Spatial Soliton Deflection Mechanism Indicated by FD-TD Maxwell's Equations Modeling

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Abstract— **We present first-time calculations from the time-domain vector Maxwell's equations of spatial optical soliton propagation and mutual deflection, including carrier waves, in a 2-D homogeneous Kerr-type nonlinear dielectric. The nonlinear Schrödinger equation predicts that two co-propagating, in-phase spatial solitons remain bound to each other, executing a periodic separation. This disagrees with our new extensively tested finite-difference time-domain (FD-TD) solution of Maxwell's equations. FD-TD shows that co-propagating in-phase spatial solitons become unbound, i.e. diverge to arbitrarily large separations, if the ratio of soliton beamwidth to wavelength is order 1 or less. Not relying upon paraxial approximations or analogies to temporal soliton interactions, FD-TD appears to be a robust means of obtaining detailed models of the interaction of sub-picosecond pulsed light beams in nonlinear media directly in the space-time domain.**

I. Introduction

Spatial solitons have been observed in planar waveguides where self-focusing provides confinement of the beam in one transverse dimension and the refractive index profile of the waveguide provides confinement in the other transverse dimension [1]. The behavior of spatial solitons in Kerr-type nonlinear materials has also been predicted by nonlinear Schrödinger equation (NLSE) models and numerical beam propagation models [2] that generally make the paraxial approximation.

In principle, the behavior of electromagnetic fields in nonlinear dielectrics can be determined by solving Maxwell's equations subject to the assumption that the electric polarization has a nonlinear relation to the electric field. However, recent work using the finite-difference time-domain (FD-TD) methods [3]–[7], the nonlinear Maxwell's equations have not been solved directly. In this letter, new results are presented for interactions of spatial solitons in Kerr-type media using the FD-TD Maxwell's solver. It has been found for an important case that the FD-TD simulation of spatial solitons does not behave as described by NLSE. Further, FD-TD modeling allows simulation of the dynamics of pulsed spatial solitons and associated switching devices.

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II. 2-D Algorithm for Instantaneous Nonlinearity

Consider a 2-D transverse magnetic (TM) problem. Maxwell's equations for the electric and magnetic field intensities, E_z, H_x, and H_y are given by:

$$\frac{\partial \mu_0 H_x}{\partial t} = -\frac{\partial E_z}{\partial y}, \qquad \frac{\partial \mu_0 H_y}{\partial t} = \frac{\partial E_z}{\partial x}, \qquad \frac{\partial D_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \tag{1}$$

Kerr nonresonant virtual electronic transitions in silica occur at time scales on the order of about 1 fs or less. In this letter, the FD-TD model for Kerr-type materials assumes instantaneous nonlinear response. The nonlinearity is modeled in the relation $D_z = \varepsilon_o \varepsilon E_z$, where

$$\varepsilon = n^2 = \left(n_o + n_2|E_z|^2\right)^2 \cong n_o^2 + 2n_o n_2 |E_z|^2 \tag{2}$$

In (2), n_o is unitless and n_2 has units of m^2/V^2. Equations (1) are first solved for H_x, H_y, and D_z by using the standard second-order accurate FD-TD scheme [8]. Then, the latest value of E_z can be obtained by iteration, using the new value of D_z and the old value of E_z:

$$E_z = \frac{D_z}{n_o^2 + 2n_o n_2 |E_z|^2} \tag{3}$$

Knowledge of E_z then permits another updating of H_x, H_y, and D_z, and the process repeats cyclically until time-stepping is completed. This procedure comprises the complete solution method for the space-time behaviour of the 2-D optical electromagnetic field.

