Channel and Receiver State Feedback for Frequency-Selective Block Fading Channels

Manish Agarwal, Dongning Guo, and Michael L. Honig
Dept. of Electrical Engineering and Computer Science
Northwestern University
2145 Sheridan Road, Evanston, IL 60208 USA
{m-agarwal, dGuo, mh}@northwestern.edu

***Abstract*—Given a frequency-selective fading channel, feedback can be used to convey *Channel State Information (CSI)* and *Receiver State Information (RSI)* to the transmitter. RSI refers to information about the receiver's estimate of the message, which can be used to improve reliability, e.g., through retransmissions. This paper studies the trade-off between these two types of feedback under total feedback constraint, assuming multi-carrier transmission through a doubly-selective Rayleigh fading channel. The objective is to maximize the error probability exponent. The CSI feedback scheme specifies particular groups of sub-channels over which uniform power is allocated, and the RSI feedback determines retransmissions of the codeword. The optimal trade-off exhibits phase transitions which depends critically on the coherence time and the total amount of feedback. Specifically, the first few feedback bits should be allocated to CSI up to a critical amount. Additional feedback bits, if available, should first be allocated to RSI only, and then allocated to both CSI and RSI. This is because as the feedback rate increases beyond a certain amount, additional RSI feedback is not beneficial unless CSI feedback increases simultaneously.**

I. Introduction

A feedback link from the receiver to the transmitter can be utilized in several ways. For example, the receiver can inform the transmitter about its estimate of the message, based on which the transmitter might choose to re-transmit part or all of the original codeword. These schemes include automatic repeat request (ARQ) protocols [1]. The main benefit is improvement in data reliability, or equivalently, a reduction in average decoding delay for a target probability of error. Several generalizations, including full feedback of received symbols [2], [3] and limited feedback schemes [4], [5] have been studied. We refer to this type of feedback as *Receiver State Information (RSI)* feedback.

Alternatively, given a fading channel, which is known at the receiver, but is initially unknown at the transmitter, feedback can convey *Channel State Information (CSI)*. This can increase the capacity of the channel through power and rate adaptation [6]. Limited feedback schemes for CSI have been recently studied in [7]–[9] for multi-carrier transmission.

This work formulates a model and studies the relative benefits of CSI and RSI. Specifically, given fixed forward and feedback rates, the problem is to optimize the split between CSI and RSI. To ease the analysis, we maximize the error exponent instead of minimizing the error probability directly. Note that allocating feedback bits to CSI increases the error exponent by increasing the achievable rate. In contrast, RSI does not increase the achievable rate, but increases the error exponent by reducing error events [2], [4].

The channel is assumed to be doubly-selective block Rayleigh fading. That is, there are parallel block fading sub-channels each having the same coherence time and bandwidth. The channel realizations are independent across all time-frequency coherence blocks. The CSI feedback occurs at the beginning of each coherence block. To accomodate a continuous range of feedback rates, which can be arbitrarily small, we assume that the set of sub-channels is partitioned into equal-size *groups* (see [7], [10]). The CSI feedback indicates which groups should be activated, i.e., the groups in which all sub-channel gains exceed a threshold. The transmitter uniformly spreads the data power over the active groups. The feedback rate is therefore controlled by the group size and threshold.

The RSI feedback scheme is taken from [4]. Namely, a codeword spans multiple coherence blocks and limited RSI is fed back at the end of each codeword transmission. Note that CSI and RSI are fed back at different times and the only constraint is on the average number of feedback bits per channel use. Also, as in [4], there are no delay constraints associated with the feedback, which facilitates the analysis. Although RSI feedback gives some implicit information about the channel realized during the course of the transmitted codeword, CSI in our model pertains to the channel state corresponding to the *next* packet transmission. Note that transmission takes place in the units of coherence blocks. In particular, the channel can be regarded as memoryless conditioned on the CSI.

