

Limited Feedback for Multi-Carrier Beamforming: A Rate-Distortion Approach

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Abstract—The achievable rate of a wideband Multi-Input Single-Output channel with multi-carrier transmission is studied with limited feedback of Channel State Information (CSI). The set of sub-channel vectors are assumed to be jointly quantized and relayed back to the transmitter. Given a fixed feedback rate, the performance of an optimal joint quantization scheme can be characterized by the rate-distortion bound. The distortion metric is the average loss in capacity (forward rate) relative to the capacity with perfect channel state information at the transmitter and receiver. The corresponding rate-distortion function gives the forward capacity as a function of feedback rate, and is determined explicitly by casting the minimization of mutual information as an optimal control problem. Numerical results show that when the feedback rate is small, the rate-distortion bound significantly outperforms separate vector quantization of each sub-channel vector. Practical issues, such as quantization codebook design and implementation, are also briefly discussed.

I. INTRODUCTION

The performance of a multi-antenna communication system can be substantially increased when Channel State Information (CSI) is made available at the transmitter. For a Multi-Input Single-Output (MISO) channel this allows transmit beamforming, which can both increase the rate and potentially decrease interference to neighboring receivers. However, the associated gains are compromised when the CSI is inaccurate due to a limited-rate feedback channel.

The design and analysis of limited feedback techniques for beamforming has been extensively studied over the past few years (e.g., see [2]–[11] and the broad overview of work on limited feedback for wireless systems in [1]). Most of the previous work on limited-feedback beamforming is based on a narrow-band fading model. Optimal beamforming codebooks can be designed, in principle, by using the Lloyd algorithm. This was first proposed for narrowband beamforming in [2], and further analyzed in [7], [9]. Other approaches to codebook design have been proposed based on Grassmanian packings [4] and Random Vector Quantization (RVQ) [?], [6], [11].

Here we study the performance of limited-feedback beamforming with a wideband fading channel. Specifically, we consider multi-carrier transmission over a wideband MISO channel where the MISO sub-channel gains are assumed to be *i.i.d.* Given N_t transmit antennas and N sub-channels (assumed to be constant with frequency), the CSI then consists of the set of N *i.i.d.* sub-channel vectors (each $N_t \times 1$). The

general problem is then how to design an optimal codebook (in some sense) to represent the N beams, and how to characterize the associated performance. As in most prior work, we will assume that the CSI is perfectly known at the receiver, and a noiseless, limited-rate, zero-delay feedback link relays the CSI back to the transmitter.

To maximize the achievable forward rate given a fixed feedback rate, the set of sub-channel vectors should be *jointly* quantized. That is, each entry of the quantization codebook should contain a complete set of N vectors (each $N_t \times 1$), as opposed to quantizing each of the N sub-channel vectors separately. The asymptotic performance of an optimal joint quantization scheme as N becomes large (with fixed N_t) is given by rate-distortion theory. The source is the set of sub-channel vectors (equivalently, transmit beams across sub-channels) and the distortion metric is the difference in forward rates achieved with quantized and perfect beams, respectively. We assume an *i.i.d.* block Rayleigh fading model, and take the achievable rate to be ergodic capacity.

To calculate the rate-distortion function, the minimization of mutual information over the conditional density of the sub-channel vector, given the quantized vector, is cast as an optimal control problem. We further simplify the problem by quantizing only the sub-channel directions (ignoring the magnitudes), and applying a uniform power distribution over the sub-channels, which is near-optimal at all but very low SNRs. The resulting problem can be solved, giving an explicit characterization of achievable forward rate versus feedback rate. Numerical results show that joint quantization of the sub-channel vectors can provide a significantly higher forward rate than separate quantization of each vector.

