A Numerical Technique for Analyzing Electromagnetic Wave Scattering from Moving Surfaces in One and Two Dimensions

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Abstract—**The electromagnetic wave scattering properties of a moving, perfectly conducting mirror are analyzed using a new numerical technique based on the finite-difference time domain (FD-TD) method. This numerical technique is unique in that it does not require a system transformation where the object is at rest but gives a solution to the problem directly in the laboratory frame. First, two canonical one-dimensional cases are considered, the uniformly moving and the uniformly vibrating mirror. Numerical results for the scattered field spectrum are compared to available analytical results, and an excellent agreement is demonstrated. The ability of the FD-TD model to obtain the physics of the double-Doppler effect (for the uniform translation case), and FM-like reflected spectrum (for the uniform vibration case) is highlighted. Second, the method is extended to two-dimensions where a plane wave at oblique incidence on an infinite vibrating mirror is considered. A good agreement with published results is demonstrated for this case. This new approach based on FD-TD provides a potentially strong tool to numerically model a variety of problems involving moving and vibrating scatterers where alternative analytical or numerical modeling means are not available.**

I. Introduction

THE ANALYTICAL THEORY of electromagnetic wave scattering by moving bodies has been developed principally for canonical one-, two-, and three-dimensional structures [1], [2], [3]. Canonical problems considered include planar conducting and dielectric interfaces in uniform translation or vibration [4], uniformly moving random rough surfaces [5], uniformly moving or vibrating cylindrical and spherical shapes [6], [7], [8], and simple rotating shapes [9]. Motivation for pursuing such analyses has been provided in part by research in the generation of millimeter and submillimeter waves using the interaction of microwaves with relativistically moving ionization (plasma) fronts or electron beam fronts [10], [11].

Existing analytical theory in this area models the physics of a reflecting surface in uniform translation or vibration by employing system transformations where the surface is at rest.

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Difficulties arise when attempting such analyses for general two- or three-dimensional scatterers, since closed-form solutions cannot be obtained when the scatterer shape, composition, translation, and surface vibration are arbitrary. Yet, such general problems arise as more detailed information is required concerning microwave interactions with moving or oscillating charged particle beams of finite cross section.

This paper introduces a purely numerical approach for modeling scattering by relativistically moving perfectly conducting bodies based upon the finite-difference time-domain (FD-TD) method [12]–[24]. This approach uses no system transformation and gives the solution directly in the laboratory frame. It exploits the detailed time-domain modeling characteristics of FD-TD, and has the potential to permit computation of accurate solutions for moving/vibrating rigid body problems of substantially more complexity than existing analytical approaches. The work presented here includes derivation of the necessary modifications of FD-TD for the relativistic body case, and validations for uniform translation and vibration in one and two dimensions against existing analytical theory. The ability of the FD-TD model to obtain the physics of the double-Doppler effect (for uniform translation), and FM-like reflected spectrum (for vibration), will be highlighted. A subsequent paper will address the extension of the new approach to treat convex, conducting, two-dimensional bodies subject to uniform relativistic translation and/or vibration.

The present paper is organized as follows. Section II briefly summarizes the background of the basic FD-TD method, and then describes the basis and FD-TD numerical implementation of the required relativistic electromagnetic field boundary conditions. Section III discusses validation studies for the uniformly moving mirror in one dimension. Section IV discusses validation studies for the uniformly vibrating mirror also in one dimension. Section V presents a two-dimensional case study of the oblique incidence with comparative results. Last, Section VI provides the summary and conclusions.

II. Description of the Numerical Method

A. Background of the Basic FD-TD Method

In the mid-1960's, Yee introduced a computationally efficient means of directly solving Maxwell's time-dependent

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curl equations using finite differences [12], now designated as the finite-difference time domain method. With this approach, the continuous electromagnetic field in a finite volume of space is sampled at discrete points in a space lattice and at discrete points in time. Wave propagation, scattering, and penetration phenomena are modeled in a self-consistent manner by marching in time, that is, repeatedly implementing the finite-difference analog of the curl equations at each lattice point. This results in a simulation of the continuous actual waves and sampled-data numerical analogs propagating in a data space stored in a computer. Space and time sampling increments are selected to avoid aliasing of the continuous field distribution, and to guarantee stability of the time-marching algorithm [13]. Time marching is completed when the desired steady-state field behavior is observed.

The basic FD-TD method permits the modeling of electromagnetic wave interactions with a level of detail comparable to that of the widely used method of moments [25]. Further, its explicit nature leads to overall computer storage and running time requirements that are linearly proportional to N, the number of field unknowns in the finite volume of space being modeled. These two attributes permit FD-TD to provide detailed numerical models of wave interactions with structures having volumetric complexity, such as biological tissues [14] and loaded cavities [15], [16].

For the present work, it has been necessary to modify the basic FD-TD formulation to model moving, perfectly conducting, scatterers. The most simple, "brute-force" approach would be to simply let the scatterer occupy slightly different positions in the space lattice at each time step. This corresponds to the quasi-steady-state method [4], which has been adopted in certain analytical solution approaches. Although this method gives an approximate answer when applied to FD-TD, as will be seen in Section III, it does not completely provide the proper physics. An appropriate relativistic electromagnetic field boundary condition, discussed next, must also be incorporated into the FD-TD code at the surface of the scatterer. Fortunately, this condition is easy to derive in a form suitable for FD-TD implementation.

B. The Relativistic Boundary Conditions in the FD-TD Code

There are a number of ways to solve for the scattered field from a moving object. In general, the desired analytical solution for the scattered field can be obtained by a Lorentz transformation of the incident field to the moving system, and solution for the scattered field in the frame of reference of the moving system [26]. In this reference frame, the scatterer surface is stationary and the electromagnetic boundary conditions are well defined. The inverse Lorentz transformation then provides the final answer in the laboratory frame. However, a direct solution that is more straightforward (and shorter in some cases) is possible in the laboratory frame without a Lorentz transformation if one uses what is defined as the "relativistic boundary conditions" at a moving interface between medium 1 and medium 2. The derivation of these conditions, in its general form, is well presented in [3] and yields

$$\hat{u}_n \times (\vec{E}_2 - \vec{E}_1) - (\hat{u}_n \cdot \vec{v})(\vec{B}_2 - \vec{B}_1) = 0 \tag{1a}$$

$$\hat{u}_n \cdot (\vec{D}_2 - \vec{D}_1) = \rho_s \tag{1b}$$

$$\hat{u}_n \times (\vec{H}_2 - \vec{H}_1) + (\vec{u}_n \cdot \vec{v})(\vec{D}_2 - \vec{D}_1) = \vec{J}_s \tag{1c}$$

$$\hat{u}_n \cdot (\vec{B}_2 - \vec{B}_1) = 0 \tag{1d}$$

where $\vec{E}_i, \vec{D}_i, \vec{H}_i$, and $\vec{B}_i$ are, respectively, the electric field, electric flux density, magnetic field, and magnetic flux density in medium 1 and 2; ρ_s and $\vec{J}_s$ denote the surface-charge and current densities; $\vec{v}$ is the velocity of the moving interface (assumed to be uniform), and $\hat{u}_n$ is the unit vector normal to the interface.

