Downlink OFDMA capacity analysis with channel estimation

Jieying Chen

EECS

March 9, 2006
Outline

1. Introduction
2. System Model
3. LMMSE Estimation
4. Effects of Estimation Error on channel capacity
5. Optimal Parameters
**Introduction**

- OFDMA downlink systems
- Time-variant fading sub-channels
- Training and channel estimation
- One-bit feedback
Training scheme

- The total bandwidth is divided into $N$ equal subchannels.
- The channel is assumed to be constant in $M$ channel uses.
- Average power constraint $P$ per channel use.

\[
T + D = M
\]
\[
\alpha \times P_T + (1 - \alpha) \times P_D = P
\]
\[
\alpha = \frac{T}{M}
\]
Pilot signals

- Known to both the transmitter and the receiver
- Transmitted during the training time, $T$ channel uses

\[ Y = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_T \end{bmatrix} h + \begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_T \end{bmatrix} \]

- $X_i$ is a diagonal matrix consists training symbols
On-off power allocation

- Receiver compares the estimation of channel gain with a given threshold $t_0$
- Feedback the result to the transmitter using one bit
- A constant power allocation scheme is adopted

$$P(\hat{h}_i) = \begin{cases} P_0 & \text{if } |\hat{h}_i|^2 \geq t_o \\ 0 & \text{otherwise} \end{cases}$$
LMMSE estimation error

- The estimation of the channel
  \[ \hat{h}_T = \omega^H Y_{MF} = E(h_T Y_{MF}^H) (E(Y_{MF} Y_{MF}^H))^{-1} Y_{MF} \]

- Average channel estimation error per sub-channels
  \[ \sigma_e^2 = \frac{1}{N} E(|h - \hat{h}|^2) = \frac{1}{N} \sum_{i=1}^{N} \frac{\sigma_n^2 \lambda_i}{N \sigma_n^2 + P_T T \sigma_h^2} \]

\[ \frac{\sigma_n^2 \sigma_h^2}{N \sigma_n^2 + P_T T N \sigma_h^2} \leq \sigma_e^2 \leq \frac{\sigma_n^2 \sigma_h^2}{N \sigma_n^2 + P_T T \sigma_h^2} \]

- Since autocovariance matrix $R_h$ is Toeplitz, Durbin algorithm can be used to adaptively derive the eigenvalues of $R_h$. 

Channel capacity loss due to LMMSE error

\[ C = \sum_{i=1}^{N} \max_{p(x_i|\hat{h}_i)} I(X_i, Y_i|\hat{h}_i) \]

The input distribution \( p(x_i|\hat{h}_i) \) maximizes the mutual information is unknown

If \( p(x_i|\hat{h}_i) \) is Gaussian, and LMMSE used at the receiver,
\[ I(X_i, Y_i|\hat{h}_i) \geq E_{\hat{h}_i} (\log(1 + \frac{P_0|\hat{h}_i|^2}{P_o\sigma_e^2 + \sigma_n^2})) \]
Channel capacity loss due to LMMSE error

For one coherence time $M$ channel uses, the capacity is lower bounded by

$$C_i = \log(1 + \frac{P_o |\hat{h}_i|^2}{P_o \sigma_e^2 + \sigma_n^2})$$

For one feedback bit

$$C_{2,i} = \log(1 + \frac{P_o t_0}{\sigma_n^2}) \times \text{Prob.}(|\hat{h}_i|^2 > t_0) \times (1 - P_{\text{outage}}) \times (1 - \alpha)$$

where $P_{\text{outage}} = \text{Prob.}(\log(1 + \frac{P_o t_0}{\sigma_n^2}) > C | |\hat{h}_i|^2 > t_0) = 1 - e^{-\frac{P_o \sigma_e^2}{\sigma_n^2 \sigma_h^2} t_o}$
Channel capacity loss due to LMMSE error

\[
\frac{C_{2,i}}{C_i} = e^{-\frac{t_0\sigma_e^2}{\sigma_n^2 - \sigma_e^2} \left( \frac{P_o}{\sigma_n^2} + \frac{1}{\sigma_h^2} \right)} \ast (1 - \alpha)
\]

If \( a \triangleq \frac{\sigma_e^2}{\sigma_h^2} \ll 1 \), \( \frac{C_{2,i}}{C_i} \equiv 1 - \left( \frac{P_o}{\sigma_n^2} + \frac{t_0}{\sigma_h^2} \right) a + o(a^2) \)
Optimization

The objective function when total power constraint is imposed

\[ C_{2,i} = \log(1 + \frac{P'_D t_0}{\sigma_n^2}) \times \text{Prob.}(\hat{h}_i^2 > t_0) \times (1 - P_{\text{outage}}) \times (1 - \alpha) \]

subject to

\[ T + D = M \]
\[ \alpha \times P_T + (1 - \alpha) \times P_D = P \]
\[ P'_D = \frac{P_D}{\text{Prob.}(\hat{h}_i^2 > t_0)} \]

(2)
Optimization

- \( C_{2,i} \) is a function of \( \alpha, t_o, \gamma \triangleq \alpha * P_T \)

- \( \alpha_o = \frac{1}{M} \)

- \( t_o \) and \( \gamma \) should satisfy the following

\[
\log(\log(1 + \frac{(P-\gamma)t_o}{(1-\alpha_o)\sigma_n^2} e^{\frac{t_o}{\sigma_h^2}}))) = \frac{t_o}{\sigma_h^2-\sigma_e^2} \left(1 - \frac{(P-\gamma)\sigma_e^2}{(1-\alpha_o)\sigma_n^2} e^{\frac{t_o}{\sigma_h^2}}\right)
\]

solution: numerically search
Thank you