

NORTHWESTERN UNIVERSITY
Department of Electrical and Computer Engineering

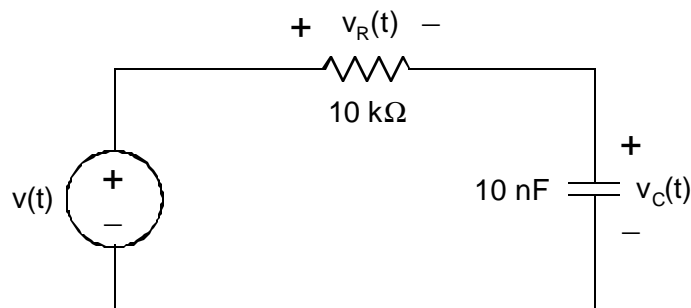
ECE-221 Fundamentals of Circuits

LAB 5: FREQUENCY RESPONSE OF RC CIRCUITS

This lab deals with the frequency domain. These several concepts are included: Transfer Function $H(j\omega)$, cut off frequency f_c , and phasor diagrams. These concepts ought to have already been introduced in lectures. What is provided here is intended to be a supplement.

1. Theory

Consider the following series RC circuit.



We wish to determine the responses $v_R(t)$ and $v_C(t)$ to a sinusoidal input $v(t)$. We will do this by analyzing the circuit in the frequency domain and then converting our results back into the time domain. When working in the frequency domain we are concerned with the complex impedance of a circuit element. We must use complex number arithmetic.

- 1.1** Determine the transfer functions $H_R(j\omega) = \frac{V_R(j\omega)}{V(j\omega)}$ and $H_C(j\omega) = \frac{V_C(j\omega)}{V(j\omega)}$ for the above circuit by using voltage division and express each as a rational polynomial in $j\omega$.

In this context, the transfer function can be thought of as the fraction of the total amount of voltage from an AC source that is “transferred” to a particular circuit element. The magnitude of this fraction is a function of frequency. Thus, we refer to a transfer function $H(j\omega)$. To obtain $H_R(j\omega)$ and $H_C(j\omega)$, use voltage division to find the voltage across the resistor and the capacitor, respectively, and then divide each voltage [$V_R(j\omega)$ and $V_C(j\omega)$] by the total voltage $V(j\omega)$.

- 1.2** For $v(t) = 5 \cos(500t)$, $R=10k\Omega$ and $C=10nF$, determine $V_R(j\omega)$ and $V_C(j\omega)$.
First convert $v(t)$ into a phasor or complex number, and multiply by your transfer functions.
- 1.3** In previous labs you observed capacitors and resistors in the time domain. It was noted that the voltage across a resistor can change instantaneously, but that capacitor voltages have rise and fall time. An AC source alternates between positive and negative voltages. For high frequencies the voltage level of the source alternates faster than the capacitor voltage is able to change. The higher the frequency the smaller the voltage-drop across the capacitor. Therefore, the impedance of a capacitor ($1/j\omega C$) decreases with increasing frequency. Earlier in this course you dealt with the RC time constant ($\tau = RC$). The time constant has units of seconds. Find τ for this circuit. There is a radian frequency ($\omega_c = 1/\tau$) known as the cut off frequency ω_c . At ω_c the magnitude of the voltage across the capacitor is $2^{-1/2}$ times its maximum value. Substitute ω_c for ω and verify that your transfer functions from 1.1 reduce to a phasor with magnitude $2^{-1/2}$.
- 1.4** Give the time domain responses $v_R(t)$ and $v_C(t)$ from the phasors in **1.2**. That is, convert your phasors ($|V_R(j\omega)| \angle \mathbf{f}_R(j\omega)$ and $|V_C(j\omega)| \angle \mathbf{f}_C(j\omega)$) into cosine functions $v_R(t)$ and $v_C(t)$.
- 1.5** By KVL, $V(j\omega) = V_R(j\omega) + V_C(j\omega)$. Draw a phasor diagram, which demonstrates this relationship for frequency $\omega=500$.

2. Procedure

We will now attempt to verify experimentally each of the above theoretical results. Connect the circuit as shown above, making sure that source drives the circuit with the function $5\cos(2\pi ft)$. After setting this, only the frequency should be varied throughout the experiment.

- 2.1** Using the frequencies provided, fill in the data table on the worksheet. They range from 100–5000Hz. Your corner frequency, f_c , must be added to your list. Set the function generator to the first frequency on the list, using the scope to verify the period of the waveform. Measure $|V_R|$, $|V_C|$, \mathbf{f}_R , and \mathbf{f}_C directly from a dual trace display on the scope. The relative phase angle \mathbf{f} between the two input signals of the oscilloscope can be measured by first pressing the time button, twice selecting the “next menu” option (lower right corner of the screen), and finally choosing phase. \mathbf{f}_R is the relative phase between $V(j\omega)$ and $V_R(j\omega)$. Likewise, \mathbf{f}_C is the relative phase between $V(j\omega)$ and $V_C(j\omega)$. Repeat this process for each of the frequencies on your list.

- 2.2 Put the scope in the x - y mode. Connect the probes such that the scope displays $v(t)$ horizontally and either $v_R(t)$ or $v_C(t)$ vertically (a Lissajous figure). Note that because we have put the scope in the x - y mode, we cannot measure the frequency of any of these waveforms, and will therefore set the frequency of the function generator using its display only. For frequencies of 800Hz, f_c , and 2000Hz, observe the Lissajous figures and record sufficient data to produce phasor diagrams like those of section 1.4. The phase difference can be determined from the Lissajous figure by:

$$\sin \phi = \text{second } DY / \text{first } DY$$

First DY is the change in Y between the greatest and least Y value on the Lissajous figure.

Second DY is the change in Y between the two Y -intercepts. [See the Oscilloscope User and Service guide (2-36) for more information.]

- 2.3 Adjust the frequency until the voltage is at $\mathcal{Z}^{1/2}$ times its maximum value. Record this frequency and compare it with the f_c that you calculated in 1.3.

3. Analysis

- 3.1 Use the results of section 1.2 (without plug in the frequency) to produce plots of the theoretical magnitude responses $|V_R(j\omega)|$ and $|V_C(j\omega)|$ and theoretical phase responses $\phi_R(j\omega)$ and $\phi_C(j\omega)$ versus ω . Use a separate pair of axes for each. Compare these plots to what was plotted in 2.1. What are the differences and why do you think they occurred?