# Jan 09, 2008 - Continue on Turing Machine

Lecture note of EECS 395 Winter 2008 (Prof. Fortnow) taken by Bach Ha

Quick Review – See the previous notes about

- Concept of a Turing Machine (TM)
- Formal definition of a TM (Note: everything is finite except the tape)
- Some terminologies such as alphabet, string, language...

Example of a TM

- Problem: Consider the alphabet $\Sigma = \{0\}$ and the language $L = \{0^{2^n} \mid n \geq 0\} = \{0, 00, 0000, \ldots\}$. We need to create a TM that accepts this language.
- Idea: The idea is to go from the beginning of the tape and cross out every other 0. It rejects when we see a non-0 or the the end of the tape is crossed out while there are still 0s left.

Computably Enumerable and Computable.

- A language $L$ is **computably enumerable** (c.e.) if there is a TM $M$ such that $L = L(M)$. For any $x$:
  - if $x \in L$ then $M(x)$ accepts,
  - if $x \notin L$, then either $M$ rejects or $M$ doesn't halt
- Note: we cannot distinguish between a TM that takes a long time to run and TM that goes into an infinite loop
- $L$ is **computable** if there is a TM $M$ such that, for all $x \in Z^*$, we have:
  - if $x \in L$ then accepts
  - if $x \notin L$ then rejects

Some equivalent terminologies

- computably enumerable: recursive enumerable (r.e.), acceptable, recognizable
- computable: recursive, decidable.

Church-Turing Thesis

- Church-Turing Thesis: Everything computable is computable by a TM.

- This thesis still holds today.
- Elaboration on why this thesis seems to be true by consider some alternative computation models: Multi-tape TM and Random Access Machine (RAM).

Multi-tape TM

- A multi-tape TM is a TM that has $k$ tapes instead of 1 with multiple heads.
- We define this machine the same way except for the $\delta$ function:

$$\delta : Q \times \Gamma^k \to Q \times \Gamma^k \times \Sigma^k$$

.

- We will argue that this multi-tape TM is not more powerful than our original TM.
- The basic idea is to consider an original TM that has a "fat" tape with a lot of tracks on it. This will take care of the multiple tape aspect. To take care of the multi head, we use marks to write the position these heads instead using the actual heads.

Random Access Machine

- Random access means memory location can be accessed directly.
- Here we consider a RAM that has a RAM memory and some registers. These this registers point to locations in RAM.
- We need to simulate this using TM to argue that it is not powerful.
- In fact, we will simulate it using a multi-tape TM: Let each tape to be a register, and the last one to be the RAM, and a scratch tape (for counter...).

Next class: Things that we can't do using TM and the notion of Universal TM.