Computational Modeling of Femtosecond Optical Solitons from Maxwell's Equations

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***Abstract*—A new algorithm has been developed that permits, for the first time, the direct time integration of the full-vector nonlinear Maxwell's equations. This new capability permits the modeling of linear and nonlinear, instantaneous and dispersive effects in the electric polarization in material media. The modeling of the optical carrier is retained in this approach. The fundamental innovation of the present approach is to notice that it is possible to treat the linear and nonlinear convolution integrals, which describe the dispersion, as new dependent variables. Using this observation, a coupled system of nonlinear second-order ordinary differential equations can be derived for the linear and nonlinear convolution integrals, by differentiating them in the time domain. These equations, together with Maxwell's equations form the system that is solved to determine the electromagnetic fields in nonlinear dispersive media. Using this algorithm, results are presented of first-time calculations in one dimension of the propagation and collision of femtosecond electromagnetic solitons that retain the optical carrier. The nonlinear modeling takes into account such quantum effects as the Kerr and Raman interactions. The present approach is robust and should permit modeling 2-D and 3-D optical soliton propagation, scattering, and switching directly from the full-vector nonlinear Maxwell's equations for integrated optical structures having complex engineered inhomogeneities.**

I. Introduction

In computational electromagnetics, there is a substantial activity in solving the linear Maxwell equations by finite-difference methods in the time domain (FD-TD), for applications to aeronautics, electronics, and biology [1], [2]. During the 1980's, the primary interest in Maxwell's equations solvers of all types centered on defense applications, primarily the prediction and mitigation of radar cross section (RCS). Grid-based Maxwell's solvers, implemented on supercomputers, are rapidly becoming this country's primary means of modeling the RCS of advanced aerospace vehicles [2]. The FD-TD method implements the spatial derivatives of the curl operators in Maxwell's equations by using finite differences on a regular Cartesian space mesh, and employs a simple leapfrog time integration scheme.

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Now, in the 1990's, initial efforts are being made to use the Maxwell's equations to explore the physics of devices having substantial commercial applications, especially ultrahigh-speed electronic, electrooptic, and all-optical devices, which are useful in the construction of advanced digital signal processors and computers. To date, these investigations are being conducted almost exclusively by using the FD-TD algorithm, which permits a means of combining the physics of the Maxwellian electromagnetic field and energy transport with the physics of electronic charge transport and volume-averaged quantum effects. Applications include modeling high-data-rate passive interconnects for digital circuits [3], active electronic devices [4] and all-optical devices [5].

However, these efforts use the linear Maxwell's equations. To adequately model optical switches, nonlinear effects in dielectric materials [6], [7] both instantaneous and dispersive, must be included. Experimental researchers have produced switches [6] capable of switching short (100 fs) optical pulses in nonlinear directional couplers. Also nonlinear responses in dielectric materials are important in the generation of very short, intense, fast rise-time pulses. Such pulses are important in ultrawideband (UWB) technology [8] in aeronautics. These pulses have very high peak power and a frequency spectrum that extends from near-direct current to several gigahertz.

The nonlinear behavior of electromagnetic fields in materials is determined by solving the nonlinear Maxwell's equations. The equations become nonlinear because the electric polarization is now determined by a nonlinear relation to the electric field intensity. Up to now, these nonlinear Maxwell's equations have not been solved exactly. Rather, various approximations to the governing equations have been made. The least approximate methods solve nonlinear scalar equations for the slowly varying envelope of the optical pulses. This class of equations, known as the generalized nonlinear Schrodinger equations (GNLSE) [9], [10] has been solved by the split-step Fourier method [10] and by the Propagating beam method [11] (PBM). For example, the split-step Fourier method [10] is often used to simulate the propagation of optical pulses in low-loss fibers over very long optical distances, and the propagating beam method [11] has been used to model directional couplers.

However certain effects are neglected when Maxwell's equations are approximated by the GNLSE, including scattering and diffraction effects, and short pulse effects

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in the 10 fs regime. For example, optical wave scattering and diffraction effects relevant to all-optical switches integrated in microchip form will be difficult or impossible to obtain with GNLSE because its formulation discards the optical carrier. Engineered inhomogeneities in nonlinear optical circuits should be fairly complicated. Very likely, these inhomogeneities will be at short distance scales in the order of 0.1–10 optical wavelengths, and all assumptions regarding slowly varying parameters (which run throughout GNLSE theory) will be unjustified. In fact, material inhomogeneities such as crossing optical paths critically affect optical pulse scattering even in the absence of nonlinearities. There is no preferred direction of scattering for general inhomogeneities, and the primary scattering physics occurs at the optical carrier time scale with the enforcement of vector electromagnetic field boundary conditions at all material interfaces. The only way to model this situation is to set up and solve Maxwell's vector-field equations for the material geometry of interest, rigorously enforcing the vector-field boundary conditions and the physics of nonlinear dispersion.