It is important to note from (1) that a scatterer motion transverse to the surface plane (perpendicular to the surface normal) results in boundary conditions similar to that of a fixed object, simply because the term $\hat{u}_n \cdot \vec{v}_n$ is now equal to 0. It should further be noted that (1) implies that the tangential E-field at the surface of a perfectly conducting moving boundary can be finite. However, this does not result in an infinite surface current density because the usual expression, $\vec{J} = \sigma\vec{E}$, for current density in a material of conductivity σ is no longer valid. Instead, for a uniformly moving object, the total induced current is the result of a conduction current plus an extra term. Defining β as the ratio v/c, c being the velocity of light in free space, the total current is given by

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})\frac{1}{\sqrt{1-\beta^2}} \tag{2}$$

where, for a perfect conductor, $\vec{E} + \vec{v} \times \vec{B} = 0$ from (1); and therefore the surface current density $\vec{J}_s$ remains finite. In many references, only small velocities are considered and the term β^2 is neglected compared to 1.

In the derivation of the above equations, no assumption is made on the speed v relative to the speed of light c, hence the name relativistic boundary conditions. The only assumption made is that the speed v is uniform. However, the same relativistic boundary conditions derived for uniform v have been widely applied to study accelerating bodies, under certain conditions where the acceleration is sufficiently low [4], [27]. Here, a new reference frame called the "co-moving frame" or "instantaneous frame" is introduced. The difference is that now the velocity v in (1) represents the instantaneous velocity instead of the uniform velocity. The term "Doppler approximation" [2] is also used to denote analyses wherein it is assumed that the instantaneous velocity equals a uniform velocity. It is not within the scope of this paper to discuss the details of this theory. Its validity in rotating coordinates has been investigated by Shiozawa [27]. The reader can also refer to the presentation given in [4] and [9].

For a perfectly conducting moving surface, the boundary condition (1a) relates linearly the local values of the instantaneous total tangential E- and H-fields at the surface of the conductor (lit side). This relation, similar in form to that of a surface impedance, presents a problem for implementing in the FD-TD code which computes E and H values separated by half-step intervals in time and space. It is necessary to derive an equivalent form of the relativistic boundary condition for perfectly conducting surfaces that is not contradictory with this half-step nonlocalization of field values in the FD

TD code. Derivation of such an equivalent form is given in the Appendix.

Using the results of the Appendix, the relativistic boundary condition for a moving mirror (in a form appropriate for FD-TD implementations) is given by

$$E = 2\frac{-\hat{u}_n \cdot \vec{v}}{c - \hat{u}_n \cdot \vec{v}} E^i \tag{3}$$

or

$$B = \frac{2c}{c - \hat{u}_n \cdot \vec{v}} \cdot B^i \tag{4}$$

where E and E^i are, respectively, the total tangential electric field and the incident tangential electric field values at the mirror surface. B and B^i are, respectively, the total tangential magnetic field and the incident tangential magnetic field values at the mirror surface.

Now, the value of the total tangential electric field at the mirror surface is given in terms of the incident electric field value at the boundary. The latter is easily obtained from a parallel one-dimensional grid already built into the FD-TD code as a look up table. Implementation of the boundary condition for a moving mirror now becomes a simple matter. At each half-time step when the E-field and the H-field are computed, respectively, the position of the reflecting mirror in the grid is first determined. Then, the relativistic boundary conditions (3) or (4) for the field values at the surface of the mirror are implemented.

C. Approximation of E and H Adjacent to a Moving Surface

The question arises as to the value of the incident electric field when the position of the mirror does not coincide with a point in the grid. For this purpose, linear interpolation is used. From the geometry of Fig. 1,

$$E^i \text{ at mirror} = \frac{(\delta_y - \Delta) \cdot E^i(j+1) + \Delta \cdot E^i(j)}{\delta_y}. \tag{5}$$

The value of the total electric field at the mirror surface is stored at the total electric field grid point closest to the surface. No extra grid points are introduced. In Fig. 1 for example, the value of the total electric field at the boundary is stored at the $E(j+1)$ point if $\Delta < \delta_y - \Delta$ and at the $E(j)$ point if $\Delta > \delta_y - \Delta$.

Next, a Faraday's law contour integral is used to compute the total H-field adjacent to the mirror surface. (The idea of contour integral subcell models has been previously used in the FD-TD analysis of wave penetration through narrow slots in thick conducting screens [22] and coupling to wires and wire bundles [23].) Applying Faraday's law, given by

$$\int \vec{E} \cdot \vec{dl} = -\frac{\partial \int \vec{B} \cdot d\vec{S}}{\partial t} \tag{6}$$

along the path defined in Fig. 1, and assuming that the H-field is almost uniform in the shaded region, we obtain

$$(E(j+1) - E(j)) \cdot \delta_z = -\mu \frac{\partial B}{\partial t} \delta_z \cdot (\delta_y - \Delta). \tag{7}$$

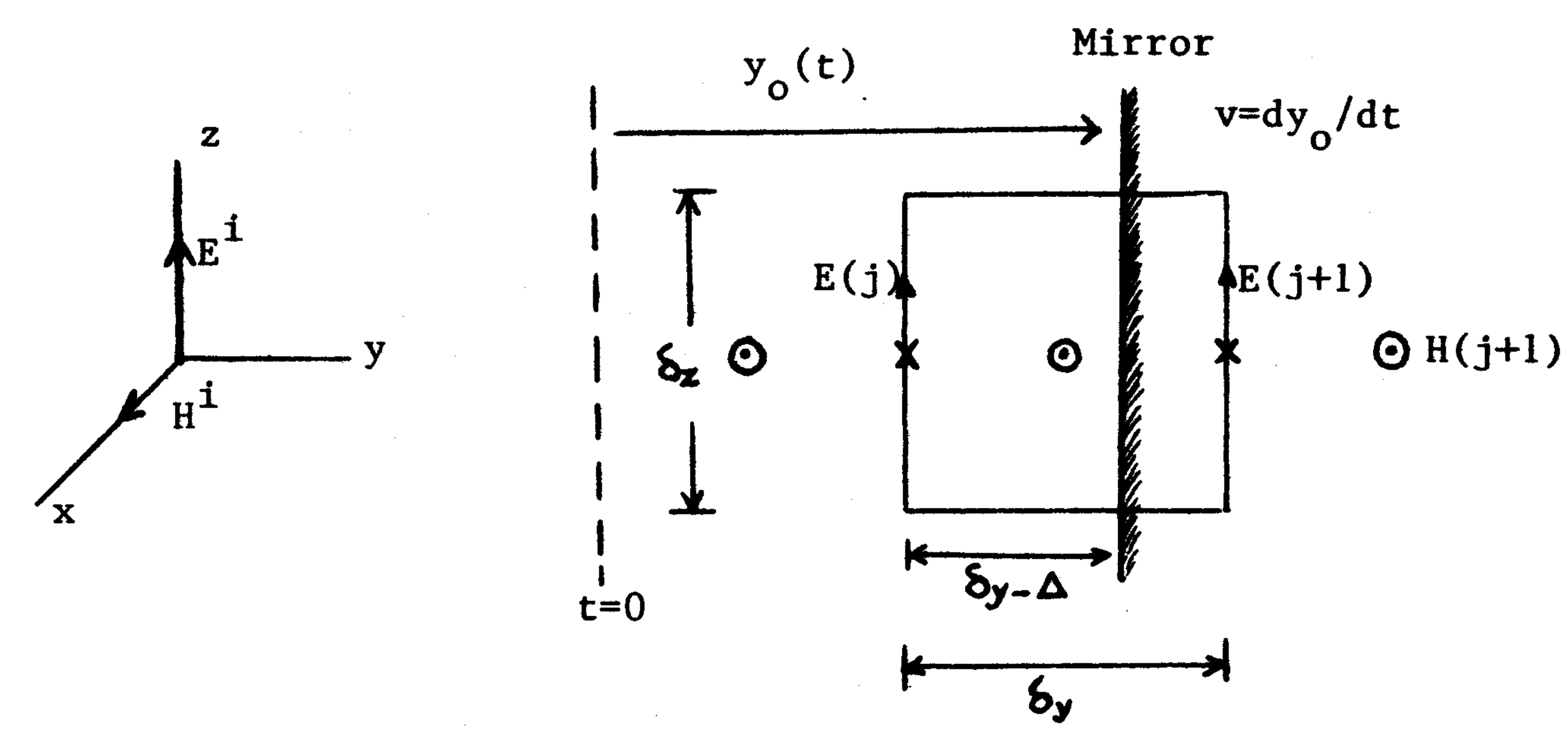

Fig. 1. FD-TD field component geometry for the moving mirror case in a one-dimensional FD-TD grid.