III. Spatial Soliton Results

The modeling capabilities of this algorithm are demonstrated by 2-D calculations of propagating and mutually attracting and deflecting optical spatial solitons. The calculations are for a propagating sinusoidal beam that is switched on at $t = 0$ in Type-RN Corning glass with $n_o = 2.46$ and $n_2 = 1.25 \times 10^{-18}$ m^2/W [9]. The beam carrier frequency is 2.31×10^{14} Hz ($\lambda = 1.3$ μm); the initial peak electric field intensity is 6.87×10^9 V/m; and the fields have an initial hyperbolic secant transverse distribution with an intensity beamwidth (FWHM) of 0.65 μm. The computational domain is 95×31 μm.

The first Maxwell's equations calculation (Fig. 1(a)) selects the amplitude of the beam to balance its spreading and self-focusing. This provides a spatial soliton that retains its

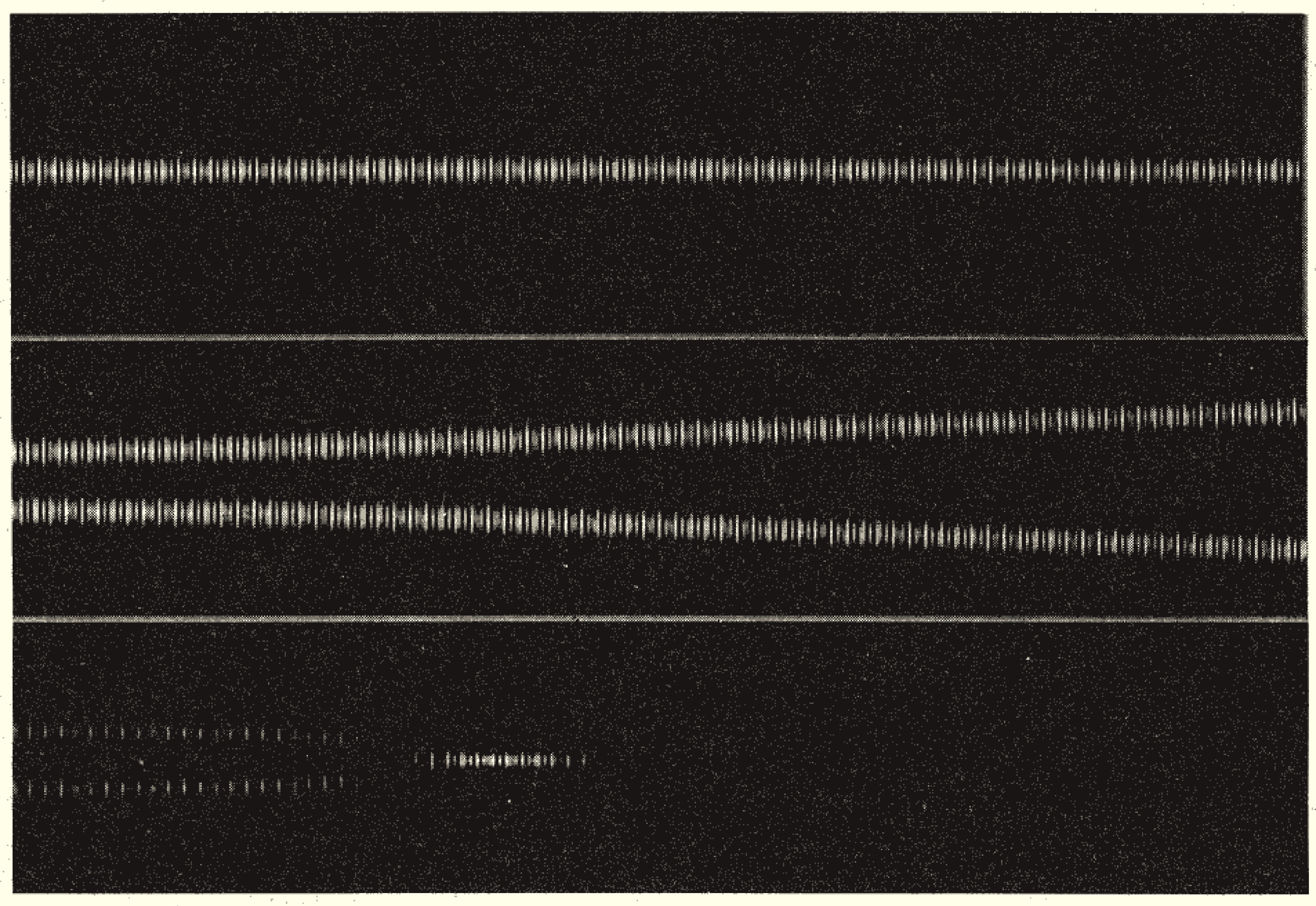

Fig. 1. FD-TD Maxwell's equations calculations of spatial solitons in Type RN Corning glass. Each beam has a 1.3-μm wavelength, an initial intensity beamwidth (FWHM) of 0.65 μm, and an initial peak electric field of 6870 V/μm. (a) Single spatial soliton; (b) repulsion of co-propagating spatialsolitons (initial center-to-center separation = 1.05 μm, relative carrier phase = π); (c) single coalescence and subsequent divergence for co-propagating spatial solitons (initial separation = 1.05 μm, relative carrier phase = 0).

Fig. 2. FD-TD simulations showing restoration of the beam re-coalescence behavior after doubling the intensity beamwidth and separation parameters ofthe simulated beams, keeping the wavelength constant.

TABLE I
PROGRESSIVE AGREEMENT OF FD-TD AND NLSE RESULTS FOR PERIODICITY OF CO-PROPAGATING IN-PHASE SPATIAL SOLITIONS AS $B_I\lambda_d$ INCREASES

B_I, FWHM (μm)	B_I / λ_d	z_p (μm) NLSE	z_p (μm) FD-TD	Difference
0.65	1.22	9	∞	∞%
1.3	2.46	34	47	38%
2.6	4.9	135	153	13%

transverse electric field and magnetic field distributions. The second Maxwell's equations calculation (Fig. 1(b)) simulates the parallel co-propagation of two equal-amplitude spatial solitons separated by 1.05 μm center-to-center, where the solitons have a carrier phase difference of π radians. This computation provides the beam-to-beam repulsion expected from NLSE [10], [11].