In this paper, a new approach will be presented for solving the nonlinear Maxwell equations. The nonlinear relation between the polarization and the electric field will be modeled by a nonlinear convolution relation [7]. This system of equations will be solved exactly, using finite-difference methods in the time domain (FD-TD). Wave scattering and diffraction effects that are not accounted for in the GNLSE approach are included here. Also, unlike the split-step Fourier method, the effects of nonlinearity and dispersion are not treated separately in this approach. The inclusion of some of the nonlinear terms in the algorithm for Maxwell's equations employs techniques that were developed in computational fluid dynamics for the solution of nonlinear equations [12]. However, the treatment of the nonlinear convolution integral has required the development of a new computational technique [13]–[15].

This approach will provide direct solutions to Maxwell's vector-field equations suitable for modeling the propagation and scattering of optical pulses, including solitons, in inhomogeneous nonlinear dispersive media. The modeling of the pulses will include the optical carrier. The study of solitons [9], [16] is of fundamental importance. Approximately one hundred different types of nonlinear partial differential equations have been found to have solitons or soliton-like solutions mathematically. Solitons are a fundamental feature of nonlinear equations in a manner similar to Fourier modes for linear equations. During the past two decades [17] the study of solitons has become a fundamental part of many different areas of nonlinear science, including optics, plasma waves, molecular biology, nerve conduction, nucleic acids, quantum gauge fields, and cosmology. For nonlinear optics, this approach has the potential to provide a modeling capability for millimeter-scale integrated optical circuits beyond that of existing techniques using the generalized nonlinear Schrodinger equation.

In Section II and III, the new formulation of the governing differential equations and the new finite-difference equations will be given, respectively. In Section IV, results of the calculations of propagating and colliding solitons will be presented. Finally, in Section V, concluding remarks will be given.

II. Governing Equations

An efficient FD-TD numerical approach has been developed [5] for the direct time integration of Maxwell's equations to model *linear* media having arbitrary-order chromatic dispersions. This approach was based upon a suggestion by Jackson [18] to relate $\vec{D}(\vec{x}, t)$ the electric field displacement, to $\vec{E}(\vec{x}, t)$, the electric field intensity, via an ordinary differential equation in time. That equation is integrated concurrently with the Maxwell's equations. The differential equation that relates $\vec{D}(\vec{x}, t)$ to $\vec{E}(\vec{x}, t)$ is derived by taking the inverse Fourier transform of the complex permittivity relation, $\epsilon(\omega) = \vec{D}(\vec{x}, \omega)/\vec{E}(\vec{x}, \omega)$. Using that approach, initial calculations were made of femtosecond pulse propagation and scattering interactions for a Lorentz medium by a direct time integration of Maxwell's equations. The computed reflection coefficients [5] were accurate to better than 6 parts in 10 000 over the frequency range dc to 3×10^{16} Hz for a single 0.2 fs Gaussian pulse incident upon a Lorentz half space, and new results were obtained for the Sommerfeld and Brillouin precursors, agreeing very well with previous published Laplace transform theory.

In this paper, we report a generalization of the above approach to deal with the *nonlinear* terms of the electric polarization. The FD-TD direct time integration of Maxwell's equations can now incorporate nonlinear instantaneous and dispersive effects as well as linear instantaneous and dispersive effects, thereby permitting the modeling of optical solitons having very large instantaneous bandwidths.

As in [5], again consider a one-dimensional problem with electric and magnetic field intensities, E_z and H_y, respectively, propagating in the x direction. Assuming that the medium is nonpermeable and isotropic, Maxwell's equations in one dimension are written as

$$\frac{\partial \mu_0 H_y}{\partial t} = \frac{\partial E_z}{\partial x} \tag{1}$$

$$\frac{\partial D_z}{\partial t} = \frac{\partial H_y}{\partial x} \tag{2}$$

$$D_z = \epsilon_0 \epsilon_\infty E_z + P_z. \tag{3}$$

Here μ_0 and ϵ_0 are the permeability and permittivity coefficients for free space, ϵ_∞ is the relative material permittivity at infinite frequency, D_z is the electric field displacement, and P_z is the electric polarization.

Assume that the polarization P_z consists [10] of two parts: a linear part P_z^L and a nonlinear part P_z^{NL}.

$$P_z = P_z^L + P_z^{NL}. \tag{4}$$

The linear polarization P_z^L is given by a linear convolution of $E_z(x, t)$ and the first-order susceptibility function $\chi^{(1)}$:

$$P_z^L(x, t) = \epsilon_0 \int_{-\infty}^{\infty} \chi^{(1)}(t - \bar{t})E_z(x, \bar{t})\, d\bar{t} \tag{5}$$

and P_z^{NL} is given by a nonlinear convolution of $E_z(x, t)$ and the third-order susceptibility function, $\chi^{(3)}$:

$$P_z^{NL}(x, t) \epsilon_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^{(3)}(t - \bar{t}, t - \bar{t}_2, t - \bar{t}_3,) \cdot E_z(x, \bar{t}_1)E_z(x, \bar{t}_2)E_z(x, \bar{t}_3)\, d\bar{t}_1\, d\bar{t}_2\, d\bar{t}_3. \tag{6}$$

This last integral models the physics of a nonlinearity with retardation or memory, i.e., a dispersive nonlinearity that can occur due to quantum effects in silica at time scales of 1 to 100 fs. Note that $\chi^{(3)}$ may differ from $\chi^{(1)}$ in physical properties such as resonances and dampings.