We determine the optimal allocation of feeback bits to CSI and RSI asymptotically as the number of sub-channels, feedback rate, coherence time (T channel uses) and transmission rate (R nats per channel use) are scaled appropriately. For the model considered, the first few feedback bits, denoted as R_{csi}^{crit}, should be allocated to CSI. Additional feedback bits, if available, should first be allocated to RSI only, and then allocated to both CSI and RSI. This is because as the amount of feedback increases, allocating additional feedback bits to RSI may not be beneficial unless a certain proportion is allocated to CSI. The results also show that the relative

This work was supported by the U.S. Army Research Office under grant DAAD19-99-1-0288 and the DARPA IT-MANET program Grant W911NF-07-1-0028.

benefits of CSI and RSI depend critically on the coherence time. Given N sub-channels and signal-to-noise ratio (SNR) S, R_{csi}^{crit} grows as $ST/4$ for $T < \log N$ with large enough N. As T increases beyond $\log N$, R_{csi}^{crit} grows more slowly and eventually decreases.

II. Multi-Carrier Model with Feedback

Consider multi-carrier transmission over N independent Rayleigh fading sub-channels, so that the $N \times 1$ vector of channel outputs across sub-channels is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} \tag{1}$$

where $\mathbf{H} = \text{diag}\,[h_1, h_2, \ldots, h_N]$ is the channel matrix in which the diagonal entries are independent, circularly symmetric complex Gaussian (CSCG) random variables with mean zero and variance σ_h^2. The $N \times 1$ input vector $\mathbf{x}$ satisfies the average power constraint $E\left[\mathbf{x}^H\mathbf{x}\right] \leq P$ and the noise vector $\mathbf{z}$ has CSCG entries with mean zero and variance σ_z^2. The channel $\mathbf{H}$ is assumed to be known perfectly at the receiver. We assume a block fading model so that $\mathbf{H}$ remains constant for T channel uses and then changes to an independent value. The time dependence is suppressed to simplify notation.

A. CSI Feedback

At the beginning of each coherence block, we assume that TR_{csi} nats are fed back, where R_{csi} is the CSI feedback rate.[1] The CSI feedback scheme presented in [7], [10] is assumed, in which the total set of sub-carriers is partitioned into G nonoverlapping groups each containing $N_G = N/G$ consecutive sub-channels. Let the $N_G \times 1$ vector of sub-channel gains corresponding to the g^{th} group be $\mathbf{h}_g = [h_{g1}, h_{g2}, \ldots, h_{gN_G}]^T$. Given a threshold t_o, the receiver informs the transmitter to use this group if $|h_{gi}|^2 \geq t_o$ for all $i = 1, 2, \ldots, N_G$. The probability of this event is $p = e^{-N_G t_o/\sigma_h^2}$, so that for large N the average amount of feedback required per coherence block can be compressed to the entropy rate.[2] The CSI feedback constraint is therefore $GH(p) \leq TR_{csi}$. where $H(p) = -p\log(p) - (1-p)\log(1-p)$. Clearly, the larger the coherence time, the less CSI is required per channel use to achieve a target rate.

Given the CSI feedback, the transmitter allocates power P_o uniformly over the set of active sub-channel groups. Subject to this constraint, the maximum achievable rate (ergodic capacity) is given by

$$C(R_{csi}) = GE_{\mathbf{h}_g}\left[\mathbf{1}_{\{|h_{gi}|^2 \geq t_o, \forall i\}} \sum_{i=1}^{N_G} \log \frac{\sigma_z^2 + P_o|h_{gi}|^2}{\sigma_z^2}\right] \tag{2}$$

$$= \frac{P}{P_o}\int_{t_o}^{\infty} \frac{e^{-(t-t_o)/\sigma_h^2}}{\sigma_h^2} \log\frac{\sigma_z^2 + P_o t}{\sigma_z^2} dt \tag{3}$$

[1]This is an abstraction for the scenario in which the channel is estimated by the receiver at the beginning of the coherence block. Here we ignore the overhead due to training and feedback.