The third Maxwell's equations calculation (Fig. 1(c)) simulates the parallel co-propagation of two equal-amplitude, in-phase spatial solitons (carrier phase difference of 0). NLSE predicts that the two in-phase solitons remain bound, executing a periodic relative motion [12] if the two beams have the appropriate amplitudes and spacing. Aitchison, *et al.* [13] indicate that two in-phase fundamental solitons with an input amplitude distribution of

$$A(z) = \frac{1}{kw}\left(\frac{n_o}{n_2}\right)^{1/2}\left[\operatorname{sech}\left(\frac{x-x_o}{w}\right) + \operatorname{sech}\left(\frac{x+x_o}{w}\right)\right] \tag{4}$$

oscillate with a period of

$$z_p = \frac{2z_o \sinh(2x_o/w)\cosh(x_o/w)}{2x_o/w + \sinh(2x_o/w)} \tag{5}$$

based on the NLSE theory of Desem and Chu [14]. Here, w is the characteristic width of the hyperbolic secant; $x_o = 1.42$ w; $2x_o$ is the center-to-center separation of the two beams; and $z_o = \pi^2 n_o w^2/\lambda$ is the usual soliton period. For the choice of parameters used in the FD-TD Maxwell's equations simulations, the predicted repetition period is $z_p = 9$ μm. However, as shown in Fig. 1(c), the FD-TD calculations show only a single beam coalescence and then subsequent beam divergence to arbitrarily large separations, yielding an effective $z_p = \infty$.

It was desired to understand why the nonlinear FD-TD Maxwell's equations model did not agree with the NLSE prediction in this case. The first possibility considered was that the FD-TD simulation was flawed because of inadequate grid resolution and/or inadequate decoupling of the beam interaction region from the weakly reflecting outer grid boundaries. In a series of exploratory modeling runs to address these issues, the space-time resolution of the FD-TD grid was progressively refined and the grid enlarged. These changes gave results identical to those of the original FD-TD model. Therefore, the original FD-TD model was concluded to be numerically converged and sufficiently free of the outer boundary artifact to yield plausible results.