We consider a material having a Lorentz linear dispersion characterized [18] by the following $\chi^{(1)}$:

$$\chi^{(1)}(t) = \left(\frac{\omega_p^2}{\nu_0}\right) \exp(-\delta t/2) \sin(\nu_0 t). \tag{7}$$

The corresponding linear permittivity as a function of frequency is

$$\epsilon(\omega) = \epsilon_\infty + \chi^{(1)}(\omega) = \epsilon_\infty + \frac{\omega_0^2(\epsilon_s - \epsilon_\infty)}{\omega_0^2 - j\delta\omega - \omega^2} \tag{8}$$

where $\omega_p^2 = \omega_0^2(\epsilon_s - \epsilon_\infty)$ and $\nu_0^2 = \omega_0^2 - \delta^2/4$.

Further, the material nonlinearity is assumed to be characterized [7] by the following nonlinear single time convolution for P_z^{NL}

$$P_z^{NL}(x, t) = \epsilon_0 \chi^{(3)} E_z(x, t) \int_{-\infty}^{\infty} g(t - \bar{t})E_z^2(x, \bar{t})\, d\bar{t} \tag{9}$$

where $\chi^{(3)}$ is the nonlinear coefficient. The causal response function $g(t - \bar{t})$ is normalized so that

$$\int_{-\infty}^{\infty} g(\bar{t})\, d\bar{t} = 1. \tag{10}$$

Equation (9) only accounts for nonresonant third-order processes. the processes considered in (9) are phonon interactions and nonresonant electronic effects. The modeling [7] of those responses is given by the third-order response function $g(t)$, where

$$g(t) = \alpha\, \delta(t) + (1 - \alpha)g_R(t) \tag{11}$$

and

$$g_R(t) = \left(\frac{\tau_1^2 + \tau_2^2}{\tau_1 \tau_2^2}\right) \exp(-t/\tau_2) \sin(t/\tau_1). \tag{12}$$

In (11), $\delta(t)$ is the instantaneous delta function response and models Kerr nonresonant virtual electronic transitions in the order of about 1 fs or less. The function $g_R(t)$ models transient Raman scattering and α parameterizes the relative strengths of the Kerr and Raman interactions. Effectively, $g_R(t)$ models a single Lorentzian line centered on the optical phonon frequency $1/\tau_1$ and having a bandwidth of $1/\tau_2$ (the reciprocal phonon lifetime). See [7] for a more detailed explanation of the modeling of the nonlinear optical fiber.

We will now describe a solution procedure [15] for evolving the electric polarization in time by solving a system of coupled nonlinear ordinary differential equations. Equations (4) through (12) describe the electric polarization in the nonlinear medium. Those equations together with (1) to (3) will provide a system of equations for evolving the electromagnetic field in nonlinear media. The procedure will be a generalization of the approach that was used in [5]. However that approach used the complex permittivity relation in the frequency domain to obtain an ordinary differential equation that related E_z to D_z. Such an approach is not applicable to nonlinear polarizations. Nevertheless, the same ordinary differential equation can be derived by an alternative approach, which is to start in the time domain with (3) and differentiate it. The key property that is used in that derivation is the fact that the kernel function $\chi^{(1)}(t)$, as given by (7), satisfies a linear second-order ordinary differential equation. Note that the kernel function $g_R(t)$ as given by (12) for the nonlinear convolution integral, also satisfies a linear second-order ordinary differential equation. The fundamental innovation of the present approach is to notice that those properties make it possible to treat the linear and nonlinear convolution integrals as new dependent variables. Using this observation, a coupled system of nonlinear second-order ordinary differential equations can be derived for the linear and nonlinear convolution integrals, by differentiating them in the time domain.

Those equations determine the polarization P_z by using (4). Since those equations are second-order ordinary differential equations, a solution procedure only requires two time levels of storage for each equation. By comparison, a direct evaluation of the convolution integrals in (4) would require the storage of the electromagnetic fields from an initial time to the current time and an evaluation of the integrals at each time level. As the following calculations will show, typical calculations require several tens of thousands of time levels of evolution. Hence the present method is more efficient both in storage and in operation count than a direct method. For a 2-D or 3-D full-vector nonlinear optics model, this improvement reduces the computer memory required for a first-principles computation from an impossible level of 100–1000 Gwords to a presently feasible level of 0.1–1 Gword.

