 10^{8} \mathrm{s} + 1.5 \times 10^{13}}{9 \mathrm{s}^{2} + 2.5 \times 10^{9}}$$
$$\mathbf{Z}_{c} = \frac{4000 \mathrm{s} + 10^{8}}{3 \mathrm{s}}$$

At this point, the transfer function can be obtained simply through the voltage divider equation.

$$\mathbf{H}_{b}(\mathbf{s}) = \frac{\mathbf{Z}_{c}}{\mathbf{Z}_{b} + \mathbf{Z}_{c}} = \frac{9\mathbf{s}^{2} + 2.5 \times 10^{9}}{9\mathbf{s}^{2} + 4.5 \times 10^{5} \mathbf{s} + 2.5 \times 10^{9}}$$

- 1.b.1 Solve for the poles (denominator = 0) and zeroes (numerator = 0) of $\mathbf{H}_b(\mathbf{s})$. What do the poles and zeroes tell us about the behavior of $H_{b \, dB}(\mathbf{w})$?
- 1.b.2 Plot $H_{b,dB}(\mathbf{w})$ just as you plotted $H_{b,dB}(\mathbf{w})$ in part (a).

2: Experiment

- 2.1 Connect the circuit for filter (a) as pictured above. Attach the function generator to the input terminals, and use the oscilloscope to observe both the input and output voltages. Record sufficient data to plot $H_{a,dB}(\mathbf{w})$ for ten frequencies in the range 10 < f < 4,500 Hz. Suggested frequency values are already entered into the lab worksheet.
- 2.2 Repeat the above procedure for filter (b). Use 12 frequencies in the range 10 < f < 16,000 Hz. Suggested frequency values are already entered into the lab worksheet.

3: Analysis

3.1 Comment on the degree to which your theoretical and experimental results are in agreement. What do you think are the most significant sources of error? What properties of these two filters have we not investigated that have a significant effect on their performance? What is a desirable state for these properties to ensure optimum filter performance?