Direct time integration of Maxwell's equations in two-dimensional dielectric waveguides for propagation and scattering of femtosecond electromagnetic solitons

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We present what are to our knowledge first-time calculations from vector nonlinear Maxwell's equations of femtosecond soliton propagation and scattering, including carrier waves, in two-dimensional dielectric waveguides. The time integration efficiently implements linear and nonlinear convolutions for the electric polarization, and the nonlinear convolution accounts for two quantum effects, the Kerr and Raman interactions. By retaining the optical carrier, the new method solves for fundamental quantities—optical electric and magnetic fields in space and time—rather than a nonphysical envelope function. It has the potential to provide an unprecedented two- and three-dimensional modeling capability for millimeter-scale integrated-optical circuits with submicrometer engineered inhomogeneities.

Experimentalists have produced all-optical switches capable of 100-fs responses.[1] To model such switches adequately, nonlinear effects in optical materials[2] (both instantaneous and dispersive) must be included. In principle, the behavior of electromagnetic fields in nonlinear dielectrics can be determined by solving Maxwell's equations subject to the assumption that the electric polarization has a nonlinear relation to the electric field. However, until our previous research,[3,4] the resulting nonlinear Maxwell's equations have not been solved directly. Rather, approximations have been made that result in a class of generalized nonlinear Schrödinger equations (GNLSE's) that solve only for the envelope of the optical pulses.[5]

Two-dimensional (2-D) and three-dimensional (3-D) engineered inhomogeneities in nonlinear optical circuits will probably be at distance scales on the order of 0.1–10 optical wavelengths, and all assumptions regarding slowly varying parameters (which run throughout GNLSE theory) will be unjustified. For such devices, optical-wave scattering and diffraction effects relevant to integrated all-optical switches will be difficult or impossible to obtain with GNLSE, because its formulation discards the optical carrier. The only way to model such devices is to retain the optical carrier and solve Maxwell's vector-field equations for the material geometry of interest, rigorously enforcing the vector-field boundary conditions and the physics of nonlinear dispersion.

In this Letter we describe what are to our knowledge first-time solutions of 2-D vector nonlinear Maxwell's equations for material media with linear and nonlinear instantaneous and Lorentz-dispersive effects in the electric polarization. We use the finite-difference time-domain (FD-TD) method in an extension of our previous research in one dimension.[3,4] The optical carrier is retained in this approach. The fundamental innovation is the treatment of the linear and nonlinear convolution integrals that describe the dispersion as new dependent variables. By differentiating these convolutions in the time domain, an equivalent system of coupled, nonlinear, second-order ordinary differential equations (ODE's) is derived. These equations together with Maxwell's equations form the system that is solved to determine the electromagnetic fields in nonlinear dispersive media. Backstorage in time is limited to only that needed by the time-integration algorithm for the ODE's (two time steps) rather than that needed to store the time history of the kernel functions of the convolutions. Thus, a 2-D nonlinear optics model from Maxwell's equations is now feasible.

Now we present the theoretical development. Consider a 2-D transverse-magnetic problem. Maxwell's equations for the electric- and magnetic-field intensities, E_z, H_x, and H_y, are given by

$$\frac{\partial \mu_0 H_x}{\partial t} = -\frac{\partial E_z}{\partial y}, \qquad \frac{\partial \mu_0 H_y}{\partial t} = \frac{\partial E_z}{\partial x},$$

$$\frac{\partial D_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}. \tag{1}$$

We allow for dielectric nonlinearity by assuming that the electric polarization, P_z, consists of the sum of a linear part, P_z^L, and a nonlinear part, P_z^{NL}.[5] Then we have

$$D_z = \epsilon_0 \epsilon_\infty E_z + P_z, \qquad P_z = P_z^L + P_z^{\mathrm{NL}}, \tag{2}$$

where P_z^L is given by a convolution of $E_z(x,y,t)$ and

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the Lorentz susceptibility function, $\chi^{(1)}$:

$$P_z{}^L(x,y,t) = \epsilon_0 \int_{-\infty}^{\infty} \chi^{(1)}(t - t')E_z(x,y,t')\mathrm{d}t' ,$$

$$\chi^{(1)}(t) = \left(\frac{\omega_p{}^2}{v_0}\right)\exp(-\delta t/2)\sin(v_0 t) . \qquad (3)$$

