Superior signal-to-noise ratio of a new AA1 sequence for random-modulation continuous-wave lidar

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In an earlier work [Proc. SPIE **4484,** 216 (2001)] we proposed a new AA1 modulation sequence for random-modulation continuous-wave lidar. It possesses significantly better signal properties than other pseudorandom codes (the M, A1, and A2 sequences). We derive and compare the signal-to-noise ratio (SNR) of the new AA1 sequence with those of previous modulation sequences. Using a figure of merit proposed for pseudorandom sequences in additive (and generally colored) noise, we show that the SNR of the AA1 sequence in $1/f$ noise can be as much as 50 times better than that of the commonly used M sequence. This improved SNR should permit as much as a 7:1 increase of the maximum lidar sensing range in baseband-modulation direct-detection infrared lidar with no significant changes to the transmitter and receiver.

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Laser radar (lidar) is a valuable tool for range-resolved remote sensing in a broad range of civilian and military applications ranging from hard-target detection and identification to detection of biological and chemical agents. However, applications of lidar that require compact size, light weight, and portability are usually constrained by the laser source. To date, the smallest devices appropriate for such applications are diode lasers, followed by other semiconductor lasers. Of these, the quantum cascade laser is promising.[1] Unfortunately, compact semiconductor lasers have output power levels well below those of their counterparts found in typical lidar systems, thereby yielding much shorter maximum sensing ranges.

To improve lidar range given a specific laser type and basic optical detection scheme (noncoherent–direct versus coherent–heterodyne optical detection), three strategies can be pursued: (1) increasing the laser output power, (2) improving the detector or its associated optics, or (3) improving the modulation scheme. Some progress can be expected in increasing the output power or efficiency of mid-infrared lasers operating near or at room temperature, as well as in improving the detectivity of semiconductor detectors operating above $\sim 1\ \mu$m owing to the increased, more stable internal gain, but such technological advances come at a great cost. Thus, compact lidar systems call for better modulation schemes. In this Letter we concentrate on improving the modulation scheme by employing a new modulation sequence.

Since the ratio of peak to average power is relatively low (typically 1 to 10) for semiconductor lasers that are appropriate for compact lidar applications, such sources are optimally utilized in high-duty-cycle, ideally continuous-wave (CW) modulation schemes. In such schemes, however, the detector receives backscattered returns from all distances at the same time, thereby requiring a more complex algorithm to recover range data than needed to process the time-delayed return signals generated in pulsed lidars. The basic solution to this problem has been known in the fields of radar and spread-spectrum communications and was applied to lidar for the first time by Takeuchi *et al.*[2] Here the fundamental idea is that range resolution in CW operation is preserved if the emitted signal is modulated such that its cross correlation with a demodulating signal is a cyclically repeated deltalike function of time. This allows retrieval of lidar returns from specific distances despite temporal overlap at the detector.

Apart from frequency modulation lidar (which requires excessively high modulation frequencies and detection bandwidth), the CW technique for achieving lidar range resolution is known as random-modulation continuous-wave (RM-CW) lidar,[2,3] sometimes also referred to as pseudorandom or pseudonoise modulation lidar. RM-CW lidar is defined by its modulation and demodulation sequences, collectively referred to as pseudorandom codes (PRCs) or pseudonoise codes and (or) sequences. On demodulation, this type of lidar yields values of the atmospheric response function at discrete points along the sensing dimension. In particular applications this function can be proportional to a hard target's reflectivity or to the position-dependent gas concentration or differential backscattering coefficient of an aerosol.[4]

However, not all pseudorandom modulation sequences have the same signal-to-noise performance. In earlier work we proposed the new AA1 modulation sequence for RM-CW lidar.[4,5] Each element of the AA1 sequence is defined as the sum of two adjacent elements of the A1 sequence:

$$a_i^{\mathrm{AA1}} \equiv a_i^* + a_{i+1}^*, \tag{1}$$

where a_i^{AA1} denotes the ith element of the AA1 sequence and a_i^* denotes the ith element of the A1 sequence.[6] Therefore the new AA1 sequence is ternary; i.e., its elements have three possible levels as opposed to the A1 sequence, which is binary. The demodulation sequence assigned to AA1, denoted by $a^{*'}$, is a bipolar replica of A1; i.e., it assumes the values -1 and $+1$ when the unipolar A1 assumes the values 0 and $+1$, respectively.

