Lecture 6

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$$Dtime(t(n)) \subseteq Ntime(t(n)) \subseteq Dspace(t(n)) \subseteq Nspace(t(n)) \subseteq \bigcup_{c>0} Dtime(C^{t(n)})$$

$$Ntime(t(n)) \subseteq Dspace(t(n))$$

Let there be a D.T.M. M with multiple tapes simulating a N.T.M N. One tape of M can be a "guess tape" which will hold a specific set of choices for the the value of the delta function chosen from the power set of choices in N. M will use its other tapes to simulate the behavior of a D.T.M similar to N in all respects except the delta function, which will instead be the value of the guess tape. M will run through all possible values of the guess tape, thus adequately simulating N. The space taken will be the maximum size of any of the tapes:

$$|guess\ \ tape| = O(t(m))$$

$$|work\ \ tapes| = O(t(n))$$

So :

$$Ntime(t(n)) \subseteq Dspace(t(n))$$

Savitch's Theorem

$$Nspace(S(n)) \subseteq Dspace(S^2(n))$$

Proof by Tableau:

$Conf_0$
$Conf_1$
$Conf_2$
...
$Conf_{final}$

A Tableau is proper for M(x) if:

1. Configuration 0 on the Tableau is the initial configuration of M(x)
2. The final configuration on the Tableau is an accepting configuration of M(x)
3. Configuration i + 1 is one step of the delta function away from configuration i for all i

A T.M. M will accept if and only if there is a proper Tableau for M(x)

Define a function Check, taking as arguments two configurations (a and b)and a number of steps (t), which will return true if there is a path from a to b in t steps or less.

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Pseudo Code:
if (t = 0) accept if a = b else reject;
if (t = 1) accept if b is one step of the delta function from a; //easy to check
for all configurations m where a < m < b {
        accept if Check(a, m, t / 2) && Check(m, b t/2)
} else reject;
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$$S_t = S(n) + S(t/2) = S(n)\log(t)$$

$$t = C^{S(n)}$$

$$S(n)\log(C^{S(n)}) = S^2(n)$$

Check must be called once for all accepting configurations to see if a valid Tableau exists for a path from the initial configuration of the machine to the aforementioned accepting configuration, a process that requires less space than Check itself, and may thus be ignored.

Other Notes:

if $L \in Dtime(t(n))$ then $\bar{L} \in Dtime(t(n))$ (not true for Ntime)

$$Nspace(S(n)) = co \cdot Nspace(S(n))$$