Related work on multi-carrier beamforming with correlated sub-channels is considered in [12]. There an interpolation approach is used to extend quantization of single vectors for narrowband channels to the wideband scenario. ****do they assume uniform power also?*** Here our focus is on demonstrating the benefits of optimal (joint) quantization relative to separate quantization of each vector. We consider *i.i.d.* sub-channels primarily for tractability, and because the performance gains shown here serve as a lower bound on the corresponding gains with correlated sub-channels. (Also, with *i.i.d.* sub-channels interpolation cannot improve the performance of separate sub-channel quantization.)

Rate-distortion results for narrowband limited-feedback beamforming are presented in [2], [7], although the distortion metrics are different from that considered here. Also, the previous approaches to codebook design rely on an exhaustive search of the quantization codebook. For the wideband MISO channel model considered with a large number of *i.i.d.* sub-channels, this is clearly impractical even for relatively small feedback rates. We therefore briefly discuss the possibility of using graphical based codes, which can approach the rate-distortion bound with reasonable complexity.

II. MULTI-CARRIER MISO MODEL

Consider a single-user MISO channel with N_t transmit antennas ($N_t \geq 2$ is assumed) and one receive antenna. Suppose there are N *i.i.d.* sub-channels and denote the i -th sub-channel vector as $\mathbf{h}_i^\dagger = [h_{i1} \ h_{i2} \ \dots \ h_{iN_t}]$, which is assumed to be *i.i.d.* complex Gaussian with unit variance per complex dimension, where $(\cdot)^\dagger$ denotes Hermitian transpose. For sub-channel i we assume that the transmitter array transmits a single symbol x_i . Before transmission on antenna k , the symbol is weighted by a complex number \hat{v}_{ik} . Then the vector $\hat{\mathbf{v}}_i = [\hat{v}_{i1} \ \hat{v}_{i2} \ \dots \ \hat{v}_{iN_t}]^T$ is the beamforming vector on sub-carrier i with power constraint $\|\hat{\mathbf{v}}_i\|^2 = 1$, where $(\cdot)^T$ denotes transpose and $\|\cdot\|$ denotes the vector two-norm. The received signal on sub-carrier i can be written as

$$r_i = \mathbf{h}_i^\dagger \hat{\mathbf{v}}_i x_i + n_i. \quad (1)$$

where n_i is the noise term with zero mean and variance N_0 . Without loss of generality, we assume $N_0 = 1$. Define $E_s \stackrel{\text{def}}{=} \mathbb{E}[|x_i|^2]$, where $\mathbb{E}[\cdot]$ stands for ensemble average. From (1), the received SNR on sub-carrier i is given by $E_s |\mathbf{h}_i^\dagger \hat{\mathbf{v}}_i|^2$.

Given the number of feedback bits for each sub-carrier, denoted as B , the approach used for narrow-band beamforming can be directly applied here, which is to design a vector quantization codebook $\mathcal{V} = \{\mathbf{v}_1, \dots, \mathbf{v}_{2^B}\}$ that is known to both the transmitter and the receiver. Using this codebook, for each sub-carrier the receiver then chooses the SNR-maximizing beamforming vector, yielding the quantization rule

$$\hat{\mathbf{v}}_i = \arg \max_{\mathbf{v}_k \in \mathcal{V}} |\mathbf{h}_i^\dagger \mathbf{v}_k|^2. \quad (2)$$

The set of indices are represented by a total of $N \times B$ bits, which are conveyed to the transmitter through a noiseless and zero-delay feedback link from the receiver to the transmitter.

The preceding approach quantizes each sub-channel vector independently, so that it will induce a performance loss from the optimal (joint) vector quantizer. Specifically, *joint* vector quantization refers to a codebook with $2^{N \times B}$ entries, where each element in the codebook is a complex vector of length $N \times N_t$ and is composed by stacking the beamforming vectors across sub-channels. For a given channel realization ideally the entry in the codebook is selected, which maximizes the sum capacity over all sub-carriers. The large number of degrees of freedom (sub-carriers, antennas) naturally suggests the application of rate-distortion theory, which provides a fundamental tradeoff between the number of feedback bits and the distortion (loss in achievable rate).