The signal properties of a given PRC are determined by the cross correlation between the modulation and the demodulation sequence. Examination of the cross-correlation functions of all pseudorandom sequences previously used in lidar (namely, the M, A1, and A2 sequences) and the AA1 sequence shows that the signal properties of the latter are superior.[4,5] Specifically, the AA1 sequence has zero cross-correlation values between its principal peaks, a property it shares with the M sequence. This results in zero cross talk between backscattered signals originating from different distances along the sensing range. In contrast, the A1 and A2 sequences possess nonzero values (corrugation or ripple) between the principal peaks of their respective cross-correlation functions. The resulting cross talk (or clutter) has been shown to degrade lidar performance unacceptably when one is using either of these sequences during cloudy conditions.[7]

At the same time the new AA1 sequence has a balanced demodulation sequence, a property it shares with the A1 and A2 sequences. That is, its demodulation sequence consists of the same number of high and low chips. Consequently, the AA1 sequence (as well as A1 and A2 sequences) is immune to vertical shifts of its modulation levels.[4] This property is important in situations in which the modulation levels undergo linear scaling, for example, as a result of unavoidable offsets in practical systems, undesirable residual amplitude modulation in semiconductor lasers with wavelength constraints, or in devising schemes for (range-resolved) differential absorption lidar. The M sequence does not possess this desirable feature since one of its modulation levels has to be zero to maintain its desirable cross-correlation (signal) properties. Balance in a demodulation sequence is important for yet another reason: to improve its noise performance.[4,8]

The alternating polarity of the principal peaks in the cross-correlation function of the AA1 sequence (as well as in A1 and A2)[4,5] is of little or no practical importance, because the minimum length of the pseudorandom (cyclic) sequences is always chosen such that there is no measurable backscattering signal originating from two consecutive cross-correlation peaks. Their truncated shape in the case of the AA1 sequence[4,5] is also practically inconsequential.

Therefore, the new AA1 sequence possesses both the desirable cross-correlation property of the M sequence and the balanced demodulation sequence property of the A1 and A2 sequences. This combination provides the AA1 sequence with nearly ideal signal characteristics for RM-CW lidar.

One expects that, in a compact, portable lidar based on a semiconductor laser and detector and optically noncoherent (direct) photodetection, the system noise is additive and dominated by the detector. This is particularly true at infrared wavelengths greater than approximately 1–2 μm where signal shot-noise-limited photodetection would be practically impossible, primarily because of technological barriers. Namely, mid-infrared detectors with sufficient and stable gain to overcome thermal noise do not exist, and their low shunt resistance and (or) high dark current typically yield greater noise than the thermal noise of the following amplifier–load.[4,8]

We have derived the (postdemodulation) signal-to-noise ratio (SNR) in lidar operating at this regime, based on fundamental lidar component specifications: the maximum instantaneous laser power, the detector's area and detectivity with an arbitrary noise power spectral density distribution (colored noise), the autocorrelation and cross-correlation properties of the PRC, and the atmospheric response function.[4,8] The atmospheric response function, originally defined for distributed aerosol sensing, can also be defined and (or) generalized to cover hard target (surface) sensing[9] and other types of lidar application. As the only relevant signal property in the derivation of the SNR is the height of the main cross-correlation peak, this formula cannot describe limitations (such as the different immunity to cross talk) of PRCs associated with their cross-correlation function irregularities. These limitations–signal properties can be deduced from the cross-correlation function of a given PRC and have been summarized above for the known RM-CW lidar PRCs.

