EECS 395 - Complexity Theory Notes

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1 Recap

So far everything in class has been computability theory. Now we will deal with computational complexity: Model of what can be done efficiently in time and memory.
Model of TM: usual TM with one read-only tape (input) and several work tapes (memory). Call this one DTM D for deterministic.
By time we mean the number of applications of the TM's δ function.
By space or memory we mean the size of the work tapes.
$t_M(x)$ is the time TM M takes on x.
$s_M(x)$ is the amount of space TM uses on x.

2 DTIME and DSPACE

$$DTIME(t(n)) = \{L \mid \exists\, DTM\ M \text{ such that } L(M) = L \text{ and } t_M(x) = O(t_M(|x|))\}$$

$$DSPACE(s(n)) = \{L \mid \exists\, DTM\ M \text{ such that } L(M) = L \text{ and } t_M(x) = O(s_M(|x|))\}$$

3 Non-determinisitc

N is for non-deterministic.
NTM is just like usual TM except that δ function now maps to the power set of $Q \times \Gamma \times \{L, R\}$.
Deterministic: $\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$
Non-deterministic: $\delta : Q \times \Gamma \rightarrow P(Q \times \Gamma \times \{L, R\})$
We say that NTM accepts if there exists some sequence of applications of δ leading to q_{acc}
c.e. by non-deterministic implies c.e. by deterministic.
NTIME and NSPACE are defined analogously to DTIME and DSPACE only with NTMs instead of DTMs.

4 Main Results

$$DTIME(t(n)) \subseteq NTIME((t(n)) \subseteq DSPACE(t(n)) \subseteq NSPACE(t(n)) \subseteq \bigcup_{c>0} DTIME(c^{t(n)})$$

We will always assume that $t(n) \geq n$ since reading the input is $O(n)$
Similarly we will always assume that $s(n) \geq log(n)$ since the amount of memory needed to specify the position of the head of the input tape is $O(log(n))$.

$$DTIME(t(n)) \subseteq DSPACE(t(n))$$
$$NTIME(t(n)) \subseteq NSPACE(t(n))$$

We will call the tuplet (state, tape contents (except for input tape), head positions) the configuration of a TM.
Space s(n): how many configurations? $|Q| \times |\Gamma|^{ks(n)} \times s(n)^k \leq c^{s(n)}$

Theorem $NSPACE(s(n)) \subseteq \bigcup_{c>0} DTIME(c^{s(n)})$
Proof Suppose $L \in NSPACE(s(n))$ via NTM M. Consider a directed graph as defined as follows: each configuration of M(x) is a node and $(c_a, c_b) \in Edges$ if M can get from config c_a to config c_b. M(x) will accept if there is a path from the initial configuration to an accept configuration. Do a standard search (eg depth first) which will be of polynomial order on the vertex set v. Then

$$|v|^2 \leq (c^{s(n)})^2 = (c^2)^{s(n)}$$

and so

$$NSPACE(log(n)) \subseteq \bigcup_{c>0} DTIME(c^{log(n)}) = \bigcup_{k>0} DTIME(n^k)$$