By applying Ampere's law, a similar contour integral can be derived to compute the total E-field adjacent to the mirror surface [22].

III. The Case of a Uniformly Moving Mirror, Normal Illumination

A. Existing Analytical Formulation

An incident sinusoidal plane wave of frequency ω_i (illumination frequency) and unit amplitude is normally incident on a uniformly moving mirror. Referring to Fig. 1, a positive mirror velocity v means that the mirror is receding from the incident wave, and a negative mirror velocity means that the mirror is advancing toward the incident wave. The scattered electric field is given by [2]

$$E_z^r(y,t) = -\left[\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right] \cdot \exp j\left[\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)(\omega_i t - ky) + 2jk\left(\frac{r_0 - vt_0}{1-\frac{v}{c}}\right)\right] \tag{8}$$

where $y_0 = v(t - t_0) + r_0$ is the position of the mirror boundary with respect to a reference point, and r_0 and t_0 are some initial values. (For simplicity, we set both r_0 and t_0 equal to 0.) A "double-Doppler" effect is apparent from (8) in that both the frequency and amplitude of the scattered field are transformed by the same multiplying factor defined as $\alpha = [1 - (v/c)]/[1 + (v/c)]$.

B. FD-TD Modifications Considered

Three different FD-TD algorithm modifications for the electromagnetic boundary condition at a moving surface, discussed in Section II-B, have been considered in numerical tests of whether FD-TD can properly model the double-Doppler effect.

1) *The Quasi-Stationary Method*—Here, the mirror is assumed stationary for a complete one-time-step interval. The relativistic boundary conditions are not implemented. Only the position of the mirror is determined after each full time step. A contour integral model is used when necessary to compute more exactly the H- and/or E-field next to the mirror surface. Such a method

TABLE I
DOUBLE-DOPPLER SHIFTS AS OBTAINED BY FD-TD AND ANALYTICALLY, FOR THREE DIFFERENT MODELS AND A GIVEN VELOCITY

Case	v/c	Reflected Amplitude		Reflected Frequency	
		Analytical	FD-TD	Analytical	FD-TD
Reference	0	1	1	1	1
Quasistatic	-1/3	1 0 at $\omega \neq 2$	0.9783 0.0427 at $\omega = 9^*$	2	2
Semi relativistic	-1/3	2 0 at $\omega \neq 2$	1.994 0.1308 at $\omega = 9^*$	2	2
Full relativistic	-1/3	2 0 at $\omega \neq 2$	1.994 0.0523 at $\omega = 9^*$	2	2

* Spurious frequency components

will give the proper shift in frequency but leaves the amplitude unchanged as the theory predicts [2].

2) *The Semirelativistic Method*—Here, the relativistic boundary condition is implemented each time for the E-field only. In other words, only (3) is used. The value of the total E-field at the mirror surface is stored at the closest, total E-field, grid point to the mirror surface. No extra grid point is introduced. A contour integral model is used to compute the H-field next to the mirror surface. This method should be enough to model the proper physics of the problem. However, usage of a contour integral model makes the program more difficult to generalize for arbitrary mirror velocities.

3) *The Fully-Relativistic Method*—Here, the relativistic boundary condition is implemented each half-time step for both the E-field and the H-field using (3) and (4) respectively. This case does not require a contour integral model since now the H-field, next to the mirror, is computed from (4). This method was found to be more accurate and more general than the previous method.

In all the above three cases, the fields behind the mirror are set to zero.

C. Comparative FD-TD and Analytical Results

Let us consider the case of a mirror illuminated at normal incidence by a unit-amplitude sinusoidal plane wave having a normalized frequency, $\omega_i = 1$. The mirror is assumed to be advancing toward the incident wave at one-third the speed of light ($v = -c/3$). Table I shows double-Doppler shifts as obtained analytically and by FD-TD for the three relativistic moving surface models. The spatial frequency spectrum of the reflected wave is obtained by taking the Fourier transform of the FD-TD computed field versus position sample after 20 cycles had been stepped. The spatial frequency is scaled such that a value, $\omega = 10$, corresponds to the FD-TD grid Nyquist frequency (the maximum spatial frequency that the FD-TD grid can support as a sampled-data system).

It is seen that the quasi-stationary boundary conditions cause the FD-TD code to generate a reflected-wave spatial frequency component with the proper upward Doppler frequency shift (to $\omega = 2$) leaving the amplitude almost unchanged as predicted by the analytical theory of [2]. The semirelativistic boundary condition provides the proper Doppler shifts in both the frequency and magnitude (again of a shift of 2:1) with a small spurious frequency component. The fully relativistic boundary condition, causes a further damping of the undesired frequency component. For both the semi-relativistic and fully-relativistic cases, the error in the computed amplitude of the properly shifted spectral component at $\omega = 2$ is only 0.3 percent (0.026 dB). The FD-TD computed spurious frequency component near $\omega = 9$ is limited to 6.54 percent (−23.7 dB) in the semirelativistic case and to 2.62 percent (−31.6 dB) in the fully relativistic case.

Table II shows double-Doppler results obtained for eight different mirror velocities using only the fully relativistic boundary condition. In all of these cases, FD-TD generates a reflected wave with the proper Doppler shifts in both frequency and amplitude. The error in the FD-TD computed amplitude of the properly shifted spectral component is limited to less than 1.5 percent (0.131 dB), and the generation of spurious frequency components is limited to less than 5 percent (−26 dB). These spurious components are numerical artifacts due to the interpolation process used in computing the incident field at the mirror surface (Section II-C), and the storing of the surface field values at the closest grid point (Section II-B). As observed in Table II, these artifacts disappear when the mirror velocity equals $c/2$ where, at every time step, the mirror position corresponds exactly to a grid field point.