III. THE RATE-DISTORTION BOUND

A. Problem Formulation

For simplicity, in what follows we drop the sub-carrier index i , which appears in (1). The sub-channel vector \mathbf{h}^\dagger can be written as the product of two independent parts, the magnitude $\|\mathbf{h}^\dagger\|$ and the direction (or phase) $\mathbf{v}^\dagger = \mathbf{h}^\dagger / \|\mathbf{h}^\dagger\|$, i.e.,

$$\mathbf{h}^\dagger = \|\mathbf{h}^\dagger\| \mathbf{v}^\dagger, \quad (3)$$

where $\|\mathbf{h}^\dagger\|^2$ has a chi-square distribution and \mathbf{v}^\dagger is isotropically distributed over the N_t dimensional complex unit hyper-sphere [8], [9]. We adopt a similar approach as for narrow-band beamforming, and only quantize the channel direction without considering the channel magnitude. Since the channel magnitude is unavailable at the transmitter, the transmit power is assumed to be uniform over different sub-carriers.

The distortion metric is the average capacity loss with quantized beams relative to the capacity with perfect CSI at the transmitter, which is defined as

$$d(\mathbf{v}, \hat{\mathbf{v}}) \stackrel{\text{def}}{=} \mathbb{E}_{\|\mathbf{h}^\dagger\|^2} [\log_2(1 + E_s \|\mathbf{h}^\dagger\|^2)] - \mathbb{E}_{\|\mathbf{h}^\dagger\|^2} [\log_2(1 + E_s \|\mathbf{h}^\dagger\|^2 |\mathbf{v}^\dagger \hat{\mathbf{v}}|^2)]. \quad (4)$$

The distortion metric depends only on $|\mathbf{v}^\dagger \hat{\mathbf{v}}|$, the angle between the actual direction and the quantized direction. We will use this observation to evaluate the rate-distortion bound.

From the independence of the magnitude $\|\mathbf{h}^\dagger\|$ and the corresponding direction \mathbf{v} , the distortion metric can be simplified as

$$d(\mathbf{v}, \hat{\mathbf{v}}) \stackrel{\text{def}}{=} C_{E_s, N_t}(1) - C_{E_s, N_t}(|\mathbf{v}^\dagger \hat{\mathbf{v}}|), \quad (5)$$

where the function $C_{E_s, N_t}(x)$, $0 \leq x \leq 1$ is defined as

$$C_{E_s, N_t}(x) \stackrel{\text{def}}{=} \mathbb{E}_{\|\mathbf{h}^\dagger\|^2} [\log_2(1 + E_s \|\mathbf{h}^\dagger\|^2 x^2)]. \quad (6)$$

According to [8], $C_{E_s, N_t}(x)$ is given by

$$C_{E_s, N_t}(x) = \log_2(e) e^{\frac{1}{E_s x^2}} \sum_{k=0}^{N_t-1} \int_1^\infty e^{-\frac{y}{E_s x^2}} y^{-k-1} dy. \quad (7)$$

Then, the rate-distortion function can be written as

$$R(D) = \min_{f(\hat{\mathbf{v}}|\mathbf{v})} I(\mathbf{v}; \hat{\mathbf{v}}) \quad (8)$$

$$\text{subject to: } \mathbb{E}_{\mathbf{v}, \hat{\mathbf{v}}} [d(\mathbf{v}, \hat{\mathbf{v}})] \leq D, \quad (9)$$

where $I(\mathbf{v}; \hat{\mathbf{v}})$ denotes the mutual information between \mathbf{v} and $\hat{\mathbf{v}}$, $f(\hat{\mathbf{v}}|\mathbf{v})$ is the conditional probability density function (PDF) to be optimized, D denotes the distortion constraint, and the expectation is with respect to the joint probability distribution on \mathbf{v} and $\hat{\mathbf{v}}$. Note that the reproduction points $\hat{\mathbf{v}}$ are also confined to be on the surface of the N_t -dimensional complex unit hyper-sphere since only the channel direction is quantized.