[2]A practical variable-length prefix code typically requires an additional bit per coherence block. We ignore this, since a coherence block is likely to contain several hundred channel uses, so that this extra bit contributes negligible feedback overhead.

where $P_o = P/(N\,e^{-N_G t_o/\sigma_h^2})$. Note that this rate does not depend upon the coherence block length T, since the transmitter is assumed to code across many coherence blocks in frequency and/or time.

We wish to choose the feedback parameters N_G and t_o to maximize $C(R_{csi})$ subject to the feedback constraint $GH(p) \leq TR_{csi}$. Although it appears to be difficult to obtain an analytical characterization of the solution for arbitrary N, the following proposition, repeated from [10], characterizes the solution for large N and TR_{csi}.

Let $u^\star$ be the positive solution to $\log(1+u) = 2\,u/(1+u)$ (i.e., $u^\star \approx 3.92$), and ϵ_1, ϵ_2 satisfy

$$\log N - \log\left[\frac{S}{u^\star}(\log N)^{1-\frac{\epsilon_1}{2}}\right] = (\log N)^{1-\frac{\epsilon_1}{2}} \tag{4}$$

$$\log N - \log[S(\log N)^{1+\epsilon_2}] = (\log N)^{(1+\epsilon_2)/2}, \tag{5}$$

respectively, where $S = P\sigma_h^2/\sigma_z^2$ is the SNR. Note that for large N, $\epsilon_1 \in (0,2)$ and $\epsilon_2 \in (0,1)$. Moreover, as $N \to \infty$, we have $\epsilon_1 \to 0$ and $\epsilon_2 \to 1$.

Proposition 1: [10] As $N \to \infty$, if $TR_{csi} \to \infty$ with N and

$$TR_{csi} \leq \frac{S}{u^\star}(\log N)^{2-\epsilon_1}, \tag{6}$$

then the capacity, optimized over both N_G and t_o, satisfies

$$C(R_{csi}) = \sqrt{\frac{S}{u^\star}}\log(1+u^\star)\sqrt{TR_{csi}} + o(1) \tag{7}$$

where $o(1)$ stands for a vanishing term as $N \to \infty$. If instead the feedback rate is greater than (6) but satisfies

$$\frac{S}{u^\star}(\log N)^{2-\epsilon_1} \leq TR_{csi} \leq S(\log N)^{2+\epsilon_2}, \tag{8}$$

then the optimized capacity satisfies

$$C(R_{csi}) = \frac{TR_{csi}}{\log N}\log\left(1 + \frac{S\log N}{TR_{csi}}\log\frac{N\log N}{TR_{csi}}\right) + o(1). \tag{9}$$

Finally, if

$$S(\log N)^{2+\epsilon_2} \leq TR_{csi}, \tag{10}$$

then the optimized capacity satisfies

$$C(R_{csi}) = S\log N + o(\log N) \tag{11}$$

A characterization of the optimized threshold $t_o^\star$ and the optimized sub-channel group size $N_G^\star$, along with a discussion of those results, is given in [10]. In particular, assuming a large fixed N, ${N_G}^\star$ decreases with TR_{csi}. When $TR_{csi} > (S/u^\star)(\log N)^{2-\epsilon_1}$, ${N_G}^\star = 1$ and $t_o^\star$ increases as $\log N$.

B. RSI Feedback

In addition to CSI feedback, we assume an average feedback rate of R_{rsi} nats/channel use, which specifies RSI. RSI feedback bits are assumed to be transmitted after the transmission of each codeword. We adopt the approach presented in [4], [5], in which the RSI feedback determines whether or not the transmitter re-transmits the codeword. This also requires a synchronization mechanism between the transmitter and receiver via the noisy forward channel.