We will now describe the differential equations that govern the evolution of the polarization, as given by (4) to (12). Assuming zero values of the electromagnetic field and the kernel functions for $t \leq 0$, define the functions $F(t)$ and $G(t)$ as respectively the linear and nonlinear convolutions:

$$F(t) = \epsilon_0 \int_0^t \chi^{(1)}(t - \bar{t})E_z(x, \bar{t})\, d\bar{t} \tag{13}$$

$$G(t) = \epsilon_0 \int_0^t g_R(t - \bar{t}) E_z^2(x, \bar{t})\, d\bar{t}. \tag{14}$$

Then, by differentiating F and G, we have found that they satisfy the following coupled system of nonlinear ordinary differential equations:

$$\frac{1}{\omega_0^2}\frac{d^2F}{dt^2} + \frac{\delta}{\omega_0^2}\frac{dF}{dt} + \left(1 + \frac{\epsilon_s - \epsilon_\infty}{\epsilon_\infty + \alpha\chi^{(3)}E_z^2}\right)F + \left(\frac{(\epsilon_s - \epsilon_\infty)(1-\alpha)\chi^{(3)}E_z}{\epsilon_\infty + \alpha\chi^{(3)}E_z^2}\right)G = \left(\frac{(\epsilon_s - \epsilon_\infty)}{\epsilon_\infty + \alpha\chi^{(3)}E_z^2}\right)D_z \tag{15}$$

$$\frac{1}{\bar{\omega}_0^2}\frac{d^2G}{dt^2} + \frac{\bar{\delta}}{\bar{\omega}_0^2}\frac{dG}{dt} + \left(1 + \frac{(1-\alpha)\chi^{(3)}E_z^2}{\epsilon_\infty + \alpha\chi^{(3)}E_z^2}\right)G + \left(\frac{E_z}{\epsilon_\infty + \alpha\chi^{(3)}E_z^2}\right)F = \left(\frac{E_z}{\epsilon_\infty + \alpha\chi^{(3)}E_z^2}\right)D_z \tag{16}$$

where $\bar{\delta} = 2/\tau_2$ and $\bar{\omega}_0^2 = (1/\tau_1)^2 + (1/\tau_2)^2$. Equation (15) and (16) are first solved simultaneously for F and G at the latest time step by using a second-order accurate finite-difference scheme that operates on data for the current value of D_z and previous values of D_z, E_z, F, and G. Only two time levels of storage are required with this approach. Then, the latest value of E_z can be obtained via a Newton's iteration of the following equation, using the new values of D_z, F, and G:

$$E_z = \frac{D_z - F - (1-\alpha)\chi^{(3)}E_z G}{\epsilon_0(\epsilon_\infty + \alpha\chi^{(3)}E_z^2)}. \tag{17}$$

This algorithm, as given by the system (15) to (17), determines values of E_z and P_z so that (3) is satisfied. This procedure, combined with the usual FD-TD realization [19] of (1) and (2), comprises the complete solution method.

III. Algorithm

In this section, the finite-difference equations [20] will be described that are used to solve Maxwell's equations, (1) and (2), as well as the equations that account for the nonlinear and dispersive effects contained in the polarization, (15), (16), and (17). Let a region in space-time, x-t, (one-dimensional), contain a lattice of points with coordinates (i, n) for $D_z(x, t)$ and $E_z(x, t)$ and $(i + (1/2, n + (1/2)$ for $H_y(x, t)$. This computational grid will be used to implement the standard leapfrog algorithm [19], known as the FD-TD method.

Using the FD-TD approach, the finite-difference equations for (1) and (2) are given, respectively, by

$$\frac{\mu_0}{\Delta t}\left(H_y^{n+(1/2)}\left(i + \frac{1}{2}\right) - H_y^{n-(1/2)}\left(i + \frac{1}{2}\right)\right) = \frac{1}{\Delta x}\left(E_z^n(i+1) - E_z^n(i)\right) \tag{18}$$

$$\frac{1}{\Delta t}\left(D_z^{n+1}(i) - D_z^n(i)\right) = \frac{1}{\Delta x}\left(H_y^{n+(1/2)}\left(i + \frac{1}{2}\right) - H_y^{n+(1/2)}\left(i - \frac{1}{2}\right)\right). \tag{19}$$

These explicit equations are used in the solution procedure as follows. First (18) is used to determine $H_y^{n+(1/2)}$ from the previous values E_z^n and $H_y^{n-(1/2)}$. Then (19) is used to determine D_z^{n+1} from the previous values $H_y^{n+(1/2)}$ and D_z^n.