We note that the range of $d(\mathbf{v}, \hat{\mathbf{v}})$ implies that D must lie in the interval $[0, C_{E_s, N_t}(1)]$. A narrower range for D is observed by noting that without any feedback the MISO beamforming model reduces to a scalar (Single-Input Single-Output) model with capacity $C_{E_s, 1}(1)$, so that the range of D is given by

$$0 \leq D \leq C_{E_s, N_t}(1) - C_{E_s, 1}(1). \quad (10)$$

B. Evaluation of the Average Distortion

It appears to be difficult to optimize the conditional PDF $f(\hat{\mathbf{v}}|\mathbf{v})$ directly, since given \mathbf{v} , it is a function mapping from the surface of the N_t -dimensional unit complex hyper-sphere to real numbers. However, the problem can be simplified by observing that the distortion metric only depends on the angle between the actual direction \mathbf{v} and the quantized direction $\hat{\mathbf{v}}$. The average distortion therefore depends only on the probability distribution of the scalar random variable $X \stackrel{\text{def}}{=} |\mathbf{v}^\dagger \hat{\mathbf{v}}|$. Assuming that the corresponding PDF $f_X(x)$ exists, we have

$$\mathbb{E}_{|\mathbf{v}^\dagger \hat{\mathbf{v}}|}[d(\mathbf{v}, \hat{\mathbf{v}})] = \int_0^1 d(\mathbf{v}, \hat{\mathbf{v}}) f_X(x) dx. \quad (11)$$

and combining (5) and (11) gives

$$\mathbb{E}_{\mathbf{v}, \hat{\mathbf{v}}}[d(\mathbf{v}, \hat{\mathbf{v}})] = C_{E_s, N_t}(1) - \int_0^1 C_{E_s, N_t}(x) f_X(x) dx. \quad (12)$$

The average distortion in (9) is therefore expressed in a simpler form than in (12), i.e., as the integral over $f_X(x)$, a function of a single variable. In the next section, we rewrite the objective (8) as an integral over the same function.

C. Evaluation of the Mutual Information

The mutual information in (8) can be written as

$$I(\mathbf{v}; \hat{\mathbf{v}}) = h(\mathbf{v}) - h(\mathbf{v}|\hat{\mathbf{v}}), \quad (13)$$

where $h(\cdot)$ denotes the differential entropy and $h(\cdot|\cdot)$ denotes the conditional differential entropy. Since \mathbf{v} is isotropically distributed, its differential entropy is given by

$$h(\mathbf{v}) = - \int_{\mathbf{v}} f(\mathbf{v}) \log_2 f(\mathbf{v}) d\mathbf{v} = \log_2 \frac{2\pi^{N_t}}{(N_t - 1)!}, \quad (14)$$

where $2\pi^{N_t}/(N_t - 1)!$ is the surface area of the N_t -dimensional complex unit hyper-sphere evaluated in [3].

To evaluate the conditional differential entropy, we note that for any distribution $f_X(x)$ satisfying the distortion constraint (9), we have

$$h(\mathbf{v}|\hat{\mathbf{v}}) = \int_0^1 f_X(x) h(\mathbf{v} | \hat{\mathbf{v}}, |\mathbf{v}^\dagger \hat{\mathbf{v}}| = x) dx \quad (15)$$

$$\leq \int_0^1 f_X(x) h(\mathbf{v} | |\mathbf{v}^\dagger \hat{\mathbf{v}}| = x) dx \quad (16)$$

$$\leq - \int_0^1 f_X(x) \log_2 \frac{f_X(x)}{l_x} dx, \quad (17)$$

where (16) follows from that fact that conditioning reduces entropy and (17) follows from the fact that given the support of a random variable, the uniform distribution maximizes its entropy, where