The second possibility considered was that the ratio of intensity beamwidth, B_I to dielectric wavelength λ_d, was below the limit of applicability of NLSE. Because it is known that additional terms in the NLSE are required to model higher-order effects for temporal solitons, it was reasoned that basic NLSE modeling of co-propagating spatial solitons would be more physically meaningful if the two beams were widened relative to the optical wavelength while maintaining the same ratio of beamwidth to beam separation. This would reduce higher-order diffraction effects, hopefully bringing the test case into the region of validity for the simple NLSE model.

To test this possibility, two new FD-TD simulations were conducted where B_I and separation parameters of the simulated beams were each doubled keeping the dielectric wavelength, λ_d, constant. After the first doubling, the FD-TD-predicted spatial solitons began to qualitatively show the re-coalescence behavior predicted by NLSE, but with a 38% longer period of re-coalescence than the NLSE value. This FD-TD simulation is shown in Fig. 2. After the second doubling of beamwidth and beam separation, the FD-TD and NLSE predictions for z_p showed much better agreement, differing by only 13%. Results for these numerical experiments are shown in Table I.

It was concluded that there is a strong likelihood that co-propagating, in-phase optically narrow beams have only a single coalescence and then indefinite separation. The FD-TD model appears to properly predict the behavior of beams in nonlinear media both in the regime where the standard NLSE model breaks down ($B_I/\lambda_d < 1$) and the the regime

