Monotonic Convergence of Distributed Interference Pricing in Wireless Networks

Changxin Shi, Randall A. Berry, and Michael L. Honig
Department of Electrical Engineering and Computer Science
Northwestern University, Evanston, Illinois 60208
Email: cshi@u.northwestern.edu, {rberry, mh}@eecs.northwestern.edu

***Abstract*—We study distributed algorithms for allocating powers and/or adjusting beamforming vectors in a peer-to-peer wireless network which may have multiple-input-single-output (MISO) links. The objective is to maximize the total utility summed over all users, where each user's utility is a function of the received signal-to-interference-plus-noise ratio (SINR). Each user (receiver) announces an interference price, representing the marginal cost of interference from other users. A particular user (transmitter) then updates its power and beamforming vector to maximize its utility minus the interference cost to other users, which is determined from their announced interference prices. We show that if each transmitter update is based on a current set of interference prices and the utility functions satisfy certain concavity conditions, then the total utility is non-decreasing with each update. The proof is based on the convexity of the utility functions with respect to received interference, and applies to rate utility functions, and an arbitrary number of interfering MISO links. The extension to multi-carrier links is discussed as well as algorithmic variations in which the prices are not immediately updated after power or beam updates.**

I. Introduction

Achieving high spectral efficiencies in multiuser wireless networks depends critically on the application of interference mitigation techniques. This becomes especially challenging in peer-to-peer networks with distributed resource management, since an optimal allocation of resources at a particular transmitter (e.g., that maximizes total rate) generally requires information about interference at other nodes along with associated channel gains. An efficient mechanism for exchanging information is therefore needed to allocate resources while minimizing the associated overhead.

We consider a peer-to-peer network where the transmitters may have multiple antennas. Resources therefore include power across available degrees of freedom in frequency and space, and also beamforming vectors (directions). Each user is assigned a utility function, which depends on the received signal-to-interference-plus-noise ratio (SINR), and our objective is to allocate powers and/or beams to maximize the total (sum) utility over the users. Furthermore, we seek distributed resource allocation algorithms, which require relatively little information exchange.

Here we analyze the performance of distributed *interference pricing*, presented in [1] for allocating power in single-antenna wireless networks. Specifically, receivers announce *interference prices*, defined as the marginal decrease in utility per unit increase in interference power. A transmitter then updates its power by maximizing its utility minus the interference cost to others, which is determined by the set of announced interference prices. In [1], an *Asynchronous Distributed Pricing* (ADP) algorithm is given in which powers and interference prices can be iteratively updated asynchronously. Extensions of the ADP algorithm to power allocation for multi-carrier channels and beam updates for multi-input-single-output (MISO) channels are presented in [2], [3].

It is shown in [1] that with single-antenna nodes the set of powers and prices determined by the ADP algorithm converges for a suitable class of utility functions. The proof is based on relating the updates in the distributed algorithm to best response updates in a supermodular game. A limitation of this result is that the class of utility functions does not include the the Shannon rate function. Hence the convergence of the sum rate with the ADP algorithm was left as an open problem. In [3] convergence of a MISO version of the ADP algorithm was established for two users with the same class of utility functions as in [1]. The proof is again based on supermodularity.

Here we present a different approach to the convergence analysis of distributed pricing algorithms, which is based on establishing a convexity property of certain utility functions with respect to the received interference power. For utilities with this property, all interference prices are current (i.e., have been updated since the last power/beam update) then a subsequent power or beam update cannot decrease the total utility. Hence the total utility must monotonically converge to a limit. Satisfying this convexity property is a weaker conditions than those required in [1], and includes the rate utility. Furthermore, there is no restriction on the number of users in the MISO setting.[1] We also give counter-examples, which show that when either the condition on the utility functions or the condition on current prices is violated, the total utility may not converge.

Related work for single-antenna networks is presented in [4], which considers gradient updates, and [5] which con-

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[1]Note, however, that the convergence result in [1] does not require current prices, hence the results presented here are complementary to those in [1].

siders best response updates with rate utility functions and shows convergence for a restricted set of channel gains. The MISO-ADP algorithm is also presented in [6], although convergence is not addressed. The Pareto-optimal rate-pairs for a two-user MISO interference channel are characterized in [7]. The convergence of iterative waterfilling, in which nodes implement best response updates without exchanging channel or interference information, is discussed in [8]–[10] for frequency-selective channels, and in [11]–[13] for multiple-input and multiple-output (MIMO) links.