$$l_x = - \frac{d}{dx} |S_x| \quad (18)$$

and

$$S_x = \{\mathbf{v} | \|\mathbf{v}\| = 1, \|\hat{\mathbf{v}}\| = 1; X > x\}. \quad (19)$$

is this right? – please correct. That is, l_x is the derivative of the surface area of the spherical cap S_x centered at the reproduction point ($\hat{\mathbf{v}}$), containing points, which have angles

greater than x with $\hat{\mathbf{v}}$. According to [3], $|S_x| = 2\pi^{N_t}(1 - x^2)^{N_t-1}/(N_t - 1)!$, so that

$$l_x = \frac{4\pi^{N_t} x (1 - x^2)^{N_t-2}}{(N_t - 2)!}. \quad (20)$$

and (17) can be rewritten as

$$h(\mathbf{v}|\hat{\mathbf{v}}) \leq - \int_0^1 f(x) \log_2 \frac{(N_t - 2)! f_X(x)}{4\pi^{N_t} x (1 - x^2)^{N_t-2}} dx. \quad (21)$$

Equality is achieved when \mathbf{v} is uniformly distributed given X . Furthermore, equality can be achieved simultaneously in (16) and (17). The test channel **what is a "test" channel? that achieves these two equalities is defined as follows: $\hat{\mathbf{v}}$ is uniformly distributed on the surface of the complex unit hyper-sphere, corresponding to a uniform source distribution, X given $\hat{\mathbf{v}}$ has the same distribution f_X , and \mathbf{v} given X is uniformly distributed. In addition, it is easy to see that $f(\hat{\mathbf{v}}|\mathbf{v}) = f(\mathbf{v}|\hat{\mathbf{v}})$.

Combining (12-14) and (19), the rate-distortion problem in (8)-(9) can be rewritten as

$$R(D) = \min_{f(x)} \left\{ \log_2 \frac{2\pi^{N_t}}{(N_t - 1)!} + \int_0^1 f(x) \log_2 \frac{(N_t - 2)! f(x)}{4\pi^{N_t} x (1 - x^2)^{N_t-2}} dx \right\} \quad (22)$$

$$\text{subject to: } C_{E_s, N_t}(1) - \int_0^1 C_{E_s, N_t}(x) f(x) dx \leq D, \quad (23)$$

$$\int_0^1 f(x) dx = 1, \quad (24)$$

$$f(x) \geq 0. \quad (25)$$

We have transformed the original rate-distortion problem, which requires optimizing a multivariate function, into a constrained optimization problem over a single-variable function. (Here we have replaced f_X by f .) To solve this problem, we apply results from optimal control [13], [14], which are an extension of the calculus of variations.

D. Characterization of the Rate-Distortion Bound

It is obvious that the distortion constraint in (21) must be binding, otherwise, the rate can be made arbitrarily small by letting x distribute arbitrarily close to one. Therefore, the inequality in (21) can be safely replaced by equality. In addition, the last constraint (23) is redundant, since $f(x)$ is inside the logarithm function and (23) is automatically satisfied for any feasible solution.

In order to fit to the problem formulation of optimal control theory, the PDF $f(x)$ is replaced by the derivative of the corresponding cumulative distribution function $\dot{F}(x)$ ($F(x)$ is called the state variable in optimal control theory) and also another variable $u(x)$ is introduced as the control variable,

then the problem becomes

$$R(D) = \min_{u(x)} \left\{ \log_2 \frac{2\pi^{N_t}}{(N_t - 1)!} \right. \\ \left. + \int_0^1 u(x) \log_2 \frac{(N_t - 2)!u(x)}{4\pi^{N_t}x(1-x^2)^{N_t-2}} dx \right\} \quad (26)$$