Here we focus on the scheme given in [4] for which $0 \leq R_{rsi} \leq R$, i.e., increasing the feedback rate beyond the forward rate does not improve performance. Although the results in [4] assume a discrete memoryless channel, they can be extended to block fading channels by treating each block as one vector channel use. Moreover, the results in [4] can be extended to a wide class of continuous alphabet channels.

It is shown in [4] that with RSI feedback alone the probability of decoding error satisfies

$$P_e^{ub} = e^{-n\,E_1(R_{csi},R_{rsi},\lambda)} + e^{-n\,E_2(R_{csi},\lambda)} \tag{12}$$

where n is the *average* decoding delay measured in number of coherence blocks here, and the fraction $1-\lambda$ of channel uses is devoted to synchronization where $\lambda \in (0,1)$. The first exponent E_1 is determined by the probability that the receiver makes a decoding error, which the transmitter is unable to detect due to the limited RSI. The second exponent E_2 is determined by the probability of the event that the transmitter is aware that the receiver's estimate is incorrect, but is unable to synchronize within the time constraint. Note that R_{rsi} only affects the first exponent. In particular, from [4]

$$E_1 = \lambda E_g\Big(\frac{R-R_{rsi}}{\lambda}, R_{csi}\Big) + \lambda\Big|C(R_{csi}) - \frac{R-R_{rsi}}{\lambda}\Big|^+ \tag{13}$$

and

$$E_2 = \lambda E_g\Big(\frac{R}{\lambda}, R_{csi}\Big) + \lambda\Big|C(R_{csi}) - \frac{R}{\lambda}\Big|^+ + (1-\lambda)C_1(R_{csi}) \tag{14}$$

where $E_g(R, R_{csi})$ is the random coding error exponent[3] [11] and $C_1(R_{csi})$ is the distance between the two most distinguishable channel inputs.

Let $f(\bar{\mathbf{y}}, \mathbf{s}|\bar{\mathbf{x}})$ denote the joint probability density function of the length T channel output vector $\bar{\mathbf{y}}$ and the channel state $\mathbf{s}$, given the length T input vector $\bar{\mathbf{x}}$ in one coherence block. Let $f(\bar{\mathbf{x}}|\hat{\mathbf{s}})$ denote the probability density function of the input given the estimate of the channel state $\hat{\mathbf{s}}$. Then we have

$$E_g(R, R_{csi}) = \max_{0\leq\rho\leq 1}\left\{\frac{1}{T}E_o(\rho) - \rho R\right\} \tag{15}$$

where

$$E_o(\rho) = -\log\int\Big(\int_{\bar{\mathbf{x}}}(f(\bar{\mathbf{y}},\mathbf{s}|\bar{\mathbf{x}}))^{1/(1+\rho)} f(\bar{\mathbf{x}}|\hat{\mathbf{s}})d\bar{\mathbf{x}}\Big)^{(1+\rho)} d\bar{\mathbf{y}}d\mathbf{s} \tag{16}$$

and

$$C_1(R_{csi}) = \max_{\bar{\mathbf{x}}_1(\hat{\mathbf{s}}),\bar{\mathbf{x}}_2(\hat{\mathbf{s}})} \frac{1}{T}\int f(\bar{\mathbf{y}},\mathbf{s}|\bar{\mathbf{x}}_1(\hat{\mathbf{s}}))\log\frac{f(\bar{\mathbf{y}},\mathbf{s}|\bar{\mathbf{x}}_1(\hat{\mathbf{s}}))}{f(\bar{\mathbf{y}},\mathbf{s}|\bar{\mathbf{x}}_2(\hat{\mathbf{s}}))}d\bar{\mathbf{y}}\,d\mathbf{s} \tag{17}$$

where the maximization is over all codewords $\bar{\mathbf{x}}_1(\hat{\mathbf{s}})$ and $\bar{\mathbf{x}}_1(\hat{\mathbf{s}})$. Both E_g and C_1 are assumed to be continuous and increasing functions of R_{csi}.