Here $\omega_p{}^2 = \omega_0{}^2(\epsilon_s - \epsilon_\infty)$ and $v_0{}^2 = \omega_0{}^2 - \delta^2/4$. Further, we assume a dispersive (memory-type) material nonlinearity[2] characterized by the following time convolution for $P_z{}^{\mathrm{NL}}$:

$$P_z{}^{\mathrm{NL}}(x,y,t) = \epsilon_0 \chi^{(3)} E_z(x,y,t) \int_{-\infty}^{\infty} g(t - t')E_z{}^2(x,y,t')\mathrm{d}t', \qquad (4)$$

where $\chi^{(3)}$ is the nonlinear coefficient and $\int_{-\infty}^{\infty} g(t)\mathrm{d}t = 1$. Equation (4) accounts for phonon interactions and nonresonant electronic effects, as given by

$$g(t) = \alpha\delta(t) + (1 - \alpha)g_R(t) ,$$

$$g_R(t) = \left(\frac{\tau_1{}^2 + \tau_2{}^2}{\tau_1\tau_2{}^2}\right)\exp(-t/\tau_2)\sin(t/\tau_1) . \qquad (5)$$

Here $\delta(t)$ is the instantaneous delta function that models Kerr nonresonant virtual electronic transitions on the order of ~1 fs or less, and $g_R(t)$ models transient Raman scattering.

We now describe the system of coupled nonlinear ODE's that governs the time evolution of the polarization. Assuming zero values of the electromagnetic field and the kernel functions for $t \le 0$, define the functions $F(t)$ and $G(t)$ as the convolutions

$$F(t) = \epsilon_0 \int_0^t \chi^{(1)}(t - t')E_z(x,y,t')\mathrm{d}t' ,$$

$$G(t) = \epsilon_0 \int_0^t g_R(t - t')E_z{}^2(x,y,t')\mathrm{d}t' . \qquad (6)$$

Then, by time differentiating F and G, these functions satisfy the following coupled system:

$$\frac{1}{\omega_0{}^2}\frac{\mathrm{d}^2F}{\mathrm{d}t^2} + \frac{\delta}{\omega_0{}^2}\frac{\mathrm{d}F}{\mathrm{d}t} + \left[1 + \frac{\epsilon_s - \epsilon_\infty}{\epsilon_\infty + \alpha\chi^{(3)}E_z{}^2}\right]F + \left[\frac{(\epsilon_s - \epsilon_\infty)(1 - \alpha)\chi^{(3)}E_z}{\epsilon_\infty + \alpha\chi^{(3)}E_z{}^2}\right]G = \left[\frac{\epsilon_s - \epsilon_\infty}{\epsilon_\infty + \alpha\chi^{(3)}E_z{}^2}\right]D_z , \qquad (7)$$

$$\frac{1}{\overline{\omega}_0{}^2}\frac{\mathrm{d}^2G}{\mathrm{d}t^2} + \frac{\overline{\delta}}{\overline{\omega}_0{}^2}\frac{\mathrm{d}G}{\mathrm{d}t} + \left[1 + \frac{(1 - \alpha)\chi^{(3)}E_z{}^2}{\epsilon_\infty + \alpha\chi^{(3)}E_z{}^2}\right]G + \left[\frac{E_z}{\epsilon_\infty + \alpha\chi^{(3)}E_z{}^2}\right]F = \left[\frac{E_z}{\epsilon_\infty + \alpha\chi^{(3)}E_z{}^2}\right]D_z , \qquad (8)$$

where $\overline{\delta} = 2/\tau_2$ and $\overline{\omega}_0{}^2 = (1/\tau_1)^2 + (1/\tau_2)^2$. Equations (7) and (8) are first solved simultaneously for F and G at the latest time step by using a second-order accurate finite-difference scheme that operates on data for the current value of D_z and previous values of D_z, E_z, F, and G. Then, the latest value of E_z can be obtained by a Newton's iteration, using the new values for D_z, F, and G:

$$E_z = \frac{D_z - F - (1 - \alpha)\chi^{(3)}E_z G}{\epsilon_0[\epsilon_\infty + \alpha\chi^{(3)}E_z{}^2]} . \qquad (9)$$

The system of Eqs. (7)–(9) determines values of E_z and P_z so that Eq. (2) is satisfied. This procedure, combined with the usual FD-TD realization of Maxwell's equations [Eqs. (1)], comprises the complete solution method. The FD-TD algorithm used here is a generalization to two dimensions of that in Ref. 4.

The modeling capabilities of this new algorithm are demonstrated by 2-D calculations of propagating and colliding solitons. The calculations are for a propagating pulse with a carrier frequency of 1.37×10^{14} Hz ($\lambda = 2.19$ μm) and a hyperbolic-secant envelope with a characteristic time constant of 14.6 fs. The computational domain for the 2-D dielectric waveguide is 110 μm × 5 μm, with the dielectric waveguide itself 1 μm thick and with 2 μm of air on either side. The first calculation simulates Lorentz-medium linear dispersion alone [Eq. (3)]. As Fig. 1 shows, the pulse undergoes predicted pulse lengthening owing to dispersive effects.[5]