$$\text{subject to: } \dot{F}(x) = u(x), \quad (27)$$

$$\int_0^1 C_{E_s, N_t}(x)u(x)dx = C_{E_s, N_t}(1) - D, \quad (28)$$

$$F(0) = 0, \quad F(1) = 1. \quad (29)$$

The next step to transform this problem into a standard optimal control problem is to introduce another state variable

$$\dot{G}(x) = C_{E_s, N_t}(x)u(x), \quad (30)$$

with the boundary conditions

$$G(0) = 0, \quad G(1) = C_{E_s, N_t}(1) - D. \quad (31)$$

To solve this standard optimal control problem, we can apply Pontryagin's Minimum Principle [13], [14] to its Hamiltonian, which is the dynamic equivalent to the Lagrangian of a static optimization problem, to get necessary conditions for optimal solution. When applying this principle, it requires that the state variables belong to the space of piecewise continuously differentiable functions and the control variable belong to the space of piecewise continuous functions. We will first put these preconditions aside, and verify them after we get the specific solution to this problem.

For the above optimal control problem, the Hamiltonian can be written as

$$H(x, F, G, p, q, u) = u(x) \log_2 \frac{(N_t - 2)!u(x)}{4\pi^{N_t}x(1-x^2)^{N_t-2}} \quad (32) \\ + pu(x) + qC_{E_s, N_t}(x)u(x),$$

where p is the costate variable associated with the first part of the state equation ($\dot{F}(x) = u(x)$) and q is the costate variable associated with the second part ($\dot{G}(x) = C_{E_s, N_t}(x)u(x)$). Provided the set of solutions of this problem is not empty, by minimizing H , the optimal control u^* is obtained as

$$\frac{\partial H}{\partial u} = 0, \quad (33)$$

i.e.

$$\log_2 u^*(x) + \log_2 \frac{(N_t - 2)!e}{4\pi^{N_t}x(1-x^2)^{N_t-2}} + p + qC_{E_s, N_t}(x) = 0. \quad (34)$$

The costate variables are given by Hamilton-Jacobi equations [13], [14]:

$$\dot{p} = -\frac{\partial H}{\partial F} = 0, \quad \dot{q} = -\frac{\partial H}{\partial G} = 0. \quad (35)$$

Notice that p and q are two constants by (33). Therefore (32) can be solved as

$$\dot{F}^*(x) = u^*(x) = \frac{4\pi^{N_t}x(1-x^2)^{N_t-2}}{(N_t - 2)!e} 2^{-p^* - q^* C_{E_s, N_t}(x)}. \quad (36)$$

The values of p^* and q^* can be obtained by plugging (34) into the boundary conditions (27) and (29) and we obtain that q^* should satisfy the fixed point equation:

$$[C_{E_s, N_t}(1) - D] \int_0^1 x(1-x^2)^{N_t-2} 2^{-q^* C_{E_s, N_t}(x)} dx \\ = \int_0^1 x(1-x^2)^{N_t-2} 2^{-q^* C_{E_s, N_t}(x)} C_{E_s, N_t}(x) dx, \quad (37)$$

and p^* is given by

$$p^* = \log_2 \int_0^1 \frac{4\pi^{N_t}x(1-x^2)^{N_t-2}}{(N_t - 2)!e} 2^{-q^* C_{E_s, N_t}(x)} dx. \quad (38)$$

Finally, the minimum rate $R(D)$ with respect to a given distortion D is obtained by plugging \dot{F}^* (or u^*) back into (24). Since $C_{E_s, N_t}(x)$ is continuous in x , from (34) we know that the solution can satisfy the preconditions to the state variables and the control variable.

Since p^* and q^* are two constants and it is easy to show that $H(x, F, G, p^*, q^*, u)$ is convex in (F, G, u) . Thus, according to Mangasarian's sufficient condition [13], [14], the necessary conditions in (31)-(33) are actually also sufficient conditions for optimal solution. Thus, any solution to (35) and (36) is globally optimal and we do not need to worry about the number of roots of the fixed point equation (35).