[3] We use the random coding exponent E_g instead of the sphere packing exponent, as in [4], to simplify the analysis. Although this gives a slightly looser upper bound on P_e, it is unlikely to change our results.

For the block Rayleigh fading channel channel considered, we can evaluate (16) and (17) and apply Jensen's inequality to obtain the upper bound

$$E_o(\rho) \leq N\rho e^{-N_G t_o/\sigma_h^2}\log\left(1 + \frac{P_o(t_o+\sigma_h^2)}{\sigma_z^2(1+\rho)}\right) \tag{18}$$

Using (18), it is easy to show that the random coding exponent (15) satisfies

$$E_g(R, R_{csi}) \leq C(R_{csi}) - R. \tag{19}$$

Numerical results indicate that this upper bound is fairly tight. We will therefore assume that the bound is the exact value when solving for the optimal CSI and RSI trade-off. Also,

$$E_g(R-\Delta R, R_{csi}) - E_g(R, R_{csi}) \leq \Delta R \tag{20}$$

with equality if R is sufficiently below the capacity.

To evaluate C_1 in (17) we assume that the two input sequences during the synchronization phase have the elements $+\sqrt{P_o}1_{\{|h_{gi}|^2\}\geq t_o}$ and $-\sqrt{P_o}1_{\{|h_{gi}|^2\}\geq t_o}$, respectively, which maximize the distance subject to the input power constraint. We can then evaluate (17) as

$$C_1(R_{csi}) = \frac{4P(t_o+\sigma_h^2)}{\sigma_z^2}. \tag{21}$$

III. Problem Statement

Assuming that on average R_f nats/channel use are available for feedback, the problem is to determine the optimal split between CSI and RSI. The objective is the asymptotic decay rate, or exponent, of the probability of decoding error. More precisely, we wish to maximize

$$\lim_{n\to\infty} -\frac{1}{n}\log P_e^{ub} \tag{22}$$

over $(R_{csi}, R_{rsi}, \lambda)$ subject to

$$\lambda C(R_{csi}) \geq R \tag{23}$$
$$R_{csi} + R_{rsi} \leq R_f \tag{24}$$

where (23) ensures that the forward rate constraint is met with the RSI feedback scheme.

We assume that $C(R_f) \geq R$ so that the problem is feasible. Note that the objective (22) with finite n corresponds to minimizing the probability of decoding error assuming a decoding delay of n coherence blocks. An equivalent objective to (22) is to maximize the smaller one of the two exponents E_1 and E_2 given by (13) and (14) respectively.

Although our objective is to maximize (22), to simplify the analysis, we will assume that the CSI parameters N_G and t_o are chosen to maximize the achievable rate, as in Section II. Although those parameters may change according to the different objectives, we do not expect that using the capacity objective will affect our asymptotic results.

IV. Optimal Allocation of Feedback

In this section we investigate the trade-off between CSI and RSI. To simplify the analysis we make the following approximations: The bounds (19) and (20) are the exact values and the ratio $C(R_{csi})/C_1(R_{csi}) \approx \phi$ where ϕ is a constant. When R_{csi} satisfies (6), using (7) and the optimal value of t_o, we approximate $\phi = (\log(1+u^\star))/(4u^\star) \approx 0.1$, and similarly for the range (8), we take $\phi = 1/4 = 0.25$. The numerical results in the next section show that the corresponding analysis provides accurate insight into the optimized CSI and RSI trade-off.

Recall that the objective is to maximize the minimum of E_1 and E_2, which are increasing and decreasing functions of λ, respectively. If $E_2 > E_1$, then λ can be increased until $E_1 = E_2$. This is possible since $E_1 \geq E_2$ for $\lambda = 1$. On the other hand, if $E_2 < E_1$, then we wish to decrease λ until $E_1 = E_2$. Equating (13) and (14), assuming that the bound (20) is exact, we get the optimum time-sharing parameter

$$\lambda^\star = 1 - \frac{2R_{rsi}}{C_1(R_{csi})}. \tag{25}$$

This assumes that sufficient CSI is available to support the forward rate, i.e.,

$$R \leq \lambda^\star C(R_{csi}) = C(R_{csi}) - 2\phi R_{rsi}. \tag{26}$$

The minimum CSI feedback rate, corresponding to equality in (26), is denoted as R_{csi}^{min}, which is no greater than R_f.