Next, in Section II we consider convergence of distributed pricing algorithms for single-antenna nodes, and in Section III we extend these results to the MISO setting. Section IV presents variations to the price update rule, and conclusions are given in Section V.

II. Single Antenna Networks

A. System Model

In this section we consider a single antenna peer-to-peer wireless network with K distinct transmitter/receiver pairs sharing the same spectrum (i.e., a K user interference channel). Initially, we assume that the transmitted signal of each user i is spread over the entire bandwidth (normalized to 1 Hz), and so the signal at receiver i is the superposition of all transmitted signals plus additive Gaussian noise with power spectral density n_0. We assume that all interference is treated as noise and each user i's performance is determined by the received SINR

$$\gamma_i(\mathbf{p}) = \frac{p_i h_{ii}}{n_0 + \sum_{j \neq i} p_j h_{ji}}, \tag{1}$$

where h_{ij} is the channel gain from transmitter i to receiver j, p_i is the transmission power of user i, and $\mathbf{p} = \{p_1, \cdots, p_K\}$ denotes the power profile of all users.

The quality of service for each user i is measured by a utility function $u_i(\gamma_i)$, which is assumed to be a monotonically increasing, concave and twice differentiable function of γ_i. One example is the rate utility, $u_i = \log(1 + \gamma_i)$, which corresponds to the user's maximum achievable rate assuming Gaussian codebooks. Our objective is to choose $\mathbf{p}$ to maximize the utility summed over all users, i.e.,

$$\max_{\mathbf{p}} \quad \sum_{i=1}^{K} u_i(\mathbf{p}) \tag{P_1}$$
$$\text{s.t.} \quad 0 \leq p_i \leq P_i^{max}, \quad i = 1, \ldots, K,$$

where P_i^{max} denotes the power constraint for user i. Moreover, we are interested in a distributed solution, in which each user does not know the entire network topology or the utility functions of other users. One such distributed algorithm is the ADP algorithm from [1], which we review next.

B. Distributed Interference Pricing

The key idea behind the ADP algorithm is for users to exchange *interference prices*, which reflect the marginal change in their utility per unit interference. Specifically, the interference price for user i is given by

$$\pi_i = -\frac{\partial u_i(\gamma_i(\mathbf{p}))}{\partial I_i(\mathbf{p}_{-i})} = \frac{u_i'(\gamma_i(\mathbf{p})) p_i h_{ii}}{(n_0 + \sum_{j \neq i} p_j h_{ji})^2}, \tag{2}$$

where $\mathbf{p}_{-i}$ denotes the power profile for all users other than user i, $I_i(\mathbf{p}_{-i}) = \sum_{j \neq i} p_j h_{ji}$ is the interference power at receiver i, and $u_i'(\gamma_i(\mathbf{p})) = \frac{du_i}{d\gamma_i}$ evaluated at $\gamma_i(\mathbf{p})$. Given fixed interference prices and powers for the other users, transmitter i then updates its power by solving the subproblem:

$$\max_{p_i} \quad u_i(\gamma_i(p_i, \mathbf{p}_{-i})) - p_i \sum_{j \neq i} \pi_j h_{ij} \tag{P_1^i}$$
$$\text{s.t.} \quad 0 \leq p_i \leq P_i^{max},$$

which can be viewed as a user's *best response* when faced with the problem of maximizing his utility minus a cost given by π_j per unit interference caused at each receiver j.

In the ADP algorithm each user repeatedly adjusts its power and interference price according to (P_1^i) and (2), respectively. This requires users to announce a single price and to measure only "local" information. It is shown in [1] that for any number of users, if each user's utility satisfies

$$K_i(\gamma_i) := -\frac{u_i''(\gamma_i)\gamma_i}{u_i'(\gamma_i)} \in [1, 2] \tag{3}$$

for all feasible SINRs, then the ADP algorithm globally converges to the optimal power allocation with arbitrary asynchronous updates.[2] The quantity $K_i(\gamma_i)$ is referred to as the *coefficient of relative risk aversion* of the utility function u_i; larger values of this quantity indicate that u_i is "more concave." For example, (3) is satisfied by $u_i = \log(\gamma_i)$ ($K_i(\gamma) = 1$) but not by $u_i = \log(1 + \gamma_i)$ ($K_i(\gamma) < 1$).