Next (15) and (16) are used to determine the values of the linear and nonlinear convolution integrals, F^{n+1} and G^{n+1} at the new time level $n + 1$, using the new value of D_z^{n+1} and the previous values of D_z, E_z, F, and G. Note that the form of the coupling in (15) and (16) results in a finite discretization in which there is strong diagonal dominance in the resulting matrices. This feature is essential for a stable algorithm. A second-order accurate finite-difference scheme for the coupled (15) and (16) is given by the following.

$$\frac{1}{\omega_0^2}\left(\frac{F^{n+1}(i) - 2F^n(i) + F^{n-1}(i)}{(\Delta t)^2}\right) + \frac{\delta}{\omega_0^2}\left(\frac{F^{n+1}(i) - F^{n-1}(i)}{2\,\Delta t}\right) + \left(1 + \frac{\epsilon_s - \epsilon_\infty}{\epsilon_\infty + \alpha\chi^{(3)}(E_z^n(i))^2}\right)\cdot\left(\frac{F^{n+1}(i) + F^{n-1}(i)}{2}\right) + \left(\frac{(\epsilon_s - \epsilon_\infty)(1-\alpha)\chi^{(3)}E_z^n(i)}{\epsilon_\infty + \alpha\chi^{(3)}(E_z^n(i))^2}\right)\cdot\left(\frac{G^{n+1}(i) + G^{n-1}(i)}{2}\right) = \left(\frac{(\epsilon_s - \epsilon_\infty)}{\epsilon_\infty + \alpha\chi^{(3)}(E_z^n(i))^2}\right)\left(\frac{D_z^{n+1}(i) + D_z^{n-1}(i)}{2}\right) \tag{20}$$

$$\frac{1}{\bar{\omega}_0^2}\left(\frac{G^{n+1}(i) - 2G^n(i) + G^{n-1}(i)}{(\Delta t)^2}\right) + \frac{\bar{\delta}}{\bar{\omega}_0^2}\left(\frac{G^{n+1}(i) - G^{n-1}(i)}{2\,\Delta t}\right) + \left(1 + \frac{(1-\alpha)\chi^{(3)}(E_z^n(i))^2}{\epsilon_\infty + \alpha\chi^{(3)}(E_z^n(i))^2}\right)\cdot\left(\frac{G^{n+1}(i) + G^{n-1}(i)}{2}\right) + \left(\frac{E_z^n(i)}{\epsilon_\infty + \alpha\chi^{(3)}(E_z^n(i))^2}\right)\left(\frac{F^{n+1}(i) + F^{n-1}(i)}{2}\right) = \left(\frac{E_z^n(i)}{\epsilon_\infty + \alpha\chi^{(3)}(E_z^n(i))^2}\right)\left(\frac{D_z^{n+1}(i) + D_z^{n-1}(i))}{2}\right). \tag{21}$$

Finally, the new values D_z^{n+1}, F^{n+1}, and G^{n+1} are used in (17) to determine E^{n+1}, by using an approximate Newton iteration procedure as follows.

$$E_z^{p+1} = \frac{D_z^{n+1} - F^{n+1} - (1-\alpha)\chi^{(3)}E_z^p G^{n+1}}{\epsilon_0(\epsilon_\infty + \alpha\chi^{(3)}(E_z^p)^2)}$$

$$p = 0, 1, 2, \cdots . \tag{22}$$

Here $E^p = E^n$ for $p = 0$. For the results shown in this paper, the iteration process was stopped at $p = 1$, since sufficient accuracy was obtained at that step. Having determined E^{n+1}, the solution process, given by (18) to (22) is iteratively repeated to advance the electromagnetic field to successive time levels.

Results

Now results [20], [21] of the integration of Maxwell's equations will be presented, including soliton dynamics. A pulsed optical signal source is assumed to be located at $x = 0$ at the initial time $t = 0$. The pulse is assumed to have unity amplitude of its sinusoidal-carrier electric field, a carrier frequency $f_c = 1.37 \times 10^{14}$ Hz, ($\omega_c = 8.61 \times 10^{14}$ rad/s), and a hyperbolic secant envelope function with a characteristic time constant of 14.6 fs. Approximately 7 cycles of the carrier are contained within the pulse envelope, and the center of the pulse coincides with a zero-crossing of the sinusoid. To achieve soliton formation over short propagation spans of less than 200 μ, we scale values of the group velocity dispersion β_2 and the nonlinear coefficient $\chi^{(3)}$. For example, let: $\epsilon_s = 5.25$; $\epsilon_\infty = 2.25$; $\omega_0 = 4.0 \times 10^{14}$ rad/s; $\delta = 2.0 \times 10^9$ s^{-1}; $\chi^{(3)} = 7 \times 10^{-2}$ (V/m)$^{-2}$; $\alpha = 0.7$; $\tau_1 = 12.2$ fs; and $\tau_2 = 32$ fs. (The last three values are from [7].) This results in β_2 varying widely over the spectral width of the pulse, i.e., from -7 to -75 ps^2/m over the range $(1.37 \pm 0.2) \times 10^{14}$ Hz. Finally, by choosing a uniform FD-TD space resolution of 5 nm ($\approx \lambda_0/300$), the numerical phase velocity error is limited to about 1 part in 10^5, which is very small compared to the physical dispersions being modeled.