Since λ can be chosen to equate E_1 and E_2, we define $E_1^\star(R_{csi}, R_{rsi}) = E_1(R, R_{csi}, R_{rsi}, \lambda^\star)$, and rewrite the optimization problem (22)–(24) as

$$\begin{aligned} \text{maximize} \quad & E_1^\star(R_{csi}, R_{rsi}) & (27)\\ \text{subject to} \quad & R_{csi} + R_{rsi} \leq R_f & (28)\\ & 2\phi R_{rsi} \leq C(R_{csi}) - R & (29)\\ & 0 \leq R_{rsi} \leq R & (30) \end{aligned}$$

where (29) is a rearrangement of (26) which guarantees the balance of the two error exponents E_1 and E_2. Assuming the bounds (19) and (20) are tight, we have

$$\begin{aligned} E_1^\star(R_{csi}, R_{rsi}) &\approx 2\left[\lambda^\star C(R_{csi}) - R + R_{rsi}\right] & (31)\\ &= 2C(R_{csi}) + 2(1-2\phi)R_{rsi} - 2R. & (32) \end{aligned}$$

Evidently, $\partial E_1^\star/\partial R_{rsi} = 2(1-2\phi)$ is approximately constant, and $\partial E_1^\star/\partial R_{csi} = 2C'(R_{csi})$. The derivative $C'(R_{csi})$ is very large for small R_{csi} and decreases monotonically to zero as R_{csi} becomes large. As R_f increases from zero, we conclude that the benefit from CSI dominates that of RSI until the two derivatives are equal, and then the benefit from RSI becomes larger until $R_{rsi} = R$ or that the condition (29) is met with equality. From then on, more CSI is needed along with RSI to achieve the optimal allocation.

Let R_{csi}^{crit} be the solution to

$$C'(R_{csi}) = 1 - 2\phi \tag{33}$$

and $R_m = \max\left\{R_{csi}^{min}, R_{csi}^{crit}\right\}$. The optimal CSI-RSI split can now be specified as follows,

$$R_{csi}^\star = \begin{cases} R_f & \text{if } R_{csi}^{min} \leq R_f \leq R_m \\ R_m & \text{if } R_m < R_f \leq R_m + (C(R_m) - R)/(2\phi) \\ R_c & \text{otherwise} \end{cases} \tag{34}$$

where R_c is the solution to

$$R_f = R_c + (C(R_c) - R)/(2\phi). \tag{35}$$

Clearly, the trade-off exhibits three phases depending on the total amount of feedback R_f.

Proposition 1 allows us to determine R_{csi}^{crit}. Namely, if $T \leq (\log N)^{1-\frac{\epsilon_1}{2}}$ for large N, then R_{csi}^{crit} satisfies (6) and we can substitute the capacity expression (7) into (33). Taking $\phi = 0.1$ and also using the approximation $2(\log(1+u^\star))/\sqrt{u^\star} \approx 1.6$ gives

$$R_{csi}^{crit} \approx \frac{ST}{4}. \tag{36}$$

If $(\log N)^{1-\frac{\epsilon_1}{2}} < T$, then R_{csi}^{crit} satisfies (8). Substituting the capacity expression (9) into (33) and taking $\phi = 0.25$ gives the condition

$$\log\left(1 + \frac{S\log N}{TR_{csi}}a(N)\right) = S\frac{1+a(N)}{\frac{TR_{csi}}{\log N} + Sa(N)} + \frac{\log N}{2T}\ . \tag{37}$$

where $a(N) = \log[(N\log N)/TR_{csi}]$. For larger values of T, R_{csi}^{crit} can be shown to increase sublinearly with T (in contrast to (36)), and it is easy to show that $TR_{csi}^{crit} \leq S(\log N)^{2+\epsilon_2}$.