It should be noted that the method we used to solve the rate-distortion bound in this paper can be directly extended to rate-distortion bounds with other distortion metrics, such as SNR loss, etc.

IV. IMPLEMENTATION ISSUES

The rate-distortion bound evaluated in last section provides a fundamental limit for the tradeoff between the achievable forward rate (distortion) and the required backward rate (feedback bits) in a multi-carrier MISO beamforming system. Random codes with typical set encoding suffice to achieve this bound when the number of sub-carriers N goes to infinity. With finite N , the Lloyd algorithm can be used to design a near-optimal codebook for joint quantization.

Another constructive approach to design the codebook and implement joint quantization is to use the sparse graphical codes [15]–[17] and an associated set of message-passing algorithms [18]. With this approach, the joint quantization of beamforming vectors becomes similar to the iterative decoding of graphical error control codes, which in general has linear complexity with the length of the codes. Given the space constraints of this article, we will not discuss the details about sparse codes, how joint quantization can be represented in terms of sparse graphs, and the use of message-passing algorithms. For details please refer to the listed references.

V. NUMERICAL RESULTS

In Fig. 1, we plot the rate-distortion bounds in the same system with different transmit symbol energy along with the rate-distortion tradeoff for single vector based quantization using RVQ codebook (its performance is obtained by simulation). Note that to plot the curves with different parameters in one

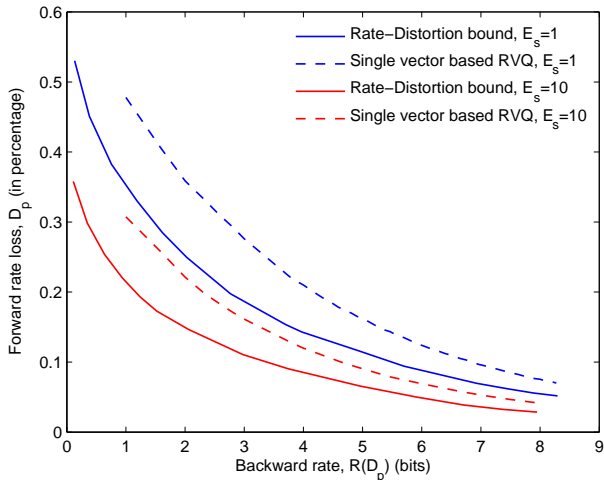


Fig. 1. Comparison of the rate-distortion bound and single vector based RVQ, $N_t = 4$.

figure we change the distortion metric to be the distortion in percentage, which is defined as $D_p \stackrel{\text{def}}{=} D/C_{E_s, N_t}(1)$. One sees that there is a large gap between the rate-distortion bound and the rate-distortion tradeoff of single vector based RVQ in the range of low-rate feedback. For instance, to achieve a target distortion of 20% in the case $E_s = 10$, the required feedback bits of joint quantization is only half of that with single vector based RVQ. In addition, Fig. 1 indicates that to achieve the same distortion (in percentage), the required rate decreases as the transmit symbol energy increases. This can be explained by the fact that the capacity function is a concave function, and as the symbol energy increases, the capacity function becomes more insensitive to the quantization errors.

VI. CONCLUSIONS

In this paper, we proposed a rate-distortion approach which jointly quantizes the channel vectors on different sub-carriers to improve the sum capacity in a multi-carrier MISO beamforming system with limited feedback. We exactly characterized the associated rate-distortion bound by formulating it as an optimal control problem. The optimal conditional distribution of the reproduction points is determined by a single variable function which determines the probability distribution on the angle between the source point and the reproduction point. Numerical results show that to achieve a certain moderate distortion the required feedback bits can be reduced in a large extent by jointly quantizing the beamforming vectors on different sub-carriers. Of course, the results in this paper correspond to the particular channel model (i.i.d. sub-carriers). For future work, extending the channel model to the more general case (correlated sub-carriers) and extending beamforming to MIMO precoding should be more interesting.

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