First a calculation was performed for the linear case ($\chi^{(3)} = 0$), in which only the effects of dispersion would act on the pulse. As shown in [15], the results showed significant effects of dispersion after a propagation distance of 150 μm. These effects included pulse broadening, diminishing amplitude, and carrier frequency modulation ($>f_c$ on the leading side, $<f_c$ on the trailing side) which causes an asymmetrical shifting of the envelope, a higher-order dispersive effect. These qualitative features of the effect of anomalous dispersion have been predicted [16], but until now have not been computed by directly integrating Maxwell's equations.

Fig. 1 shows the results for $\chi^{(3)} = 7 \times 10^{-2}$ (V/m)$^{-2}$. The pulse is plotted at $n = 2 \times 10^4$ and 4×10^4 time steps, corresponding to propagation to $x = 55$ μm and 126 μm. The nonlinear effects balance the linear dispersive effects, yielding a rightward propagating soliton that retains its amplitude and width. Also, a second low-amplitude pulse is seen to move out ahead of the soliton. The carrier frequency of this "daughter" is upshifted to $\approx 4.9 \times 10^{14}$ Hz, approximately 3.6 times that of the soliton. Also, in both the linear and nonlinear cases, by observing a video of the pulse evolution, it was noted that the phase velocity of the carrier was substantially greater than the group velocity of the envelope. In the video, waves would appear at the rear (left) side of the pulse, advance through the pulse and disappear from the front (right) side of the advancing pulse. Such observations of the carrier properties are readily observable in the present formulation.

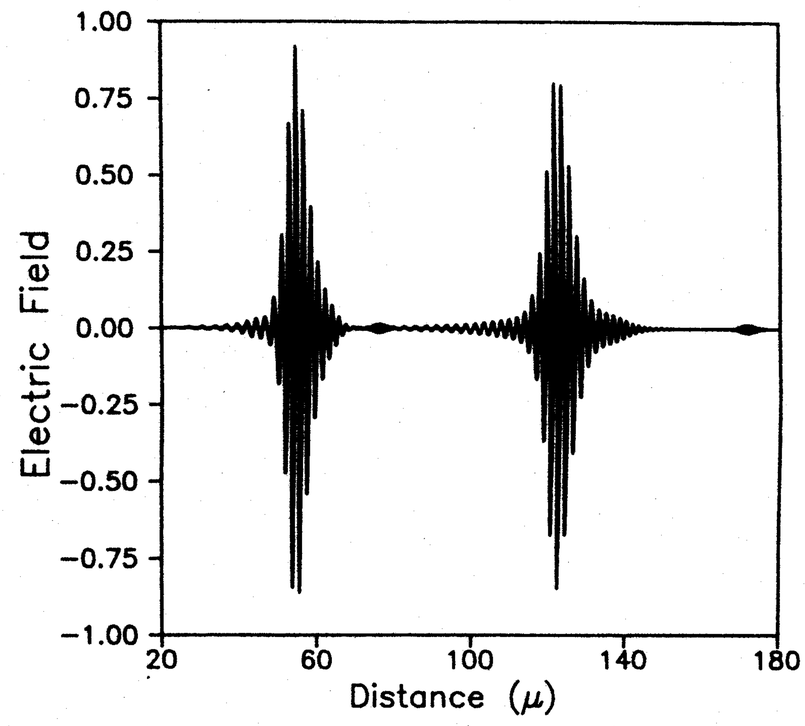

Fig. 1. FD-TD results for the optical soliton carrier pulse after propagating 55 μm and 126 μm.

To further verify that the main pulse in Fig. 1 had the characteristics of a soliton, in [15], a plot was presented of the Fourier spectrums of the main pulse at the two times that are shown in Fig. 1. The figure showed a 4 THz redshift and sharpening of the spectrum as the soliton propagates. From GNLSE theory, the redshift is predicted due to the Raman effect [7], [10] occurring as a higher-order dispersive nonlinearity modeled by the function $g_R(t)$ in (12).

Last, we consider the collision of two counter-propagating solitons. Each is identical and has all of the parameters of the previous case. As is characteristic of colliding solitons [22], we find that after the collisions both main and daughter pulses separate and move apart without changing their general appearances. Fig. 2 shows the counter-propagating solitons at 20 000 time steps; at this time the two main pulses are approaching each other. Fig. 3 shows them at 25 000 time steps; at this time they are nonlinearly interacting. A video of the collision showed violent changes in the solitons during the interaction period, with severe increases and decreases in the amplitude of the combined pulse due to the nonlinear interference effects. Finally, Fig. 4 shows them at 30 000 time steps; at this time they have passed through each other and are separating. However, there are lagging phase shifts due to the collision: 12° for the carriers in the solitons, and 31° for the carriers in the daughters. (At 30 000 time steps, the phase shift due to numerical error is approxi-