V. Numerical Results

This section presents numerical solutions to the constrained maximization of (22), and thereby demonstrates that the analytical results in this paper provide accurate insights into the optimized CSI and RSI trade-off for finite-size systems. Exact expressions for E_g and $C(R_{csi})$ are used and N_G and t_o are chosen to maximize the exponent (22) as opposed to the capacity $C(R_{csi})$. Unless stated otherwise, the SNR is normalized to $S = 1$, the number of sub-channels is $N = 1000$ and the transmission rate is assumed to be $R = 1.6$ nats per channel use.

Fig. 1 plots optimal amounts of CSI and RSI versus total feedback rate R_f for different coherence times. Due to the choice of R, here we have $R_{csi}^{crit} > R_{csi}^{min}$. Specifically, $R_m = R_{csi}^{crit} \approx 0.7$ and 0.82 for $T = 6$ and 8, respectively. As anticipated, the first R_m bits of feedback go to CSI, and zero is allocated to RSI. Thereafter, the CSI remains almost unchanged (decreases slightly) while the RSI increases. For $T = 6$, both CSI and RSI begin to increase simultaneously after the RSI reaches a certain value while for $T = 8$ the CSI remains constant (decreases slightly) as the RSI continues to increase.

Fig. 2 plots R_m (the first few bits that go into CSI before feedback bits are assigned to RSI) for different values of T corresponding to $S = 0dB$ and $5dB$. For $S = 0\,dB$ and approximately $T \leq 4$, $R_{csi}^{crit} < R_{csi}^{min}$, so that $R_m = R_{csi}^{min}$.

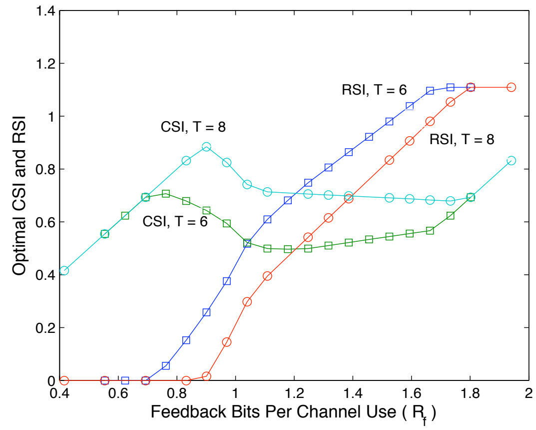

Fig. 1. Optimal CSI and RSI feedback versus total feedback rate.

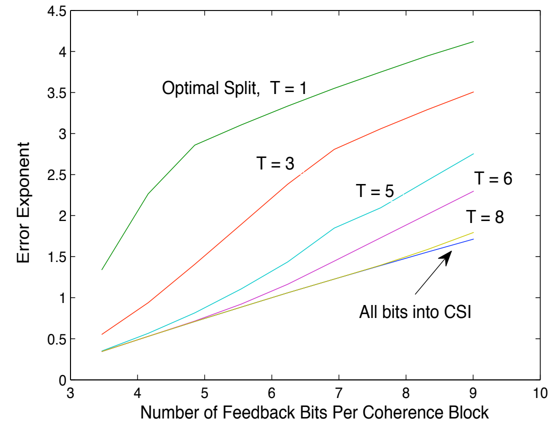

Fig. 3. Error Exponent versus feedback per coherence block (TR_f) for different values of T.

Note that R_{csi}^{min} is a decreasing function of T and hence the trend in the plot. For $4 \leq T \leq 8$, R_m increases approximately linearly and eventually decreases with T as predicted by the analytical results. Similar trends are observed for $S = 5\,dB$.

