NORTHWESTERN UNIVERSITY

Department of Electrical and Computer Engineering

ECE-221 Fundamentals of Circuits

LAB 4: SECOND-ORDER TRANSIENT RESPONSES

Part A: Second order transient response

A1. Series RLC Filter

Theory

Consider the following series RLC circuit.

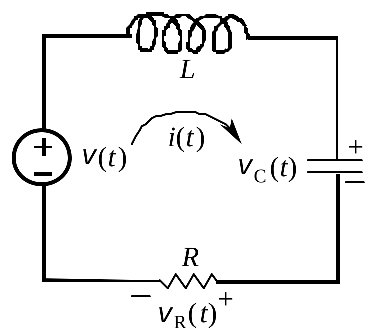

Figure 1

We will determine the complete step response $v_R(t)$ through analysis of this circuit. The fundamental integro-differential equation for this series circuit is

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int_{t_0}^{t} i\,dt - v(t) = 0$$

1.1) Carry out partial differentiation of this equation with respect to t to get a second-order differential equation.

Clearly there are three cases of interest. Depending on the choice of R, L, and C, the response may be underdamped, critically damped, or overdamped.

1.2) Write down the general form of the solution for $i(t)$ for all three cases.

1.3) What are a, w_0, and w_d for this circuit? Let ζ be defined as the damping ratio, $\zeta = a/w_0$. Express this ratio in terms of R, L, and C. Write down the exact step response $i(t)$ for the underdamped case.

As we can see from the underdamped step response, the series RLC circuit exhibits resonant behavior. The **quality or *Q* factor** of any resonant system is a ratio of the maximum energy that can be stored by the system to the rate at which energy is lost. For the series RLC circuit the *Q* factor can be expressed in two equivalent ways.

$$Q_0 = \frac{1}{2z} = \frac{w_0 L}{R}$$

1.4) For the underdamped circuit, compute t_0, which is defined to be the smallest value of $t > 0$ for which $v_R(t) = 0$.

1.5*) Imagine this is a scope trace with which you can only measure $\boldsymbol{t_0}$ and $\boldsymbol{v_R(t_0/2)=V}$ in the RLC underdamped series circuit with **Vs×u(-t)** as the voltage source. Can you think of a way to use the above measurements and the values of Vs, R, L and C to calculate w_0 and the general solution for the **i(t)** (without using the definition of a and w_0)?

Procedure

1.6) Choose values for *R, L, C* from the following sets: R = {1200, 2000, . . .4000}Ω; L = 10mH; C = 0.01μF, so that the following equations are satisfied: w_0 = 100,000 rad/s; ζ= 0.6, 1.0, 1.5, and 2.0.

For each value of ζ, you should choose *R* a unique value. *L* and *C* should remain constant.

1.7) Construct the series RLC circuit, using the values computed above, and a square wave (**Vs=5V**) of sufficiently long period (**f<5kHz**) in place of $v(t)$. By using the data ζ= 0.6 and corresponding R, L and C, calculate A1 and A2 for your theoretical $v_R(t)$, plot the waveform.

Questions

1.8) For each value of *R*, observe the step response $v_R(t)$, and record sufficient data to plot the four waveforms on the same set of axes. Classify each response as underdamped, critically damped, or overdamped, based on the appearance of the associated plot. Do these classifications agree with those predicted by theory? Choose the case when ζ= 2 and plot the theoretical and measured results on the same set of axes

1.9) Suppose that the positions of *L* and *R* were interchanged in the circuit. What practical difficulty would you then face in observing $v_R(t)$ on the oscilloscope? How would you solve this problem?

Part B: OpAmps in PSPICE

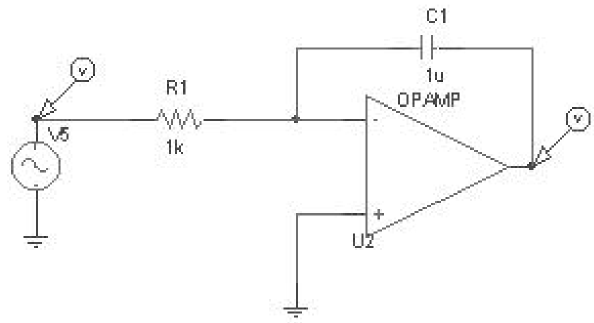

Figure 2

Theory

2.1) The circuit of Fig. 2 consists of a sinusoidally varying voltage source (VSIN) feeding through a resistor and op-amp. Model this circuit and do the transient analysis in PSpice using the following values:

For VSIN DC=0, AC=5, VOFF=0, VAMPL=5, and FREQ=1k

R=1000Ω C=1μF

Final time = 2ms

2.2) Repeat 2.1 but switch the positions of the resistor and capacitor. Observe the results.

For OPAMP VPOS = +50V, VNEG = -50V

R=1000Ω C=1μF

Final time = 2ms

2.3) From these simulations, which would you say is the “integrator” and which is the “differentiator?” Please explain why.