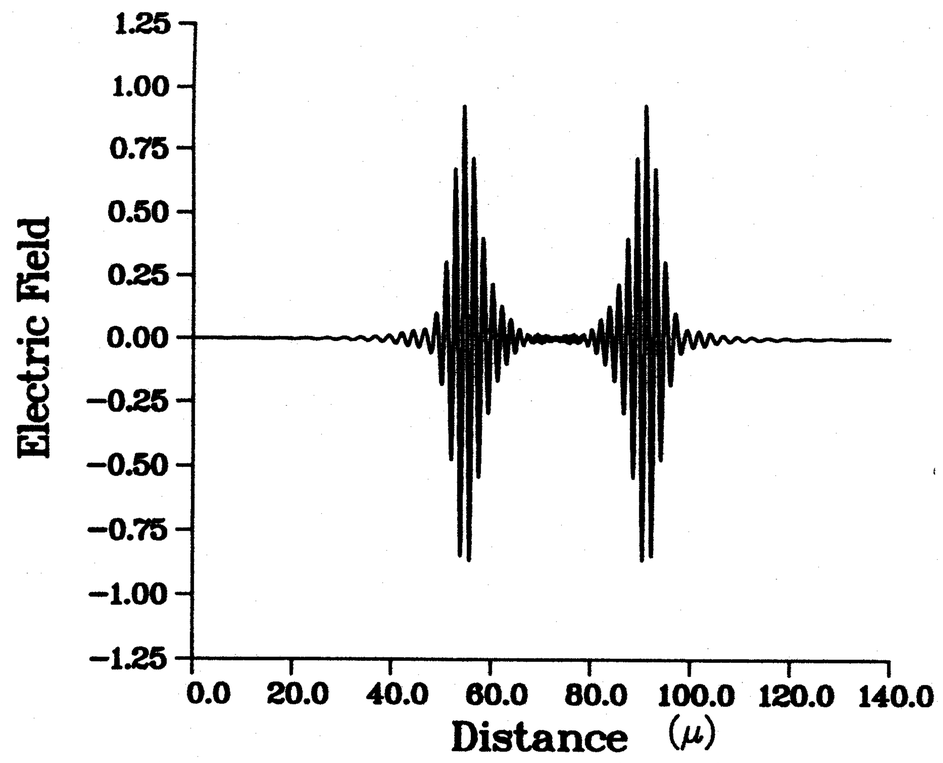

Fig. 2. Counter-propagating solitons at 20 000 time steps.

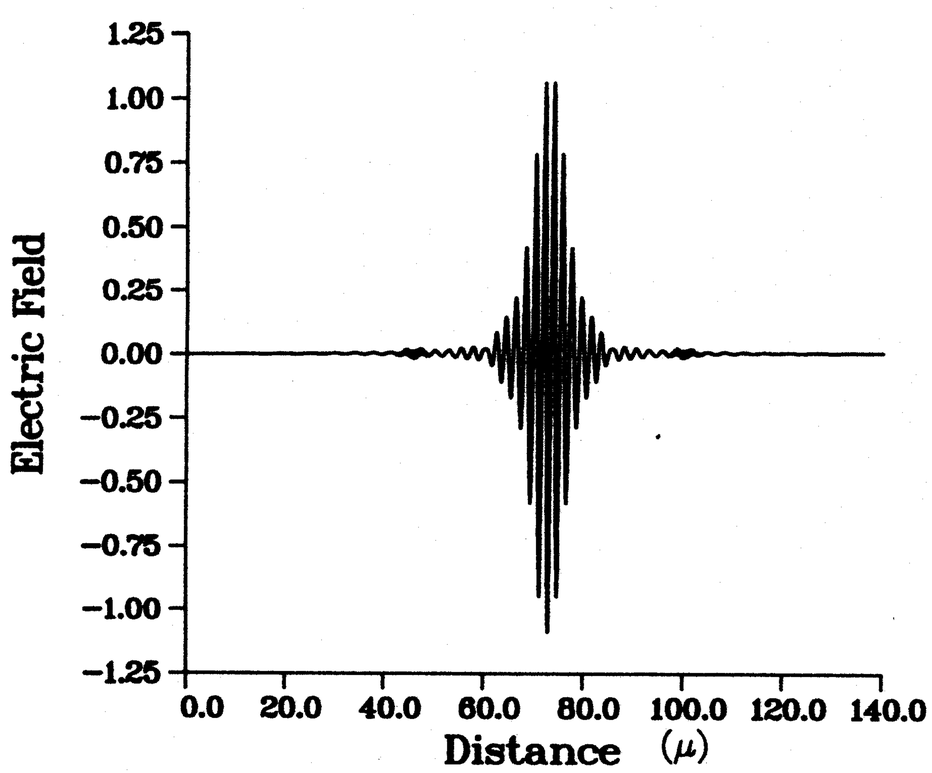

Fig. 3. Counter-propagating solitons at 25 000 time steps.