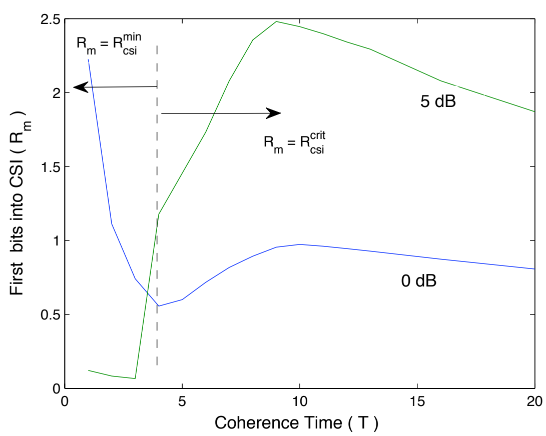

Fig. 2. R_{csi}^{min} versus T.

Fig. 3 compares the error exponent with the optimal allocation of feedback bits to the exponent achieved when all the feedback is allocated to CSI. The range of the horizontal axis is roughly the minimum to maximum values of CSI per coherence block. Clearly, allocating bits to RSI can provide substantial gains in the exponent for the smaller coherence times. For the larger coherence time it is optimal to allocate all bits to CSI.

VI. Conclusion

The trade-off between CSI and RSI has been studied in the context of block-fading multicarrier channels. Either CSI or RSI dominates the allocation as additional feedback becomes available, where the phase transition of the trade-off depends critically on the coherence time.

Of course, the results in this paper correspond to the particular channel model and feedback schemes assumed. Extending this framework to other channel models and feedback schemes may provide an interesting direction for future work.

References

[1] G. D. Forney, “Exponential Error Bounds for Erasure, List, and Decision feedback Schemes“, *IEEE Trans. Inform. Theory*, 14:206-220, March 1968.

[2] M. V. Burnashev, “Data transmission over a discrete channel with feedback. Random transmission time,” *Problems of Information Transmission*, vol. 12, pp. 10–30, 1976.

[3] J. M. Ooi, G. W. Wornell, ”Fast Iterative Coding Techniques for Feedback Channels”, *IEEE Transactions on Information Theory* 44(7): 2960-2976, November 1998.

[4] S.Draper, K. Ramchandran, B. Rimoldi, A. Sahai, D. Tse, “Attaining maximal reliability with minimal feedback via joint channel-code and hash-function design,” *Allerton Conference in Communication,Control and Computing*, Monticello IL, September 2005.

[5] S. Draper, A.Sahai, “Beating Burnashev in Delay with Noisy Feedback,” *Allerton Conference in Communication,Control and Computing*, Monticello IL, September 2006.

[6] D. Tse and P. Viswanath, “Fundamentals of Wireless Communication,” *Cambridge University Press*, 2005.

[7] J. Chen, R. Berry, M. L. Honig, “Performance of Limited Feedback Schemes for Downlink OFDMA with Finite Coherence Time,” *IEEE International Symposium on Information Theory*, Nice, France, June 2007.

[8] Y. K. Sun and M. L. Honig, “Asymptotic Capacity of Multi-Carrier Transmission with Frequency-Selective Fading and Limited Feedback”, submitted to *IEEE Trans. on Information Theory*, August 2006.

[9] S. Sanayei and A. Nosratinia, “Opportunistic downlink transmission with limited feedback,” IEEE Transactions on Information Theory, vol. 53, no. 11, pp. 4363-4372, November 2007.

[10] M. Agarwal, D. Guo, M. L. Honig, ”Multi-Carrier Transmission with Limited Feedback: Power Loading Over Sub-Channel Groups”, submitted to ICC 2008, Beijing, China.

[11] C.E.Shannon, R.G.Gallager, E.R.Berlekamp, ”Lower Bounds to Error Probability for Coding on Discrete Memoryless Channels-I”, Information and Control 10, 65-103, 1967.