IV. THE UNIFORMLY VIBRATING MIRROR

A. Existing Analytical Formulation

Referring to Fig. 1, the exact form of the scattered field from a linearly vibrating mirror is given by a set of two equations [28], [29]:

$$t = t_0 + \frac{d}{c} \sin(\omega_v t_0) - \frac{y}{c} \tag{9a}$$

$$E_z^r(yt) = -\frac{1 - \beta \cos(\omega_v t_0)}{1 + \beta \cos(\omega_v t_0)} \cos(\omega_i t_0 - kd \sin(\omega_v t_0)) \tag{9b}$$

where ω_i is the frequency of the incident wave; y_0 =

TABLE II
DOUBLE-DOPPLER SHIFTS AS OBTAINED BY FD-TD AND ANALYTICALLY, FOR UNIFORM VELOCITIES

v/c	Reflected Amplitude		Reflected Frequency	
	Analytical	FD-TD	Analytical	FD-TD
-1/3	2.000 0 at $\omega \neq 2$	1.9940 0.0523 at $\omega = 9$*	2.000	2.000
-1/5	1.5000 0 at $\omega \neq 1.5000$	1.4909 0.0660 at $\omega = 7$* 0.0751 at $\omega = 9$*	1.5000	1.5000
-1/7	1.3333 0 at $\omega \neq 1.3333$	1.3239 0.0613 at $\omega = 4$* 0.0465 at $\omega = 5$*	1.3333	1.3333
1/2	0.3333 0 at $\omega \neq 0.3333$	0.3284 0 at $\omega \neq 0.3333$	0.3333	0.3333
1/3	0.5000 0 at $\omega \neq 0.5000$	0.4939 0.0410 at $\omega = 4$*	0.5000	0.5000
1/4	0.6000 0 at $\omega \neq 0.6000$	0.5590 0.0081 at $\omega = 4$* 0.0021 at $\omega = 8$*	0.6000	0.6000
1/5	0.6666 0 at $\omega \neq 0.6000$	0.6587 0.0560 at $\omega = 5$* 0.0164 at $\omega = 6$*	0.6000	0.6000
1/7	0.7500 0 at $\omega \neq 0.7500$	0.74161 0.0514 at $\omega = 2$* 0.0153 at $\omega = 4$*	0.7500	0.7500

* Spurious frequency components

$d \sin(\omega_v t)$ describes the displacement of the mirror vibrating with a frequency, w_v and $\beta = \omega_v d/c = v_{max}/c$. Equation (9b) can also be written in a Fourier series expansion,

$$E_z^r(y,t) = -Re \sum_{m=-\infty}^{\infty} J_{-m}(\alpha_m) \cdot \left(1 + \frac{m}{m + 2\frac{\omega_i}{\omega_v}}\right) e^{j(\omega_i + m\omega_v)(t + (y/c))} \tag{10a}$$

where

$$\alpha_m = m\beta + 2kd = \beta\left(m + 2\frac{\omega_i}{\omega_v}\right). \tag{10b}$$

The scattered field spectrum thus contains the incident frequency ω_i and an infinity of sidebands located at $\omega_i + m\omega_v$ generated by the vibration of the mirror.

The scattered field spectrum for the vibrating mirror is very similar to the spectrum of an FM tone-modulated signal. In both cases, an infinity of sidebands located at $\omega_{cent} + m\omega_\Delta$ is generated, where ω_{cent} is a center frequency (the illuminating frequency for the vibrating mirror case, the carrier frequency for the FM case); and ω_Δ is the sideband separation (the vibration frequency for the mirror case, the modulating tone frequency for the FM case). Further, in both cases, the spectral amplitude of the mth sideband is proportional to J_m; a Bessel function of order m. For the vibrating mirror, the argument of the Bessel function depends on the amplitude and frequency of vibration; for FM, the argument depends upon the amplitude and frequency of the modulating tone.

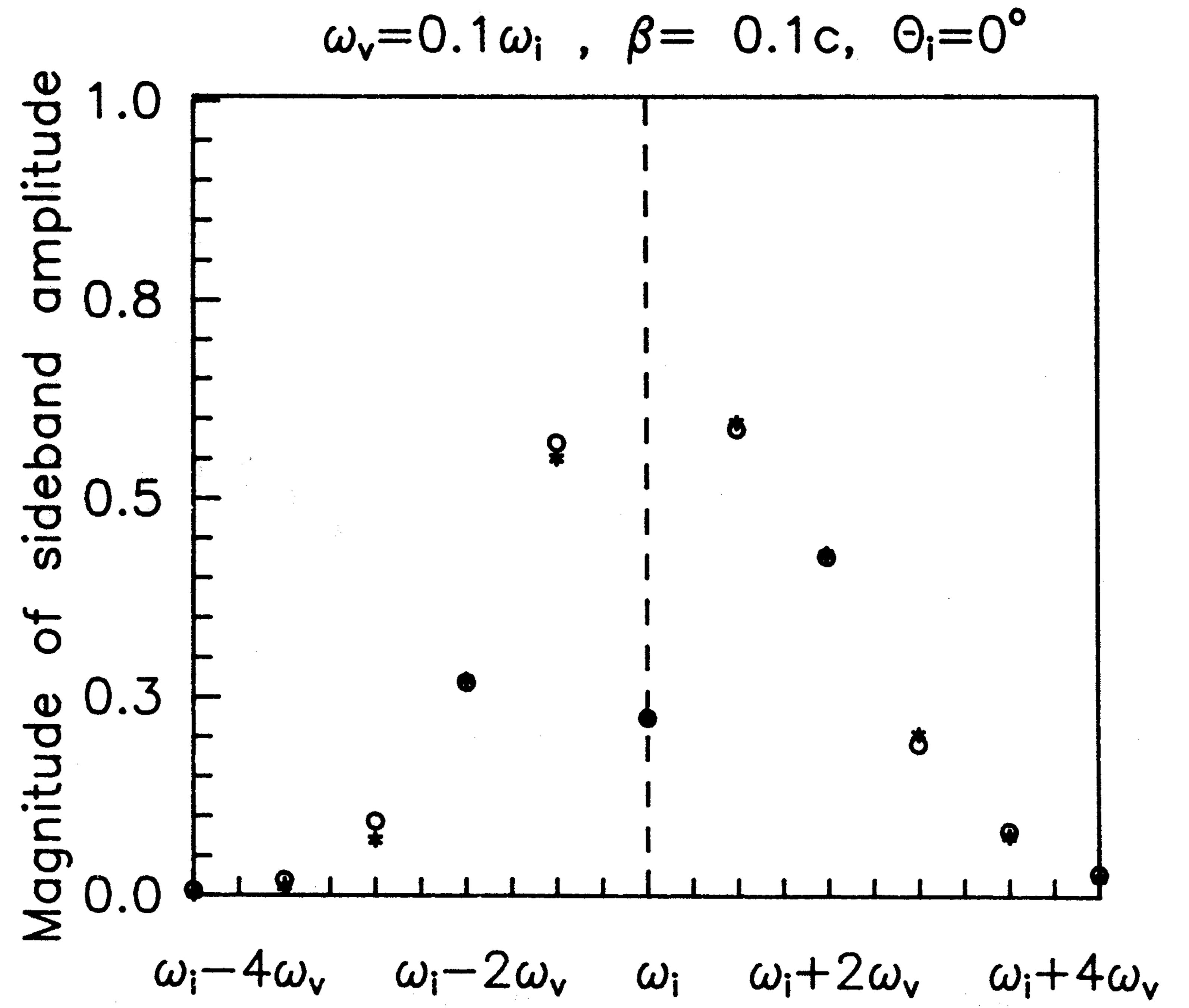

Fig. 2. Comparison of FD-TD and analytical results for the sidebands of the reflected spectrum. *: exact values; o: FD-TD values.

B. FD-TD Modeling Procedure

In modeling the vibration of the mirror with the FD-TD code, we follow the same procedure as for the uniformly moving mirror, but use only the fully relativistic boundary condition, and assume that we are in a region where the theory of the "co-moving frame" is still applicable [4]. Our interest will be mainly in the variation of the scattered field amplitude at the fundamental frequency ω_i, as a function of mirror vibration frequency ω_v, and amplitude d. It is clear from (10) that at the fundamental frequency, where $m = 0$, the exact solution for the magnitude of the scattered field leads to a $J_0(2kd)$ dependence, where $2kd = 2\beta(\omega_i/\omega_v)$.