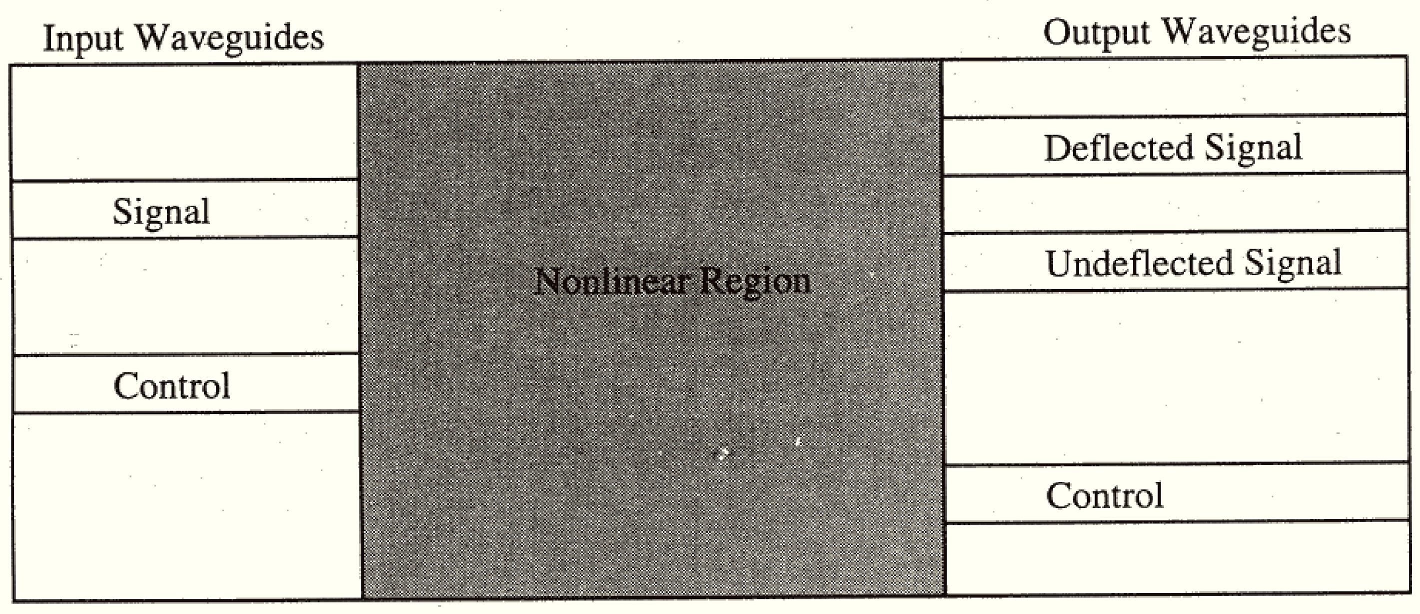

Fig. 3. Proposed all-optical switch based upon the single-time spatial soliton coalescence behavior indicated by FD-TD modeling. Pulsed optical signal and control beams are fed in at the left, interact in the Kerr nonlinear medium, and then couple into receptor waveguides. In the absence of the control, the signal propagates with zero deflection. In the presence of the control, and depending upon its carrier phase relative to the signal, there is either a single coalescence and then deflection to a collecting waveguide, or deflection without coalescence.

where the standard NLSE model is valid ($B_I/\lambda_d \gg 1$). The paraxial approximation inherent to NLSE, according to Lax *et al.* [15], accounts only for zeroth-order diffraction effects. Since the FD-TD model implements the fundamental Maxwell's curl equations, it makes no assumption about a preferred scattering direction. It naturally accounts for energy transport in arbitrary directions and should be exact for the computed optical electromagnetic fields up to the limit set by the grid resolution and Nyquist sampling theory.

IV. Pulsed Spatial Soliton Switch

The single-time spatial soliton coalescence behavior indicated by the FD-TD modeling studies discussed above provides the basis for the all-optical switch proposed in Fig. 3. This pulsed spatial soliton switch consists of a Kerr-type nonlinear interaction region (Corning glass Type-RN) with a pair of input and output waveguides on each side. Optical signal and control pulses are fed in at the left edge, interact in the nonlinear medium, and then couple into receptor waveguides. In the absence of the control pulse, the signal pulse propagates with zero deflection. In the presence of the control pulse, and depending upon its carrier phase relative to the signal pulse, there is either a single coalescence and then deflection to a collecting waveguide, or deflection without coalescence. (FD-TD studies have shown that optical pulses as short as 70 fs have the same coalescence/deflection behavior as continuous beams.) Note that the device of Fig. 3 differs from the all-optical spatial-soliton switch of Shi and Chi [10] which did not take advantage of the single-coalescence/single-divergence phenomenon, used continuous-wave excitation, and assumed a nonphysically high nonlinear coefficient.

Fig. 4 shows snapshots of the FD-TD-computed electric fields of 100-fs pulsed signal and control spatial solitons at the simulation times of 86 fs, 344 fs, and 516 fs for zero carrier phase between the pulses. The deflection of the signal pulse varies smoothly but not monotonically as the relative carrier phase is changed from 0 to π radians. The range of beam deflection complicates somewhat the design of a receptor waveguide that can efficiently capture the deflected beam for arbitrary phase differences between signal and control pulses. Yet, the principle appears proven and potentially useful.

