

EECS 395-23 – Assignment 3

1. A class of languages \mathcal{C} is closed under disjoint union if for all A and B in \mathcal{C} , the language

$$\{0x \mid x \in A\} \cup \{1y \mid y \in B\}$$

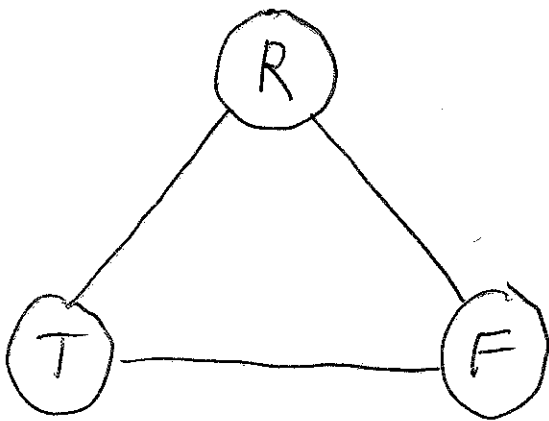
is also in \mathcal{C} . The class \mathcal{C} is closed under projections if for all L in \mathcal{C} , the languages

$$\{x \mid 0x \in L\} \text{ and } \{y \mid 1y \in L\}$$

are both in \mathcal{C} . We say a class \mathcal{C} is nice if it is closed under disjoint union and projections.

- (a) Show that NP is nice. Note the same proof applies to all complexity classes we have (and will) study.
 - (b) Show no nice class is the non-trivial union of two other nice classes, i.e., if \mathcal{C} , \mathcal{D} and $\mathcal{C} \cup \mathcal{D}$ are all nice then either $\mathcal{C} \subseteq \mathcal{D}$ or $\mathcal{D} \subseteq \mathcal{C}$.
2. Exhibit two sets in P that are not NP-complete. Show there are more than two sets in P that are not NP-complete if and only if $P \neq NP$.
 3. Define 3-COL (3-coloring) as the set of graphs $G = (V, E)$ such that there is a function $f : V \rightarrow \{1, 2, 3\}$ such that for every $(u, v) \in E$, $f(u) \neq f(v)$. Show that 3-COL is NP-complete. Hint: Use gadgets on following page.
 4. Define ONE-IN-THREE 3SAT as the set of 3CNF formulas that have an assignment that makes exactly one literal true in every clause. Show that ONE-IN-THREE 3SAT is NP-complete.
 5. A log-space computable function is a function f computable on a Turing machine with a read-only input tape, a write-only (head only moves right) output tape and has a fixed number of read-write $O(\log n)$ -size work tapes. Show that log-space computable functions are closed under composition.

PALETTE



COLORS

T = TRUE

F = FALSE

R = RED

OR-GADGET