C. Comparative FD-TD and Analytical Results

Fig. 2 shows the magnitudes of the sideband components of the reflected field spectrum for a vibrating mirror having a vibration frequency ω_v, equal to 0.1 times the illumination frequency ω_i; and a maximum mirror surface velocity equal to 0.1 times the speed of light. The plotted values are computed using both the exact solution of (10) and the FD-TD method with fully relativistic boundary conditions and a spatial resolution of 20 cells per wavelength of the illuminating wave. An excellent correspondence is noted between the exact and FD-TD numerical data. The error in computing the magnitude of the reflected component at the illuminating frequency is only 0.27 percent (0.02 dB).

As mentioned earlier, an important test for the FD-TD approach is to compare the variation of the scattered field amplitude at the illuminating frequency with the exact solution as mirror vibration parameters are changed. Noting that the exact solution states that the argument of the Bessel function weight for this spectral component is dependent upon the product of maximum normalized mirror velocity, β, and ω_i/ω_v, the FD-TD modeling procedure should trace out the same Bessel function variation of the scattered field amplitude at the illuminating frequency regardless of whether β is

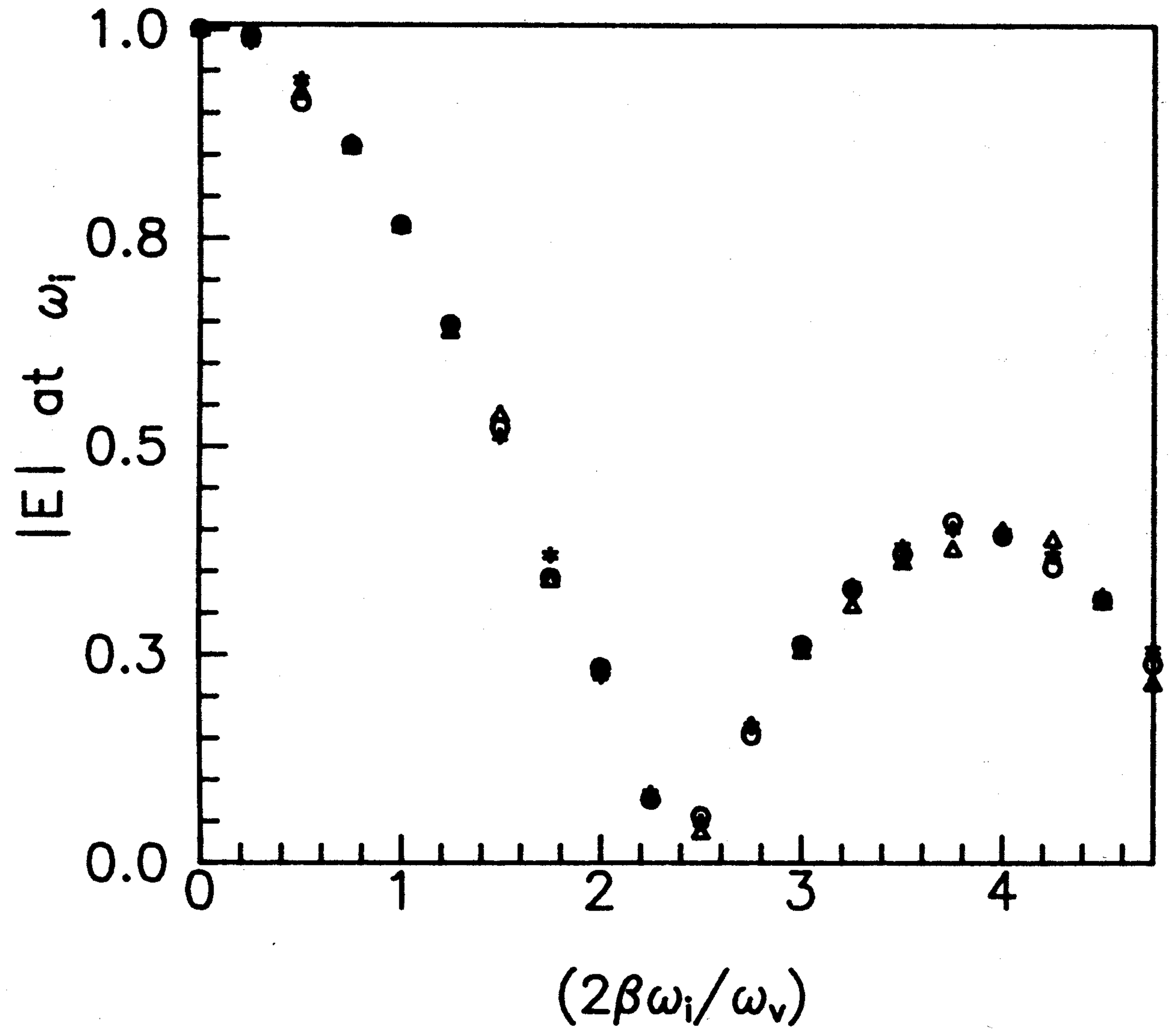

Fig. 3. Comparison of FD-TD and analytical results for the scattered field magnitude at the illumination frequency. *: exact values; o: FD-TD values for $\omega_i = 5\omega_v$; $\triangle$: FD-TD values for $\beta = 0.1c$.

varied while keeping ω_i/ω_v fixed, or whether ω_i/ω_v is varied while keeping β fixed. Fig. 3 graphs the results of numerous trials of the FD-TD procedure wherein these parametric studies (and corresponding Fourier analyses) were conducted with the fully-relativistic boundary conditions incorporated into the FD-TD code. For the first case where β is varied from 0 to 0.5, the ratio of illumination frequency ω_i, to mirror vibration frequency ω_v, is fixed at 5; and the product of $2\beta(\omega_i/\omega_v)$ varies between 0 and 5. For the second case where β is fixed at 0.1, ω_i/ω_v is varied from 0 to 25; and again the product of $2\beta(\omega_i/\omega_v)$ varies between 0 and 5.

Fig. 3 shows that the FD-TD numerical predictions for the scattered field amplitude at the illuminating frequency are very close to the Bessel function J_0 behavior given by the exact solution as the mirror vibrational parameters vary. The accuracy of the FD-TD predicted scattered field amplitude is essentially the same, regardless of whether β is fixed or ω_i/ω_v is fixed during the parametric study. These results indicate that the FD-TD code, with fully relativistic boundary conditions at the mirror surface, is properly modeling the physics of the vibrating mirror problem, including the interesting scattered field null at the first zero of the Bessel function.

V. Extension of Method to Two-Dimensions

A. Problem Description: Oblique Incidence on a Vibrating Mirror

In this section we consider the case of oblique plane wave incidence on an infinite vibrating mirror. This case, analyzed by De Zutter [30], is much more complicated than the normal incidence case in that it has no closed-form solution. The solution is written in an infinite-series form using plane-wave expansions, where the unknown coefficients in the series are obtained numerically, as described in [30]. In that paper, the field amplitude versus time is calculated at different points along the symmetry axis of the mirror, and for various angles of incidence.

B. FD-TD Modeling Procedure

An approach analogous to the one-dimensional case is adopted to implement the relativistic boundary conditions in a two-dimensional FD-TD code. The two-dimensional case approach is again based on the "Doppler approximation" [2], [28], [30], where it is assumed that the mirror moves with a uniform velocity equal to the instantaneous vibrational value. Propagation delays are accounted for by assuming that reflections are generated at the "precursor" position of the mirror. In [28], an analysis of the normal incidence case, the precursor" positions coincide for all points of the mirror. In [30], an analysis of the oblique incidence case, this feature is lost, and a similar approximate solution ignores the propagation delays. However, propagation delays are automatically accounted for in the FD- TD code by virtue of its time-domain nature.