We have shown[4,8] that SNRs of the M, A1, and A2 sequences are practically identical in the additive-noise regime for white noise, as well as in the photon shot-noise regime. However, their SNRs generally differ in the additive-noise regime for colored noise, which is typical in direct detection in the mid infrared. Their relative performance in a given colored noise depends exclusively on the autocorrelation function of the demodulation sequence.[4,8]

Therefore we propose the following definition of a SNR performance measure (SNR figure of merit) of a pseudorandom sequence in the additive-noise regime:

$$(\mathrm{S/N})_{j,\,\text{colored noise}} = \epsilon (\mathrm{S/N})_{j,\,\text{white noise}}\,, \tag{2}$$

where $(\mathrm{S/N})_{j,\,\text{colored noise}}$ and $(\mathrm{S/N})_{j,\,\text{white noise}}$ are the postdemodulation SNR in colored noise and in white noise, respectively, associated with the detection of the discrete atmospheric response function, G_j, in the jth range bin.[4,8] This equation defines ϵ, which we designate the sequence-enhancement factor. We interpret ϵ as the relative enhancement of the lidar SNR as a result of the use of a given PRC in a given colored noise compared with the corresponding SNR in white noise of given spectral density.

One of the benefits of defining ϵ is that important formulas for RM-CW lidar SNR and maximum sensing range in the general case of additive colored noise can be greatly simplified and related to the white-noise case, which is insensitive to the PRC. We also note that ϵ appears as a multiplier of the (implicit) laser power in the SNR expression. Therefore, the overall performance of RM-CW lidar can be improved as much by choosing a PRC with large ϵ as by increasing the laser output power by the same factor. Since the latter is difficult because of technological limitations, devising PRCs with large ϵ is an effective and almost penalty-free alternative means of improving the lidar performance.

Since, in our framework, signal performance is independent of noise, ϵ can be expressed as the ratio of the output (postdemodulation) noise power for white noise and colored noise:

$$\epsilon = \left[\frac{\langle N^2_{\text{out, white}}(t) \rangle}{\langle N^2_{\text{out, colored}}(t) \rangle} \right]^{1/2}, \tag{3}$$

which in the most general case further equals[4,8]

$$\epsilon = \left[\frac{\eta/2T}{\sum_{n=-\infty}^{\infty} c_n \int_{-\infty}^{\infty} \dfrac{\eta(f - nf_0)}{2} \operatorname{sinc}^2(fT)\mathrm{d}f} \right]^{1/2}, \tag{4}$$

where c_n are the Fourier coefficients of the normalized autocorrelation function of the demodulation sequence, $\eta(f)$ is the positive-frequency colored noise power spectral density, f_0 is the PRC repetition frequency (the inverse of its period T_0), T is the averaging time, and η is the constant positive-frequency white-noise power spectral density.

Since the AA1 sequence uses A1 as the demodulation sequence, and the A1 and A2 sequences have been shown to have practically identical properties,[4,8] it is sufficient in our analysis to compare all these sequences collectively with the M sequence. Also, since we are only interested in comparing their ϵ factors and not their absolute values (which would require specifying the level of white noise, η, or equivalently the detector's detectivity, D^*), we will evaluate only the following ratio:

$$\frac{\epsilon_{\mathrm{A}}}{\epsilon_{\mathrm{M}}} = \left[\frac{\langle N^2_{\text{out}}(t) \rangle_{\mathrm{M}}}{\langle N^2_{\text{out}}(t) \rangle_{\mathrm{A}}} \right]^{1/2} = \frac{(\mathrm{S/N})_{\mathrm{A}}}{(\mathrm{S/N})_{\mathrm{M}}}$$

$$= \left[\frac{\sum_{n=-\infty}^{\infty} c_n^{(\mathrm{M})} \int_{-\infty}^{\infty} \dfrac{\eta(f - nf_0)}{2} \operatorname{sinc}^2(fT)\mathrm{d}f}{\sum_{n=-\infty}^{\infty} c_n^{(\mathrm{A})} \int_{-\infty}^{\infty} \dfrac{\eta(f - nf_0)}{2} \operatorname{sinc}^2(fT)\mathrm{d}f} \right]^{1/2}, \tag{5}$$

where the subscripts and superscripts A denote the A1, A2, or AA1 sequence and the subscripts and superscripts M denote the M sequence. In Eq. (5) we assume that the basic signal performance of both classes of PRC is the same, which is true for all PRCs discussed in this Letter.