Next we give a modification of the ADP algorithm, which works for a larger class of utility functions *and* ensures monotonic convergence of the total utility. We refer to this as the *distributed pricing* (DP) algorithm and formally state it as follows:

1) Each user i chooses an initial power p_i satisfying the power constraint.
2) Using (2), each user i calculates the interference price π_i given the current power profile and announces this price to every other user.
3) One random user i solves Problem P_1^i and updates his power, given the interference prices $\{\pi_j\}_{j \neq i}$.
4) Repeat from step 2).

This has the same information requirements as the ADP algorithm in [1] but is more restrictive in the timing of each user's price and power updates. In particular, only one user is allowed to update its power at a time and after each power update, *all* users must announce new interference prices. The next proposition characterizes the convergence of this algorithm.

[2] Synchronous updates are a special case of this. Moreover, each user need not update his interference price and power allocation simultaneously.

Proposition 1: If for each user i, $K_i(\gamma_i) \in [0, 2]$ for all feasible γ_i, then the power profile of all the users under the DP algorithm converges to a limit point satisfying the Karush-Kuhn-Tucker (KKT) conditions of Problem P_1.

The condition in Proposition 1 applies to a larger set of utility functions than (3) and in particular applies to the rate utility $\log(1+\gamma)$. However, this only guarantees convergence to a point satisfying the KKT conditions of Problem P_1. If this problem has multiple local optima, then the algorithm may not converge to the global optimum.[3]

The key step in proving Proposition 1 is showing the following lemma:

Lemma 1: If each user's utility function satisfies the conditions in Proposition 1, then after each user adjusts his power in the DP algorithm, the total utility will be non-decreasing.

The proof of Lemma 1 follows from showing that any utility function in the assumed class is a convex function of the interference power term and so it is lower bounded by its tangent at any point. The slope of this tangent for each user i at the current operating point is given by that user's interference price. Using this it can be shown that the objective in Problem P_1^i is a lower bound on the total utility which is tight at the current operating point. Hence, when a user adjusts his power (by solving Problem P_1^i), this cannot decrease the total utility. See Appendix A for more details.

Since the total utility is bounded, Lemma 1 directly implies the convergence of the total utility. Furthermore, it can be shown that any limit point of the algorithm must satisfy the KKT conditions of Problem P_1. To complete the proof of Proposition 1, it suffices to show that if the total utility converges, then the power profile of each user must also converge. The details are omitted due to space limitations.

C. Generalization to Multi-channel Networks

Next we consider a generalization of the previous results to a network with M parallel channels, as in a system which uses orthogonal frequency division multiple access (OFDMA). In this case, each user i can allocate its power over the M channels subject to a total power constraint of P_i^{max}. Each channel is modeled as a Gaussian interference channel, and all users still treat any interference as noise. Let h_{ij}^m denote the gain of the mth channel between transmitter i and receiver j, and let p_i^m and γ_i^m denote user i's transmit power and SINR on this channel.

We assume that each user's quality of service is now given by a total utility that is *separable* across the channels, i.e. the total utility of user i is given by $u_i = \sum_{m=1}^M u_i^m(\gamma_i^m)$, where u_i^m is the utility that user receives from channel m.[4] Our objective in this case becomes

$$\max_{\mathbf{p}_1,\cdots,\mathbf{p}_K} \quad \sum_{i=1}^K \sum_{m=1}^M u_i^m(\gamma_i^m) \qquad (\mathsf{P}_2)$$
$$\text{s.t.} \quad \sum_{m=1}^M p_i^m \le P_i^{max}, \quad i = 1, \dots, K,$$

where $\mathbf{p}_i = \{p_i^1, \cdots, p_i^M\}$ is the power profile of user i.

