

Jan 27, 2008 - Space and Time Hierarchies

Lecture note of EECS 395 Winter 2008 (Prof. Fortnow) taken by Bach Ha

Deterministic Spaces

- If $s_1(n), s_2(n) \geq \log n$ are space-constructible and $s_1(n) = o(s_2(n))$, then $DSPACE(s_1(n)) \subsetneq DSPACE(s_2(n))$.
- Proof Idea: We want to construct a set A in $DSPACE(s_2)$, but not in $DSPACE(s_1)$ using diagonalization.
- Assumption of linear tape compression: We can create a one-tape TM for any multiple-tape TM and still use the same space.
- Define $A = \{\langle M \rangle 01^j \mid M(\langle M \rangle 01^j) \text{ rejects using space at most } s_2(|\langle M \rangle 01^j|)\}$.
- We claim that $A \in DSPACE(s_1(n))$
 - We define a TM N which has its input is $\langle M \rangle 01^j$
 - Before simulation, let N marks all of its working tape at the $s_2(n)$ space
 - If the simulation goes pass this mark, then halt and reject
 - If simulation ends in reject, we accept.
 - Thus, $A \in DSPACE(s_2(n))$
- Now we need to show that $A \notin DSPACE(s_1(n))$.
 - Suppose the contrary, then there exists M , M accepts A using $s_1(n)$ space
 - WLOG, assume that M has a single tape.
 - We have for an input $x = 1^i 0^{j-i}$, with a large enough j , the simulation of $M(x)$ succeeds in space $c \cdot space_M(j) \leq c \cdot s_1(j) \leq s_2(j)$.
 - Hence, $x \in A \Leftrightarrow N$ accepts $x \Leftrightarrow M$ rejects x . Thus $A \neq L(M)$. Contradiction.
- The same proof can be applied to Nondeterministic space case (Homework).
- This technique does not work with time since we do not have time compression.
- Without time compression, we need to have polynomial (time is not reducible). Hence, on the analysis, we should have quadratic time.
- However, we can always convert to a 2-tape machine in a logged amount of time, thus the square is now removed by log function.

Nondeterministic Time

- Due to confusion of this proof, it's better to direct to a paper from which the proof was originated. It is: S. Žák, A Turing Machine Time Hierarchy, Theoretical Computer Science, vol. 26, pp. 327-333, 1983. This journal is accessible at <http://www.sciencedirect.com/science/journal/03043975>.