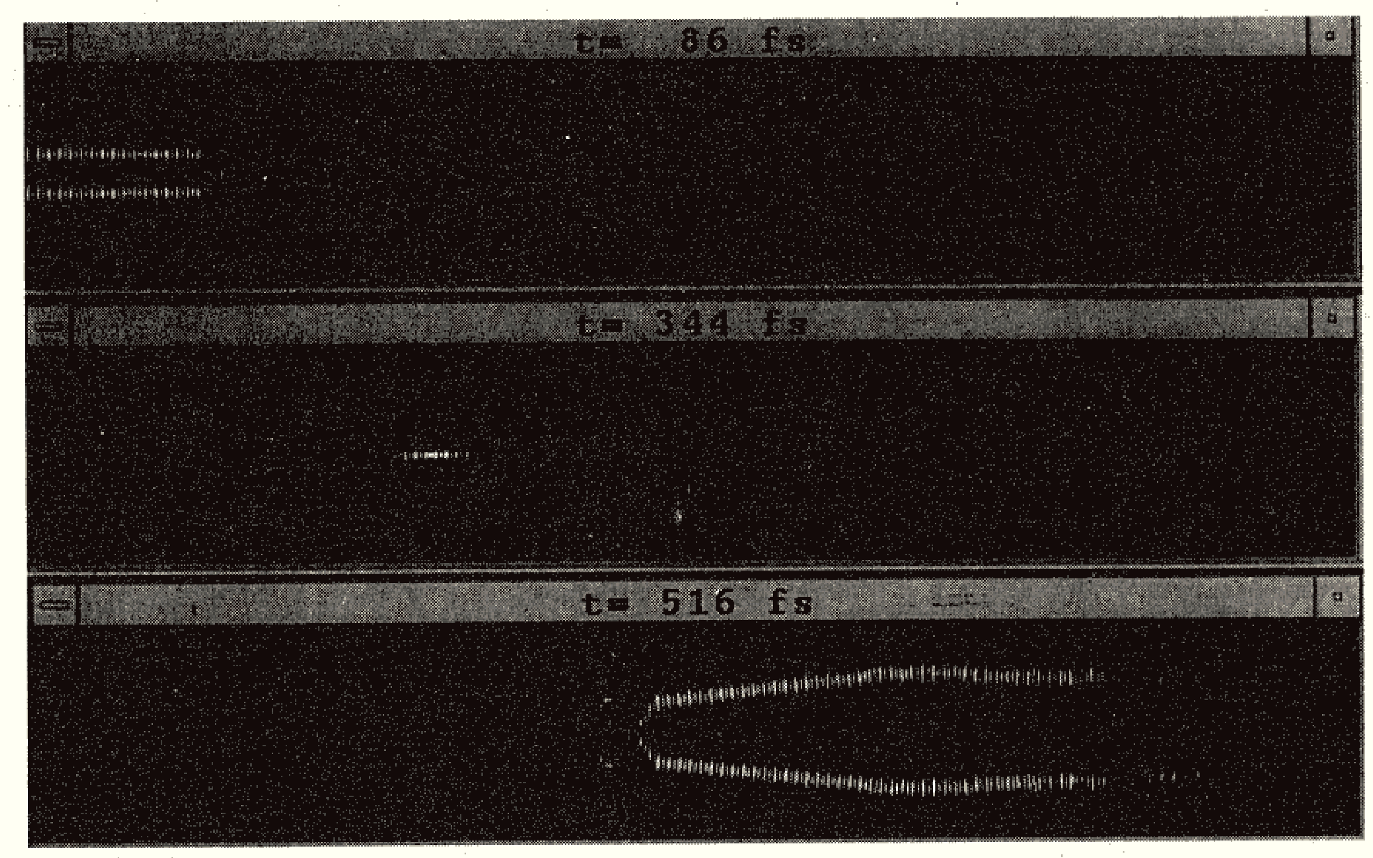

Fig. 4. Snapshots of the FD-TD-computed electric fields of 100-fs pulsed signal and control spatial solitons at the simulation times of 86 fs, 344 fs, and 516 fs for zero carrier phase between the pulses. Single-time spatial soliton coalescence behavior is indicated by FD-TD modeling for ultra-short pulses as well as continuous beams.

V. Conclusion

This letter presented FD-TD Maxwell's equations calculations of spatial optical soliton propagation and mutual deflection in a 2–D homogeneous nonlinear dielectric medium. The FD-TD results show that co-propagating, in-phase optically narrow spatial solitons undergo only a single beam coalescence before diverging to arbitrarily large separations. This phenomenon provides a possible mechanism for constructing femtosecond all-optical switches spanning less than 100 mm in length in an existing type of Corning glass.

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