The DP algorithm can be generalized to this setting as follows. First, in step 2 of the algorithm each user i calculates an interference price π_i^m for *each* channel m, where

$$\pi_i^m = -\frac{\partial u_i^m(\gamma_i^m)}{\partial I_i^m} = \frac{u_i^{m\prime}(\gamma_i) p_i^m h_{ii}^m}{(n_0 + \sum_{j\neq i} p_j^m h_{ji}^m)^2}. \qquad (4)$$

These prices are announced to every other user. Next, in step 3 of the algorithm, a random user i updates his power profile $\mathbf{p}_i$ across all channels by solving

$$\max_{\mathbf{p}_i} \quad \sum_{m=1}^M u_i^m(\gamma_i^m) - \sum_{m=1}^M p_i^m \sum_{j\neq i} \pi_j^m h_{ij}^m$$
$$\text{s.t.} \quad \sum_{m=1}^M p_i^m \le P_i^{max} \qquad (\mathsf{P}_2^i)$$
$$p_i^m \ge 0 \qquad \forall m.$$

We refer to the resulting algorithm as the *multichannel distributed pricing* (MDP) algorithm. The following corollary characterizes the convergence of this algorithm, where $K_i^m(\gamma)$ denotes the coefficient of relative risk aversion of u_i^m.

Proposition 2: If for all i and m, $K_i^m \in [0, 2]$, then the power profile of all the users converges to a limit point satisfying the KKT conditions of Problem P_2.

The proof follows a similar argument as the proof of Proposition 1; we omit the details due to space considerations. We note that in [1], a version of the ADP algorithm was given for this type of multichannel model and shown to converge when the utility per channel satisfied the stronger condition in (3); however, that convergence result requires relaxing each user's total power constraint and adjusting a "total power price" on a slower time-scale to ensure that those constraints are eventually satisfied. This is not needed here, i.e., each power constraint is satisfied during every iteration.

III. MISO NETWORKS

A. System Model

We now consider beamformer updates in a narrowband MISO network with K users, each equipped with N transmit antennas. The channel vector from transmitter i to receiver j is denoted by $\mathbf{h}_{ij} = [h_{ij}^1, h_{ij}^2, \cdots, h_{ij}^N]^T$. Let $\mathbf{v}_i$ be the complex beamforming vector for user i. Then the received SINR for user i is[5]

$$\gamma_i = \frac{|\mathbf{v}_i^\dagger \mathbf{h}_{ii}|^2}{n_0 + \sum_{j\neq i} |\mathbf{v}_j^\dagger \mathbf{h}_{ji}|^2}. \qquad (5)$$

[3] When $K_i(\gamma_i) \ge 1$ for each user i then Problem P_1 will have only one point that satisfies the KKT conditions [14]. It follows that when each user's utility satisfies (3), the algorithm must converge to the global optimum.

[4] Note that if u_i represents a maximum achievable rate (assuming Gaussian codebooks), then this satisfies our separability assumption with $u_i^m = \log(1+\gamma_i^m)$.

[5] $\dagger$ denotes Hermitian transpose.

The problem is to select the set of beamforming vectors $\{\mathbf{v}_1, \cdots, \mathbf{v}_K\}$, which maximize the sum utility, i.e.,

$$\max_{\mathbf{v}_1,\cdots,\mathbf{v}_K} \quad \sum_{i=1}^{K} u_i(\gamma_i) \qquad (\mathsf{P}_3)$$
$$\text{s.t.} \quad |\mathbf{v}_i|^2 \leq P_i^{max}, \quad i = 1, \ldots, K,$$

where γ_i is given by (5).

B. MISO Distributed Pricing (MISO-DP) Algorithm

With MISO nodes the interference price at receiver i is given by

$$\pi_i = -\frac{\partial u_i(\gamma_i)}{\partial I_i} = \frac{u_i'(\gamma_i)|\mathbf{v}_i^\dagger \mathbf{h}_{ii}|^2}{(n_0 + \sum_{j\neq i} |\mathbf{v}_j^\dagger \mathbf{h}_{ji}|^2)^2} \qquad (6)$$

and the best response update at transmitter i selects the beam $\mathbf{v}_i$ to solve

$$\max_{\mathbf{v}_i} \quad u_i(\gamma_i) - \sum_{j\neq i} \pi_j |\mathbf{v}_i^\dagger \mathbf{h}_{ij}|^2 \qquad (\mathsf{P}_3^i)$$
$$\text{s.t.} \quad |\mathbf{v}_i|^2 \leq P_i^{max}.$$

The MISO-DP algorithm is then specified as follows:

1) Each user i chooses an initial beam $\mathbf{v}_i$ satisfying the power constraint.
2) Each user i announces the interference price π_i, given by (6).
3) One random user i updates $\mathbf{v}_i$ according to the best response (solution to Problem P_3^i).
4) Repeat from step 2).

In general, there may be multiple solutions to Problem P_3^i. In that case, one of the solutions can be randomly chosen, assuming the previous beam is not a solution. Otherwise, the previous beam is kept.