The second calculation simulates the effects of the full linear [Eq. (3)] and nonlinear [Eqs. (4) and (5)]

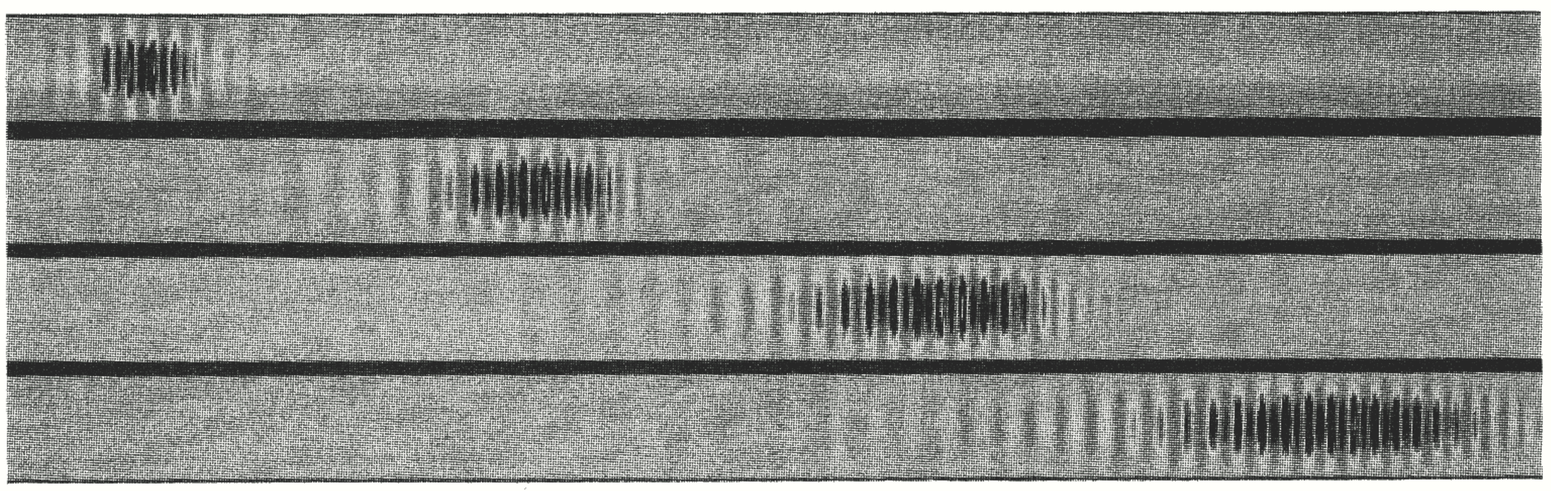

Fig. 1. Electric field of a propagating optical carrier pulse with initial hyperbolic-secant envelope ($\lambda = 2.19$ μm, $\tau = 14.6$ fs) in a 1-μm-thick linear Lorentz-medium dielectric waveguide.

