NORTHWESTERN UNIVERSITY
Department of Electrical and Computer Engineering

ECE-221 Fundamentals of Circuits

LAB 3: FIRST-ORDER TRANSIENT RESPONSES

Part A: First order transient response

A1. Transient Response of a Series RC Circuit

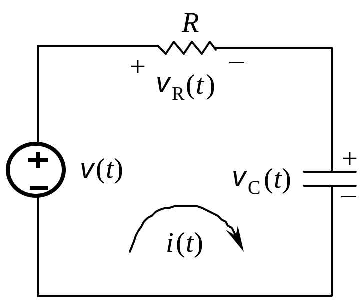

Figure 1

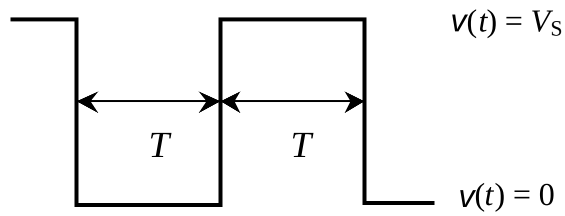

Figure 2

Theory

Consider the series RC circuit of Fig. 1. We will first determine a linear first-order differential equation describing $v_C(t)$ of the form

$$\dot{x}(t) + Px(t) = Q(t)$$

and then solve it by way of the integrating factor method. If we multiply both sides of this equation by

$$e^{\int P dt} = e^{Pt}$$

and make use of the product rule

$$\frac{d}{dt}(e^{Pt}x(t)) = e^{Pt}\dot{x}(t) + Pe^{Pt}x(t)$$

we obtain the following expression

$$d(e^{Pt}x(t)) = e^{Pt}Q(t)dt$$

After taking the indefinite integral of both sides,

$$e^{Pt}x(t) = \int e^{Pt}Q(t)dt + K$$

we obtain an algebraic expression for $x(t)$ in terms of a general forcing function $Q(t)$ (assuming we can perform the necessary integration).

1.1) Write down the following equations: a KVL equation relating $v(t)$, $v_R(t)$, and $v_C(t)$; Ohm's law for the resistor, and a differential equation relating $v_C(t)$ and the loop current $i(t)$.

1.2) Combine the above equations to obtain a linear, first-order differential equation for $v_C(t)$ in terms of $v(t)$, R, and C.

1.3) Use the integrating factor method to produce an expression for $v_C(t)$ in terms of R, C, and a general forcing function $v(t)$. Show all your work.

1.4) Let $v(t) = V_S\, u(t)$, where $u(t)$ is the unit step function. Evaluate your expression for $v_C(t)$ under this special case. Use KVL to determine an expression for $v_R(t)$.

Procedure

We will now compare the theoretical step response of an ideal RC circuit with that measured on a real circuit, which is not ideal. Design a circuit like that of Fig. 1 to have a time constant of 0.1ms. Because we do not have a source that produces the unit step function, we will drive the circuit with the unipolar square wave of Fig. 2. Choose values for $\boldsymbol{V_S}$ **= 5V** and $\boldsymbol{T}$ **= 0.2ms**. Set the function generator to produce a square wave as in Fig. 2, and connect it to the circuit. Observe $v_R(t)$, and $v_C(t)$ and record 10 points in the time scale to produce a graph of each of them over one period of the input.

1.5) Plot $v_R(t)$ and $v_C(t)$. Each plot should show the theoretical and measured waveforms on the same set of axes. List all the possible sources of discrepancy between your theoretical and measured results.

1.6) Is the time constant of your real circuit equal to 0.1ms? If not, is the time constant less than or greater than this value?

1.7) Does $v(t) = v_R(t) + v_C(t)$ for the real circuit? Is the error significant?

A2. Transient Response of a First-Order RC Filter

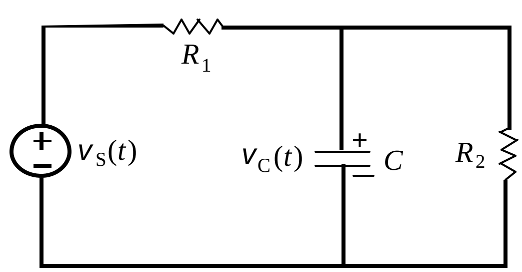

Figure 3

Theory

2.1) Now consider the circuit of Fig. 3. Write down a linear first-order differential equation for $v_C(t)$ in terms of R_1, R_2, C, and $v(t)$.

2.2) Obtain an expression for $v_C(t)$ in terms of R_1, R_2, C, and a general forcing function $v(t)$. Show all your work.

2.3) Again let $v(t) = V_S\, u(t)$. Evaluate your expression for $v_C(t)$ for this special case.

Procedure

Again we will compare theory with practice. Design a circuit like that of Fig. 3 to have a time constant of 0.1ms. Choose values for amplitude V_S = 5V and T = 0.2ms, set the function generator to produce a square wave as in Fig. 2, and connect it to the circuit. Observe the capacitor voltage waveform, and record 10 points in the time scale to produce a graph over one period of the input.

2.4) Plot the theoretical and measured waveforms for $v_C(t)$ on the same set of axes.

2.5) Consider the capacitor voltage waveforms for each of the first two circuits. Is there more agreement between theory and practice in circuit 1 or circuit 2, or is it approximately the same?

Part B: Second order transient response

B1. Second-Order RC Filter

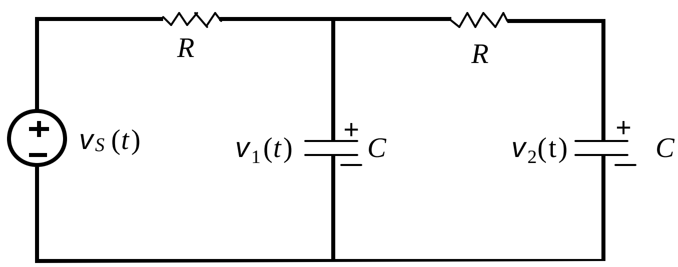

Figure 4

Theory

3.1) The circuit of Fig. 4 consists of two identical RC stages connected in cascade. For each of the two loops, write down two equations: one KVL, and one differential equation relating a capacitor voltage to the loop currents

3.2) Construct a PSpice model of this circuit and simulate a step up in voltage by using **Vpulse** as a voltage source. Measure the voltage across the second capacitor $v_2(t)$ and print out the resulting graphs. In options, choose transient from 0 to a suitable time and run the simulation with the following resistance and capacitance values:

R=1000Ω C=0.2μF

R=500Ω C=0.05μF

Procedure

Connect the circuit of Fig. 4. Use values for R, C, V_s, and T identical to those used for the circuit of Fig. 1. Observe $v_2(t)$ and collect 10 points to graph one period of the waveform.

3.3) Plot the theoretical waveform in PSpice and measured waveform for $v_2(t)$ in the excel spreadsheet.