Assuming $\eta(f) = \text{const}/|f|$ and using $T = kT_0 = k/f_0$, where k is the number of PRC cycles included in the averaging, we can evaluate the integrals in Eqs. (5), given the autocorrelation functions of the demodulation sequences[4,8] with their graphs[4,5] and Fourier coefficients c_n.[4,8] We note that these integrations cannot be performed analytically and contain a singularity. However, we can find approximate limiting values and thereby express ϵ_A/ϵ_M in terms of k and N (where N is the length of one cycle of the PRC) if we approximate all the sinc-squared functions with appropriate rectangular functions and set the lower integration limits to $1/2T$.[4] The choice of $1/2T$ as the lower cutoff frequency in integrating the $1/f$ noise is dictated by the range of validity of our stochastic analysis and removes the singularity around $f = 0$ ($n = 0$). With these approximations, we find[4]

$$\frac{\epsilon_A}{\epsilon_M} \overset{1/f \text{ noise}, N \gg 1}{\cong} \left[\frac{(N \ln N)/k + 1}{(N \ln N)/k} \right]^{1/2} \overset{k \gg N}{\cong} \left(\frac{k}{N \ln N} \right)^{1/2}. \tag{6}$$

For typical experimental conditions of $N \cong 1000$, chip length $\Delta t = 50$ ns, and $T = 5$ s, we obtain $k = 10^5$ and $\epsilon_A/\epsilon_M = 3.8$ (a 3.8-times-improved SNR). These results correspond to an approximate doubling of the lidar sensing range relative to the most commonly used PRC, the M sequence. On pushing the experimental parameters to illustrate the possibility of even better lidar performance ($N = 256$, $\Delta t = 30$ ns, and $T = 30$ s), we obtain $k = 3.88 \times 10^6$ and $\epsilon_A/\epsilon_M = 52$. This corresponds to an approximate sevenfold increase of the lidar sensing range relative to the most commonly used M sequence.

In summary, we have theoretically obtained promising results for the performance of the new AA1 modulation sequence for direct-detection RM-CW infrared lidar. We derived and compared the SNR of the new AA1 sequence with those of previous commonly used modulation sequences. Using a figure of merit proposed here for pseudorandom sequences in additive (and generally colored) noise, we showed that the SNR of the AA1 sequence in $1/f$ noise can be as much as 50 times better than that of the commonly used M sequence. This improved SNR should permit as much as a 7:1 increase in the maximum lidar sensing range with no increase in laser output power.

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References

1. C. M. Gittins, E. T. Wetjen, C. Gmachl, F. Capasso, A. L. Hutchinson, D. L. Sivco, J. N. Baillargeon, and A. Y. Cho, Opt. Lett. **25,** 1162 (2000).
2. N. Takeuchi, N. Sugimoto, H. Baba, and K. Sakurai, Appl. Opt. **22,** 1382 (1983).
3. N. Takeuchi, H. Baba, K. Sakurai, and T. Ueno, Appl. Opt. **25,** 63 (1986).
4. A. Rybaltowski, "Investigation of laser radar systems based on mid-infrared semiconductor lasers," Ph.D. dissertation (Northwestern University, Evanston, Ill., 2002).
5. A. Rybaltowski and A. Taflove, Proc. SPIE **4484,** 216 (2001).
6. Ch. Nagasawa, M. Abo, H. Yamamoto, and O. Uchino, Appl. Opt. **29,** 1466 (1990).
7. Y. Emery and C. Flesia, Appl. Opt. **37,** 2238 (1998).
8. A. Rybaltowski and A. Taflove, Opt. Express **9,** 389 (2001), http://www.opticsexpress.org.
9. A. Rybaltowski and A. Taflove, Proc. SPIE **4546,** 32 (2001).