From the special theory of relativity, a wave reflected from a uniformly moving mirror has a reflected angle θ_r, given as [26]

$$\cos\theta_r = \frac{\cos\theta_i(1+\beta^2) - 2\beta}{1 - 2\beta\cos\theta_i + \beta^2}. \tag{11}$$

A derivation similar to the one-dimensional case leads to the following relativistic boundary conditions suitable for FD-TD implementation:

$$E = \pm\frac{\beta(\cos\theta_r + \cos\theta_i)}{1 \pm \beta\cos\theta_r}\cdot E^i \tag{12}$$

$$H = \frac{(\cos\theta_r + \cos\theta_i)}{\cos\theta_i(1 \pm \beta\cos\theta_r)}\cdot H^i \tag{13}$$

where $\beta = v/c$ and the fields refer to total tangential field values. The numerical steps involved are now only slightly more complicated because of the angular dependence of the incident field values at the mirror surface. From (11) it is clearly seen that $\cos\theta_r$ is a function of v. Therefore, the reflected wave has a spread both in frequency and spatial reflection angle [30].

A validation is sought for the oblique incidence case of the infinite plane mirror modeled by De Zutter. Since it is impossible to exactly model an infinite mirror in a finite two-dimensional grid, we select a long, thin, rectangular, perfectly-conducting slab as the model for the infinite mirror, as shown in Fig. 4. The relativistic boundary conditions (12) and (13) are implemented on the front and back sides of the object. The other two sides, parallel to the velocity vector, are insensitive to the motion of the object, and therefore no relativistic boundary conditions are required there.

The use of a finite-length rectangular slab to model the infinite mirror introduces edge diffraction artifacts. To minimize the edge effect, we select a slab long enough to appear from the observation point as infinite during a well-defined early-time response when the edge effect has not yet reached the observation point. Since the transverse electric (TE) case does not provide substantially different results than the transverse magnetic (TM) case [30], only the TM case is considered. Such a test should provide us with good insight as to the ability of FD-TD to handle moving boundary problems in two dimensions.

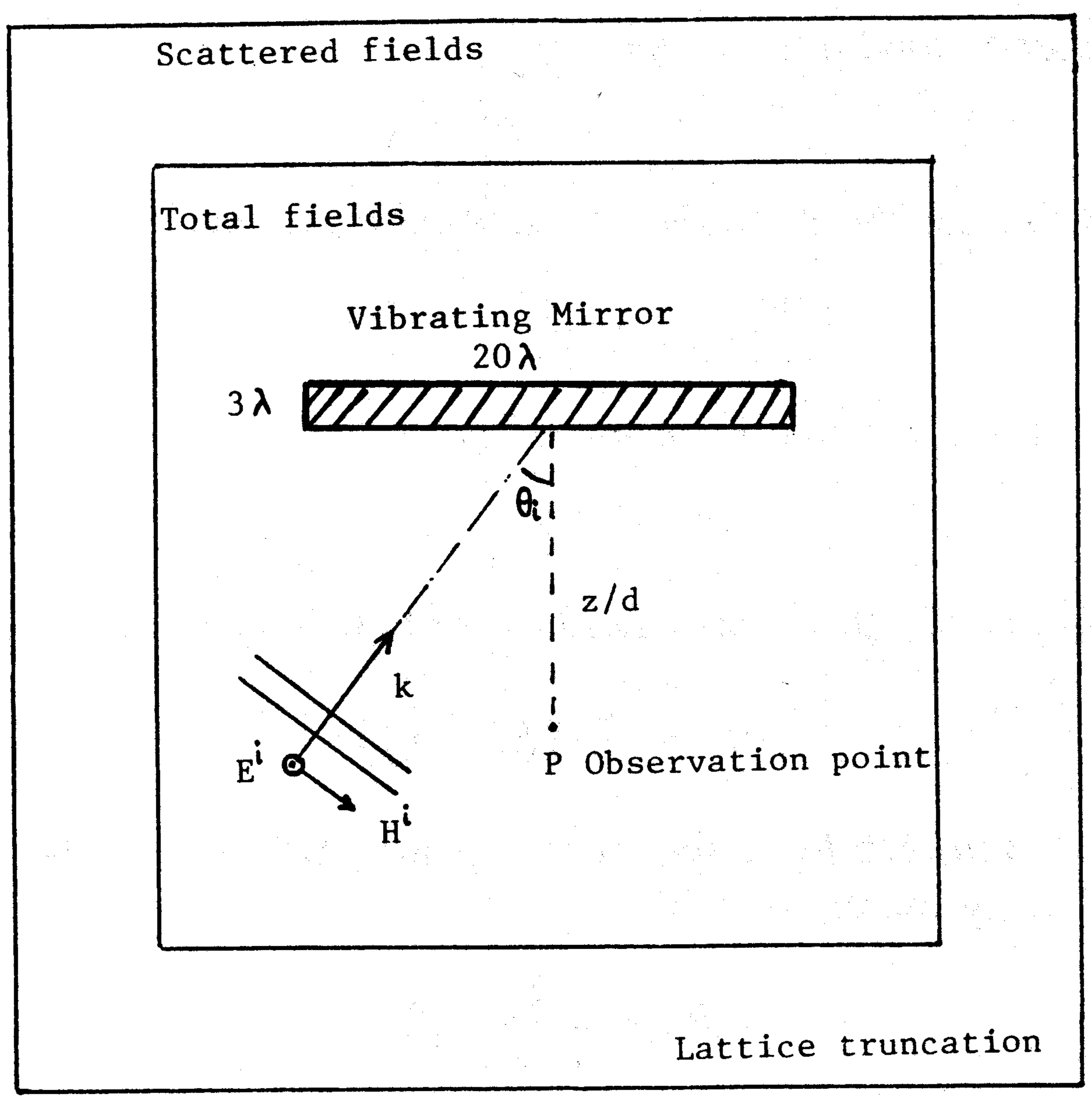

Fig. 4. Modeling of a vibrating mirror in a two-dimensional FD-TD grid.