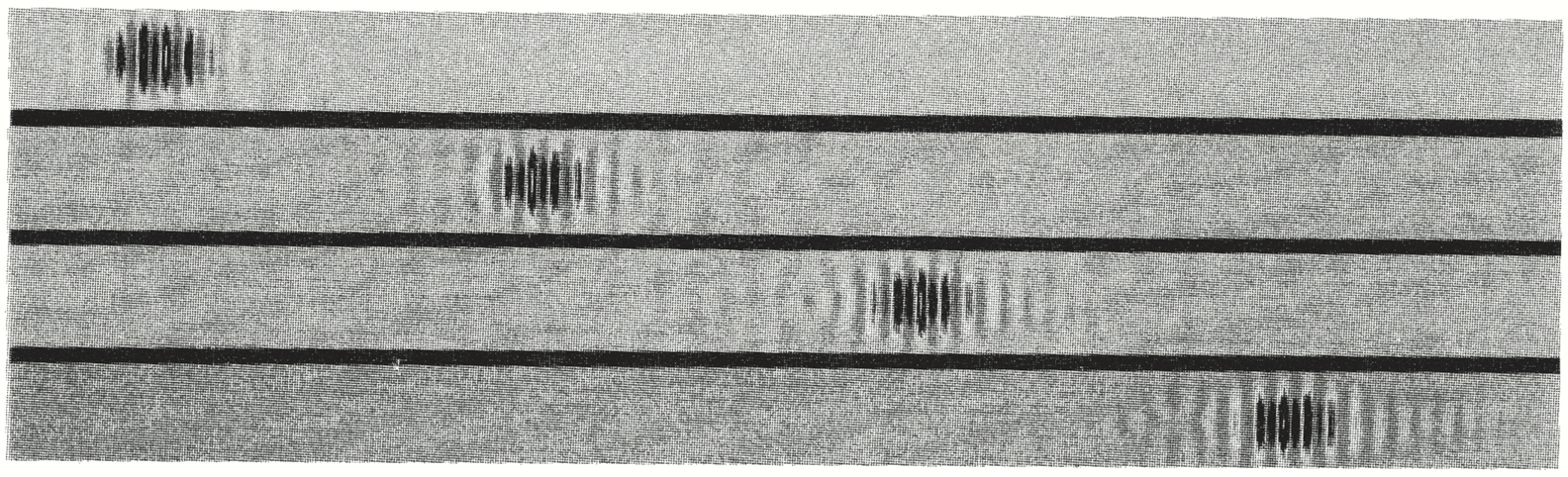

Fig. 2. Electric field of an optical soliton carrier pulse corresponding to that in Fig. 1, including quantum effects such as the Kerr and Raman interactions.

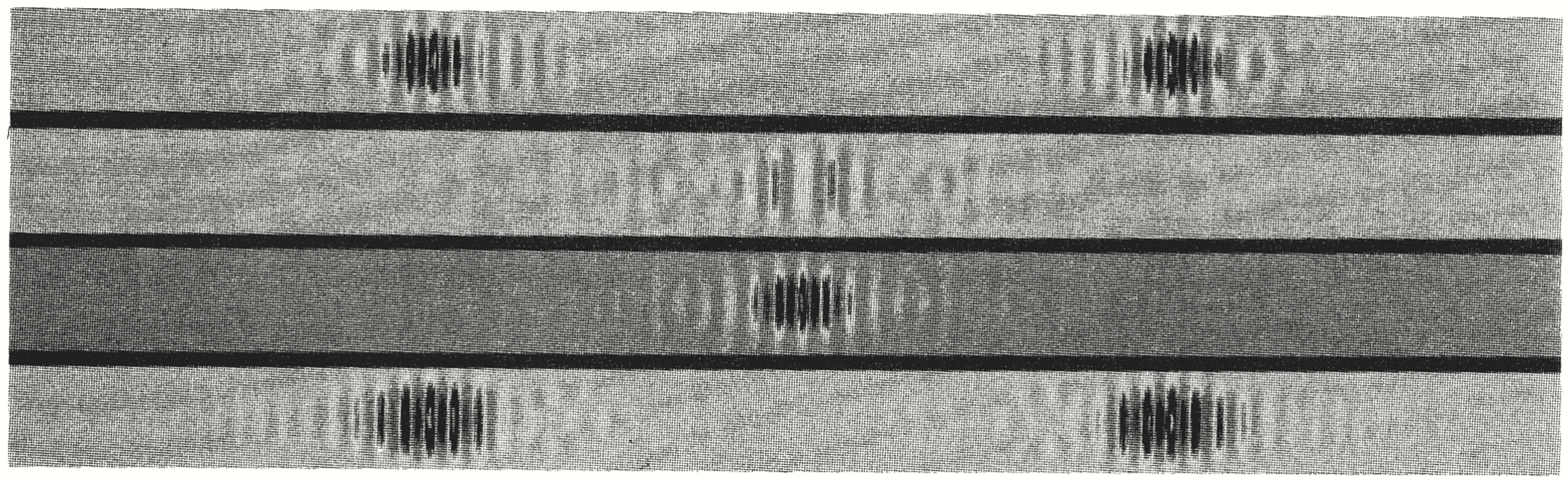

Fig. 3. Electric field of colliding counterpropagating solitons corresponding to that in Fig. 2 (approaching, destructively interfering, constructively interfering, separating).

polarizations. As shown in Fig. 2, the propagating pulse now has the features of a soliton with the retention of its length. In addition, detailed plots show that a second low-amplitude, high-frequency, daughter pulse forms and moves out ahead of the soliton.

The third calculation (Fig. 3) simulates the collision of two equal-amplitude counterpropagating solitons. The results show the solitons interacting during the collisions and then separating without general changes. However, by comparing the carriers of the collided solitons with those of the noncollided solitons, precise carrier-phase lags of the collided solitons are measured.

The novel approach discussed here achieves robustness by rigorously enforcing the vector-field boundary conditions at all interfaces of dissimilar media in the time scale of the optical carrier, regardless of whether the media are dispersive or nonlinear. As a result, the new approach is almost completely general. It assumes nothing about (1) the homogeneity or isotropy of the optical medium, (2) the magnitude of the nonlinearity, (3) the nature of the material's $\omega-\beta$ variation, and (4) the shape, duration, and vector nature of the optical pulse(s). By retaining the optical carrier, the new method solves for fundamental quantities—the optical electric and magnetic fields in space and time—rather than a nonphysical envelope function. It has the potential to provide an unprecedented 2-D and 3-D modeling capability for millimeter-scale integrated optical circuits with submicrometer engineered inhomogeneities.

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