EECS 395-23 – Assignment 1

1. Consider the variation of a Turing machine which has an infinite two-dimensional tape where the head can move up and down, as well as right and left.

   (a) Give a formal definition for this 2-D Turing machine.

   (b) Show that any 2-D Turing machine can be simulated by the classical 1-D machine.

2. For any pair of languages A and B let

   $$AB = \{x \cdot y \mid x \in A \text{ and } y \in B\}$$

   where $x \cdot y$ is string concatenation.

   (a) Show that if A and B are both computable then AB is computable.

   (b) Show that if A and B are both c.e. then AB is c.e.

3. A Turing machine M is called an enumerator if it has no input tape but has an output tape where it writes a list of strings $x_1\#x_2\#x_3\#\ldots$ possibly forever. We define

   $$L(M) = \{x_1, x_2, x_3, \ldots\}.$$

   Show that a language A is c.e. if and only if there is some enumerator M such that $A = L(M)$.

   Note: This is where the name "computably enumerable" comes from.

4. Let $L_f = \{\langle M\rangle \mid L(M)\text{is finite}\}$.

   (a) Show $L_h \leq L_f$.

   (b) Show $L_h \leq \overline{L_f}$.

   (c) Show this implies that L_f is neither c.e. or co-c.e.