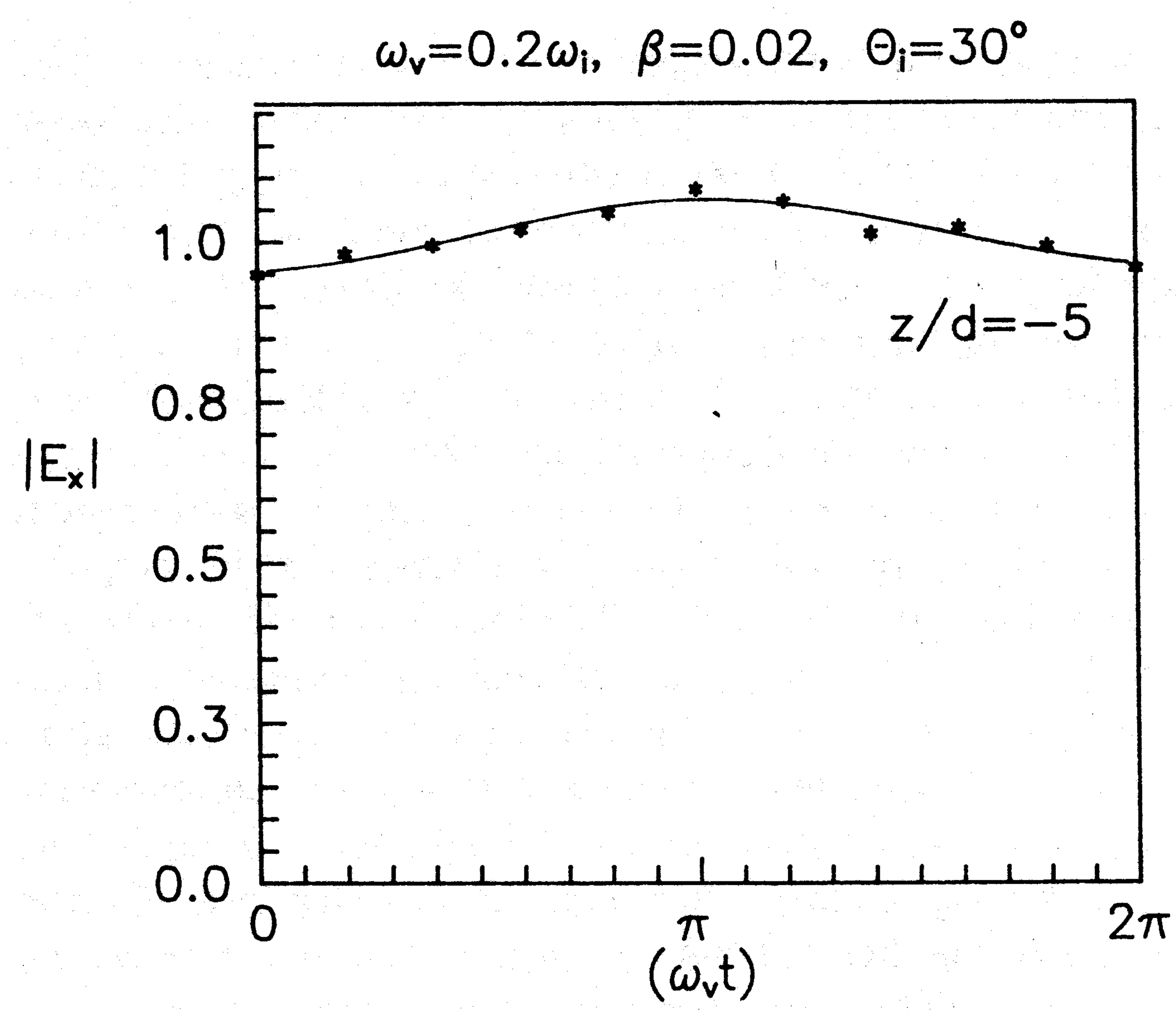

Fig. 6. Comparison of FD-TD and analytical results for the scattered field time envelope at oblique incidence. ——: analytical results; *o: FD-TD results.

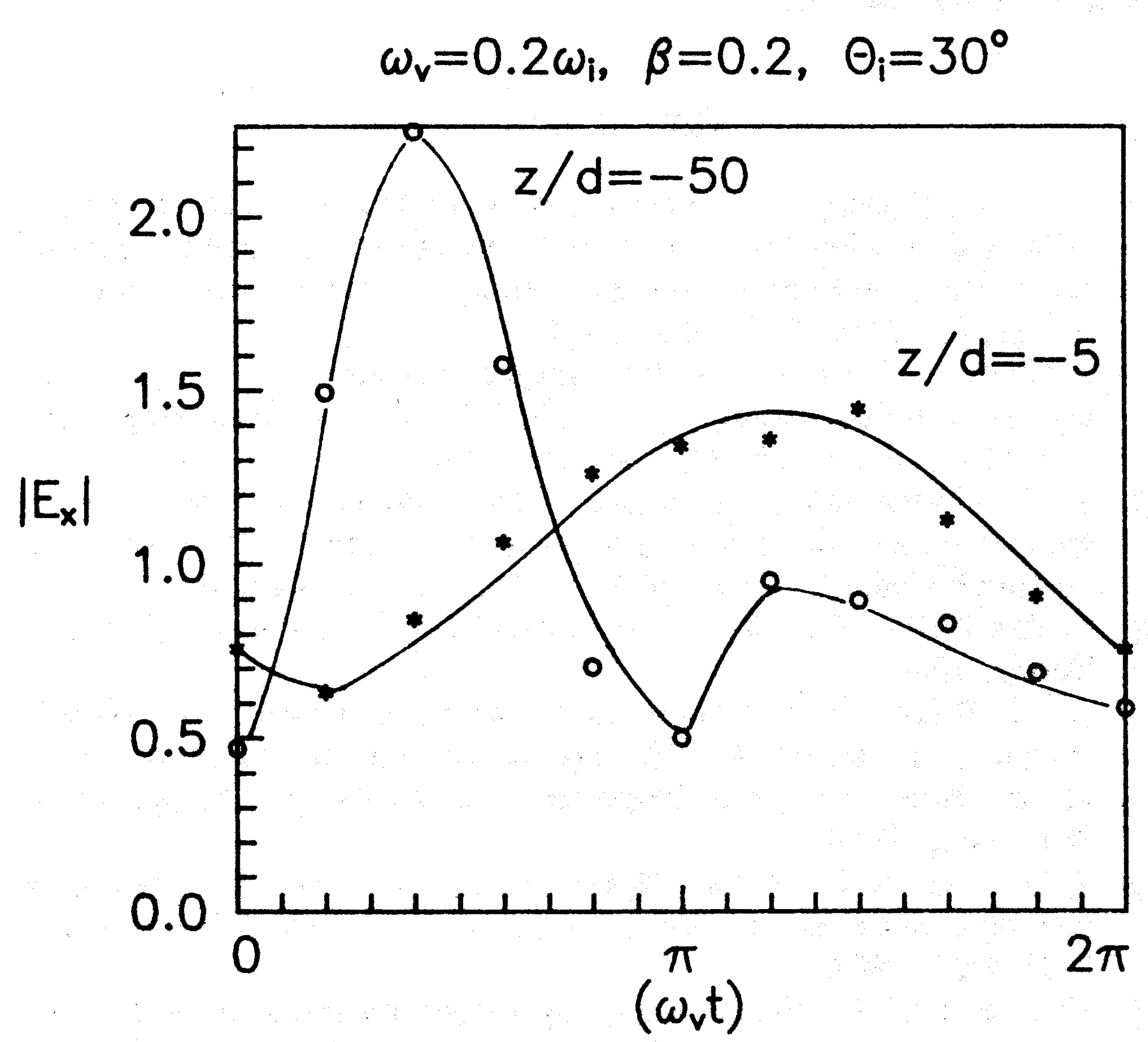

Fig. 5. Comparison of FD-TD ana analytical results for the scattered field time envelope at oblique incidence. ——: analytical results; *o: FD-TD results.