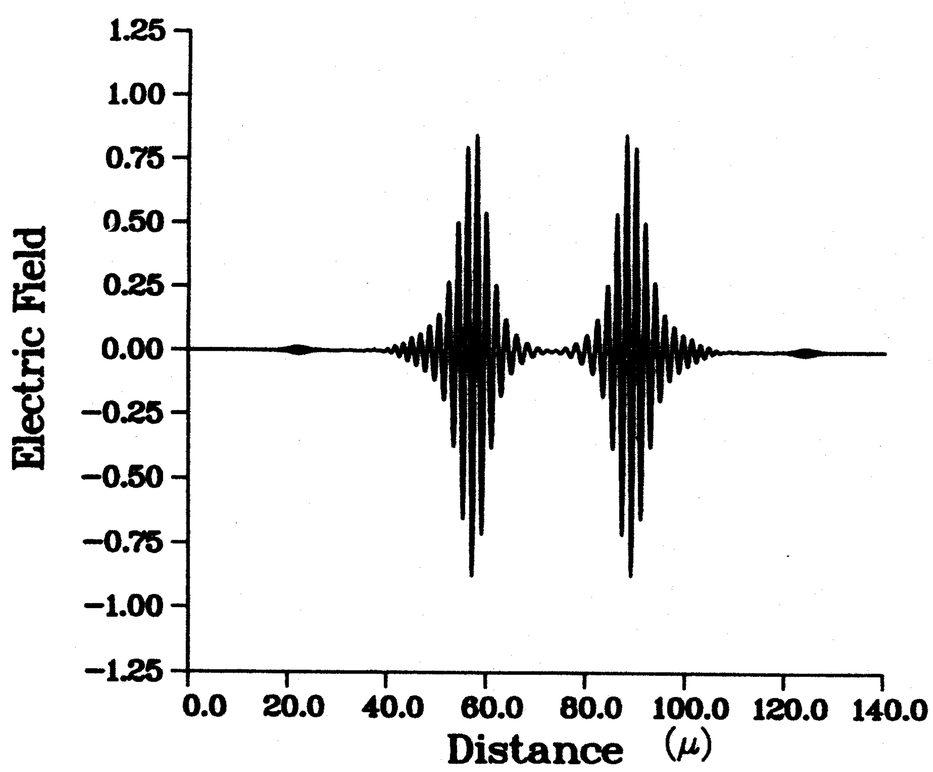

Fig. 4. Counter-propagating solitons at 30 000 time steps.

mately equal to one-third of a degree.) To illustrate the phase lag due to the interaction, in [15], a plot was presented of the space dependence of the central part of the rightward-moving daughter for the original uncollided case and the collided case, with both curves at exactly 30 000 time steps of the algorithm. It clearly illustrated the phase shift of 31°. Such precise measurements of phase shifts, not easily obtained by GNLSE theory, are readily displayed by using results from the present formulation. Also if an amplification of these phase lags could be obtained by varying the parameters in this calculation, such a phase lag might form the basis for an optical switching device.

For these calculations, less than 350 s of CPU time were required on a Cray *Y*-MP8/832, which is a small amount of time. Hence significantly larger calculations are feasible on present-day computers. For example, the propagation of larger pulses, such as picosecond pulses, may be possible. The modeling of any pulse over long distances, such as meters or kilometers, may be possible by the use of scaling on the parameters that characterize the material. In the case considered in this article, the values of the linear group velocity dispersion β_2 and the nonlinear coefficient $\chi^{(3)}$ were scaled to values that are orders of magnitude larger than those that occur naturally in optical fibers. By use of these scalings, the effects of linear dispersion, nonlinear dispersion and nonlinear instantaneous response took place over the short distance of 200 μm.

V. Concluding Remarks

Optical switching, which is based on nonlinear optical responses, has become an intense area of research [6] following the advent of the laser. Currently, many materials are being investigated [23] for their nonlinear optical properties for various applications, including optical switching.

A new algorithm has been developed that permits, for the first time, the direct time integration of the full-vector, nonlinear Maxwell's equations. This new capability permits the modeling of linear and nonlinear, instantaneous and dispersive effects in the electric polarization in material media. The modeling of the optical carrier is retained in this approach. Using this algorithm, results are presented of first-time calculations in one dimension of the propagation and collision of femtosecond electromagnetic solitions that retain the optical carrier. The nonlinear modeling takes into account such quantum effects as the Kerr and Raman interactions at distance scales larger than about 5 nm.

The novel approach discussed here is a fine-grained direct time-domain numerical solution of Maxwell's equations that rigorously enforces the field vector boundary conditions at all interfaces of dissimilar media, whether or not the media are dispersive or nonlinear. The new approach is almost completely general. It assumes nothing about 1) the homogeneity or isotropy of the optical medium; 2) the magnitude of the nonlinearity; 3) the nature of the material's $\omega - \beta$; or 4) the shape, duration, polarization, and numbers of the optical pulse(s).

By retaining the optical carrier, the new method solves

for fundamental quantities—the optical electric and magnetic fields in space and time—rather than a nonphysical envelope function. The present approach is robust and should permit modeling 2-D and 3-D boundary-value problems involving optical soliton propagation, scattering, and switching directly from the full vector, nonlinear Maxwell's equations. For nonlinear optics, this approach has the potential to provide a modeling capability for millimeter-scale integrated optical circuits beyond that of existing techniques using the generalized nonlinear Schrodinger equation.

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