Proposition 3: If $K_i(\gamma_i) \in [0, 2]$ for each user i, then each beamforming vector in the MISO-DP algorithm converges to a limit point satisfying the KKT conditions associated with Problem P_3.

Proof: (*sketch*) Comparing Problem P_3 and the associated subproblem P_3^i with their counterparts in the single-antenna case (Section II), the difference appears in the specific expressions for signal and interference powers. Treating the interference power as a function of the resources (powers or beams), it is easily verified that Lemma 1 still holds in the MISO setting. The remainder of the proof then follows from a similar argument as the proof of Proposition 1. ■

To illustrate the preceding convergence result, Fig. 1 shows plots of total utility versus iterations (each corresponding to a beam update) for the MISO-DP algorithm with five transmitter-receiver pairs, each with three transmit antennas. The channel vectors consist of realizations of *i.i.d.* complex Gaussian random variables, and the average received signal-to-noise ratio is 23 dB. Four curves are shown, corresponding to utility functions with different values of $K_i(\gamma_i)$ in (3). Specifically, they are $u_i(\gamma_i) = \gamma^\alpha/\alpha$ with $\alpha = -1, -2, -3$, for which $K_i(\gamma_i) = 1 - \alpha$, and $u_i = \log \gamma_i$, for which

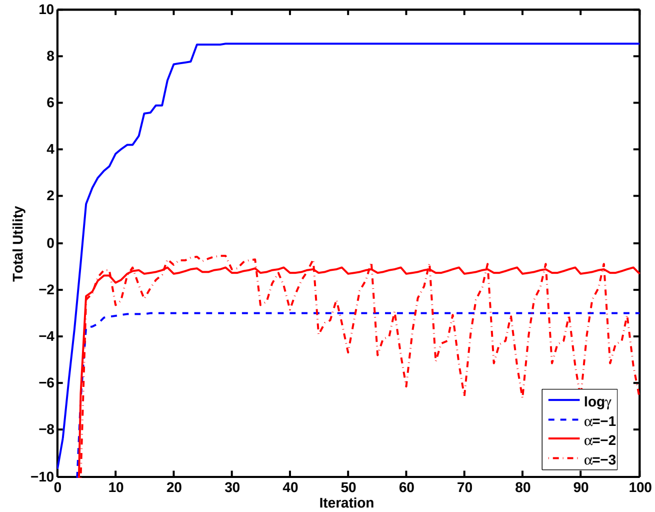

Fig. 1. Illustration of the convergence of the MISO-DP algorithm.

$K_i(\gamma_i) = 1$. The figure shows that the sum utilities corresponding to the examples with $K_i = 1$ and 2 converge, as stated by Proposition 3, whereas the examples corresponding to $K_i = 3$ and 4 oscillate.[6] Hence conditions on K_i are indeed necessary to guarantee convergence.

IV. Variations on the Update Rule

Our convergence proof for each of the previous versions of the DP algorithm relies on the fact that whenever one user updates his power(s) or beamformer, he has current interference prices from every user in the network. To reduce the overhead required to broadcast these prices it would be desirable to relax this restriction and allow multiple users to update their powers before new prices are announced. In this section we consider relaxing these restrictions.

First we consider a modification of the algorithms in which for a given set of interference prices, a group of users simultaneously updates their transmission powers in step 3. The next proposition states that with logarithmic utilities, we still have monotonic convergence.

Proposition 4: If $u_i(\gamma_i) = c_i \log(\gamma_i)$ for each user i, where c_i is a constant, then with simultaneous group updates, the total utility in the DP, MDP and MISO-DP algorithms monotonically converges.

The proof is omitted due to space considerations.

Proposition 4 does not hold for any utility which satisfies $K_i \in [0, 2]$ as the next example illustrates. Consider a single antenna network with three users sharing a single channel, and with linear utility functions $u_i(\gamma_i) = \gamma_i$ and simultaneous group updates in which at every update all three users update their powers. Suppose that for all i, $P_i^{max} = 1$, the direct channel gain $h_{ii} = 1$, and $n_0 = 1$. It can be shown that if $(1 + h_{31})h_{13} < 1$ and $(1 + h_{32})h_{23} < 1$ and h_{12} and h_{21}

[6] Interestingly, for $K_i > 2$ the underlying problem has a unique point which satisfies the KKT conditions. Apparently the best response updates used in the DP algorithm are too aggressive to find this point.

are large enough, then if every user simultaneously updates during each iteration, the DP algorithm will not converge and in fact the powers of users 1 and 2 oscillate between 0 and P_i^{max} while user 3 transmits at constant power of P_3^{max}. Numerically, we have also observed similar behavior with rate utilities ($u_i(\gamma_i) = \log(1 + \gamma_i)$).