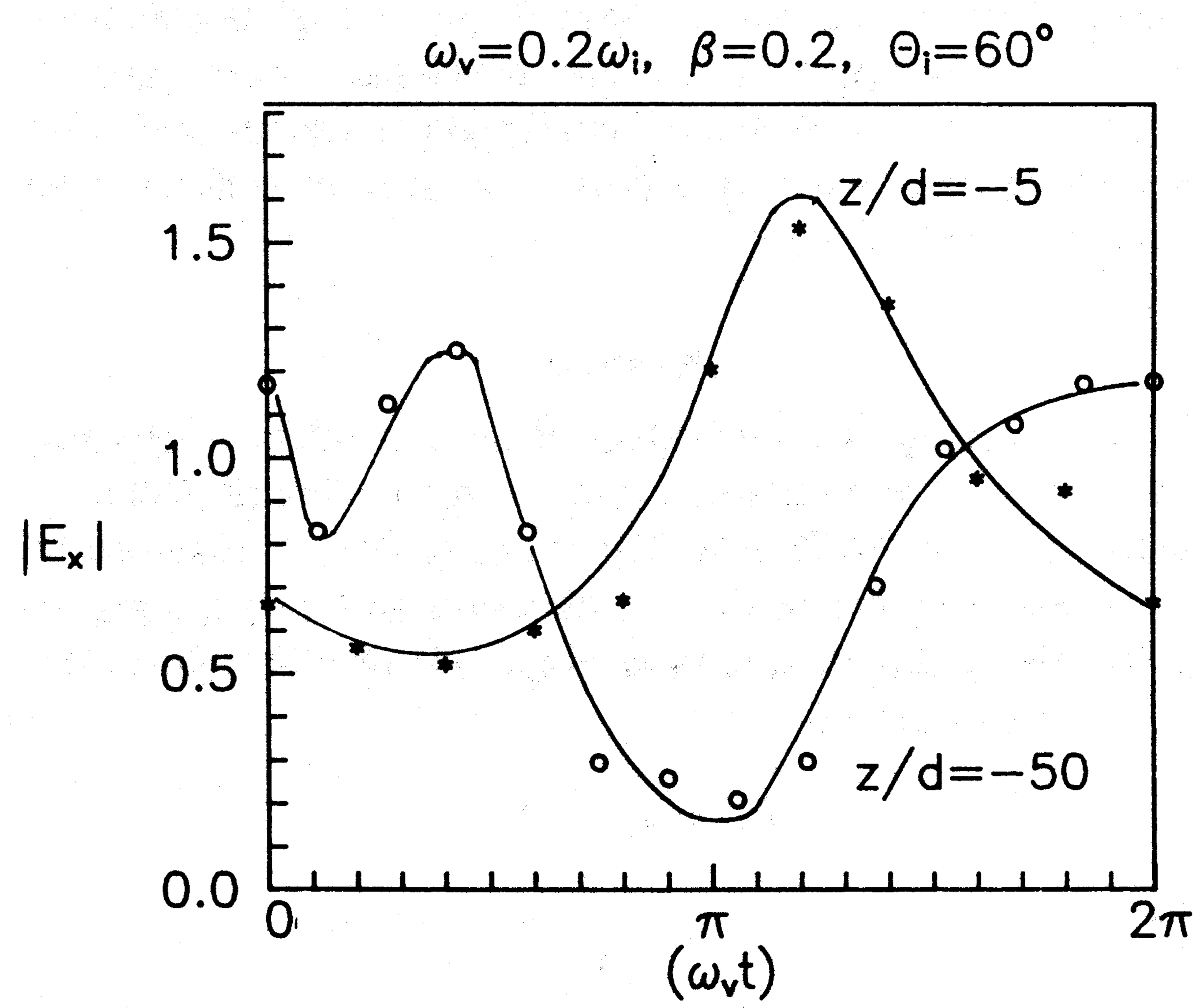

Fig. 7. Comparison of FD-TD and analytical results for the scattered field time envelope at oblique incidence. ——: analytical results; *o: FD-TD results.

C. Comparative FD-TD and Analytical Results

Fig. 5 shows good agreement between the FD-TD results and the analytical results obtained from [30] for the envelope of the scattered E-field versus time for $\theta_i = 30°, \beta = 0.2, kd = 1$, and observation points $z/d = -5$ and $z/d = -50$. Similar agreement is shown in Fig. 6 for $\theta_i = 30°, \beta = 0.02$, and $kd = 0.1$. Fig. 7 compares the FD-TD and analytical results for $\theta_i = 60°, \beta = 0.2$, and $kd = 1$. For both $z/d = -5$ and $z/d = -50$, a good correspondence is noted between the analytical and FD-TD numerical data.

In general, the FD-TD method gives good results, and it is fair to claim that this technique, unique in its approach for numerically modeling moving boundaries, is a promising strong tool to analyze more complicated problems involving arbitrary moving shapes.

VI. Summary and Conclusion

A numerical approach based on the FD-TD technique, using fully relativistic electromagnetic field boundary conditions at the surface of a conductor, has been formulated to model scattering from perfectly conducting moving mirrors in one and two dimensions. The numerical approach is unique in that it requires no system transformation, contrary to other possible numerical methods where the problem is first solved in the moving frame and then transformed back to the rest frame. For nonuniform velocities, the concept of a "Doppler-approximation" was used. Since the stability of the FD-TD code is assured by the proper selection of the space and time increments, and since no new iterative equation coupled to the original FD-TD equations is introduced, the method remains stable. Two types of one-dimensional relativistic mirror motion have been considered: uniform translation and sinusoidal vibration of the mirror surface. Comparison with the exact, analytical solutions for these types of mirror motion indicates that the new numerical approach accurately computes the magnitude and frequency of spectral components resulting from the scattering process. Physics that appears to be properly modeled includes the double-Doppler effect (uniform translation case) and FM-like spectral sidebands (sinusoidal vibration case). When extended to two dimensions, the code again shows good agreement with the available analytical results for the case of oblique incidence upon an infinite vibrating mirror. Here, the physics involved is much more complicated than in one dimension because both propagating and nonpropagating evanescent modes are generated at the mirror surface.

The FD-TD code that has been constructed can be directly adapted to model other types of moving-boundary problems involving two- and three-dimensional, perfectly conducting bodies of finite size and arbitrary shape. A logical extension of the existing approach involves developing more general, suitable relativistic boundary conditions to model scattering by moving objects having a finite conductivity without using a system transformation.

Appendix

The following is a derivation of an equivalent relativistic boundary condition suitable for modeling moving perfect conductors in the FD-TD grid. The incident wave is assumed to be polarized in the positive z-direction and propagating in the positive y-direction with an amplitude of unity. Thus, the incident fields are given by

$$E_z^i = e^{j\omega_i(t-y/c)}$$

and

$$B_x^i = \frac{1}{c} E_z^i.$$

The reflected E-field will have the form

$$E_z^r = Ae^{j\omega_r(t+y/c)}$$

so that the reflected B-field in free space will then be given by

$$B_x^r = \frac{-1}{c} E_z^r.$$

The total B-field is therefore

$$B_x^t = B_x^i + B_x^r = \frac{1}{c}(E_z^i - E_z^r)$$

but, since

$$E_z^r = E_z^t - E_z^i$$

therefore

$$B_x^t = \frac{1}{c}(2E_z^i - E_z^t).$$

For a mirror receding from the incident wave, the relativistic boundary condition is given by

$$E_z^t - v \cdot B_x^t = 0.$$

Substituting for B_x^t in the above equation we get

$$E_z^t - \frac{v}{c}(2E_z^i - E_z^t) = 0.$$

Therefore, the final form for E_z^t is

$$E_z^t = 2\frac{v}{c+v} E_z^i. \tag{14}$$

Similarly, for the total B-field at the boundary we have

$$B_x^t = \frac{1}{c}(2E_z^i - E_z^t).$$

Substituting for E_z^t in the above equation with $E_z^i = cB_x^i$, we get finally for the B-field

$$B_x^t = \frac{2c}{c+v} B_x^i. \tag{15}$$

Equations (14) and (15) are the ones used in our code to implement the proper relativistic boundary conditions at the surface of the mirror.

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