A second variation on our update rule is to consider the case where a group of users *sequentially update* their powers or beamforming vectors between price updates. This differs from the previous model in that after one user updates, the next user can update his power to account for the first user's action. With logarithmic utilities, if two users sequentially update we can again show monotonic convergence.

Proposition 5: If $u_i(\gamma_i) = c_i \log(\gamma_i)$ for all i and at most two users sequentially updated between any new price update, then the total utility in the DP, MDP and MISO-DP algorithms monotonically converges.

V. Conclusions

We have studied the convergence of distributed pricing algorithms for adjusting powers and beamforming vectors in a peer-to-peer network. In contrast to previous analysis, based on supermodularity, our results rely upon the convexity of the utility functions with respect to the received interference power. Convergence is established for a class of algorithms in which prices are updated after each (individual) power or beam update. These results complement prior work in [1], [3], which do not require such frequent price updates, but impose stricter concavity constraints on the utility functions. Examples were also presented to show that violating the conditions for convergence can lead to oscillations in total utility.

Although convergence of distributed pricing algorithms has been established for many scenarios of interest, the speed of convergence (number of iterations to reach a target utility) as a function of channel gains and system parameters has not been considered. In addition, extensions to MIMO channels (both narrowband and wideband) are left for future work.

Appendix A

Proof of Lemma 1: If $K_j(\gamma_i) \in [0, 2]$, then by direct calculation it can be verified that

$$\frac{\partial^2 u_j(\gamma_i(\mathbf{p}))}{\partial I_j^2(\mathbf{p}_{-j})} \geq 0, \tag{7}$$

which means that u_j is a convex function of I_j, given that all other parameters are fixed. Therefore, if we fix all parameters except I_j (i.e. we are considering a user j when some user $i \neq j$ is updating its power) it follows that

$$\begin{aligned} u_j(\gamma_i) &\geq u_j(\gamma_j^o) + \frac{\partial u_j}{\partial I_j}\Big|_{\mathbf{p}^o}(I_j - I_j^o) \\ &= u_j(\gamma_j^o) - \pi_j(\mathbf{p}_o)(I_j - I_j^o) \end{aligned} \tag{8}$$

where γ_j^o and I_j^o are user j's SINR and interference power at the current operating point $\mathbf{p}^o$, and γ_j and I_j are the corresponding values at any new operating point after varying I_j.

Now suppose that user i updates his power by solving Problem P_1^i given the current power profile $\mathbf{p}^o$. After the update, we have

$$u_i(\gamma_i(\mathbf{p}_i^*)) - \sum_{j\neq i} \pi_j I_{ij}(p_i^*) \geq u_i(\gamma_i(\mathbf{p}^o)) - \sum_{j\neq i} \pi_j I_{ij}(p_i^o), \tag{9}$$

where I_{ij} is the interference power from transmitter i to receiver j, and $\mathbf{p}^* = \{p_1^o, \cdots, p_{i-1}^o, p_i^*, p_{i+1}^o, \cdots, p_K^o\}$ indicates the operating point after user i's power update. Since $I_j^* - I_j^o = I_{ij}^* - I_{ij}^o$, adding constant terms to both sides of (9) and simplifying yields

$$u_i(\gamma_i^*) + \sum_{j\neq i}\Big[u_j(\gamma_j^o) - \pi_j(I_j^* - I_j^o)\Big] \geq \sum_{i=1}^{K} u_i(\gamma_i^o). \tag{10}$$

Additionally, from (8), we have

$$\sum_{i=1}^{K} u_i(\gamma_i^*) \geq u_i(\gamma_i^*) + \sum_{j\neq i}\Big[u_j(\gamma_j^o) - \pi_j(I_j^* - I_j^o)\Big]. \tag{11}$$

Combining (10) and (11) yields $\sum_{i=1}^{K} u_i(\gamma_i^*) \geq \sum_{i=1}^{K} u_i(\gamma_i^o)$, which is the desired result. ■

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