Optics Communications 393 (2017) 185–190

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Reflection statistics of weakly disordered optical medium when its mean refractive index is different from an outside medium

Prabhakar Pradhan[a,*], Daniel John Park[b], Ilker Capoglu[b], Hariharan Subramanian[b], Dhwanil Damania[b], Lusik Cherkezyan[b], Allen Taflove[c], Vadim Backman[b]

[a] Department of Physics and Materials Science, BioNanoPhotonics Laboratory, University of Memphis, Memphis, TN 38152, United States
[b] Biomedical Engineering Department, Northwestern University, Evanston, IL 60208, United States
[c] Electrical Engineering and Computer Science Department, Northwestern University, Evanston, IL 60208, United States

ARTICLE INFO

Keywords:
Scattering theory
Backscattering
Weakly disordered biological media
Cancer detection

ABSTRACT

Statistical properties of light waves reflected from a one-dimensional (1D) disordered optical medium [$n(x) = n_O + dn(x)$, $<dn(x)>=0$] have been well studied, however, most of the studies have focused on the situation when the mean refractive index of the optical medium matched with the outside medium, i.e., $n_O = n_{out} =1$. Further, considering $dn(x)$ as a Gaussian color noise refractive index medium with exponential spatial correlation decay length l_c and k as the incident wave vector, it has been shown that for smaller correlation length limit, i.e., $kl_c << 1$, both the mean reflection coefficient $<r>$ and *std* of r, $\sigma(r)$, have same value, and they follow the relation $<r> = \sigma(r) \propto <dn^2> l_c$. However, when the refractive index of the sample medium is different from the outside medium, the reflection statistics may have interesting features, which has not been well studied or understood. We studied the reflection statistics of a 1D weakly disordered optical medium with the mean background refractive index n_0 being different from the outside medium n_{out} ($\neq n_0$), to see the effect of mismatching (i.e., value of $n_O - n_{out}$) on the reflection statistics. In the mismatched case, the results show that the mean reflection coefficient $<r>$ follows a form similar to that of the matched refractive-index case, i.e., $<r(dn, l_c)> \propto <dn^2> l_c$, with a linear increased shift, which is due to 1D uniform background reflection from a slab. However, $\sigma(r)$ is shown to be $\sigma(r) \propto (<dn^2> l_c)^{1/2}$, which is different from the matched case. This change in *std* of r is attributed to the interference between the mismatched-crerated edge mediated multiple scattering that are coupled with the random scattering. Applications to light scattering from random layered media and biological cells are discussed.

1. Introduction

The statistical transport properties of one-dimensional (1D) mesoscopic disordered optical and electronic media are now well studied [1–6]. The Schrödinger equation and Maxwell's wave equation are similar in the sense that they can be projected to the Helmholtz equation; therefore, the formalisms are the same for corresponding scalar waves in both cases [7–10]. After the Landauer formalism showed that the reflection coefficient is related to the resistance/conductance of the sample, the outer scattering parameters such as the reflection and transmission coefficients became important for the study of localization and conductance fluctuations in the electronics case [7,8]. Similarly, extending the idea from the electronic systems, in most of the previous studies of light scattering and localization properties of different optical disordered media, the fluctuation part of the refractive index is primarily considered while the sample's mean refractive index is the same as the outside medium [7,9–11]. The results show that both the average reflection and the fluctuations have the same form for the mesoscopic optical sample. However, the mismatch of the refractive index between the sample and the outside medium and its effect upon the reflection statistics remains poorly understood.

In this paper, we study reflection statistics in the context of the synergistic effects between refractive index mismatched values and the fluctuation of the refractive index. For a biological medium, for example a biological cell, the spatial fluctuation of the refractive index is relatively weak (~.01) and buried in a higher uniform mean background refractive index (~1.38). Enhancement of the backscattering signals from the weakly fluctuating refractive index, as mediated by the refractive index mismatching, is also addressed. Finally, applications of the method for light scattering from biological cells are discussed in terms of the enhancement of the scattering signal from the spatial refractive index nanoscale fluctuations of a biological cell that is

* Corresponding author.
E-mail address: ppradhan@memphis.edu (P. Pradhan).

http://dx.doi.org/10.1016/j.optcom.2017.02.041
Received 9 December 2016; Received in revised form 13 February 2017; Accepted 15 February 2017

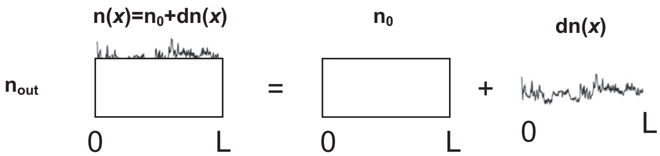

Fig. 1. Schematic of a mismatched case where outside refractive index is n_{out} and sample refractive index is $n(x) = n_0 + dn\,(x)$, with n_0 as the average refractive index of the sample and $dn(x)$ as the spatial refractive index fluctuation of the sample $<dn(x)> = 0$. For matched case, $n_0 = n_{out}$.

associated with the progress of cancer. This will improve the sensitivity of cancer detection.

2. Reflection statistics of matched disordered media

Consider a 1D sample of length L with refractive index inside the sample $n(x) = n_0 + dn(x)$ (for $0 < x < L$), where the average refractive index of the samples is $n_0 = <n(x)>$, $dn(x)$ is the fluctuation part of the refractive index with its average $<dn(x)> = 0$, and n_{out} is the refractive index of the outside medium as shown schematically in Fig. 1. The 'matched' case can be defined as *equality* between the mean refractive index of the sample and the outside medium, i.e., $n_0 = n_{out}$, whereas the 'mismatched' case can be defined as $n_0 \neq n_{out}$. Since we are interested in the reflection statistics, let us define $R(L)$ as the complex reflection amplitude of a sample of length L which is illuminated by a plane wave of wave vector k. Then, the mean and standard deviation of the reflection coefficient ($r = RR^*$) are the primary concerns of this work. For example, we will prove below that the mean reflection coefficient for the mismatched case, $<r_{mismatched}>$, can be written in terms of the reflection coefficient of a slab ($n_0 \neq n_{out}$) and the mean reflection coefficient of the matched case, r_{slab} and $<r_{matched}>$, as:

$$<r_{mismatched}> = r_{slab} + <r_{matched}> \times F(k, dn, l_c, n_0, L, r_{slab}). \tag{1}$$

In the literature, reflection statistics from disordered optical media are primarily studied assuming the matched case; however, the mismatched case, as defined above, is not well studied. Therefore, we first briefly review the results of a 1D matched case ($r_{matched}$) before describing the results of the mismatched case ($r_{mismatched}$). The statistics of spatial random refractive index fluctuation, $dn(x)$, generally represented by Gaussian color noise, i.e., $<dn(x)> = 0$ and $<dn(x) \times dn(x')> = <dn^2> exp(-|x-x'|/l_c)$, where l_c is the exponential spatial correlation decay length of the spatial refractive index fluctuation $dn(x)$. Then, using the Fokker-Planck approach, the above $<r_{matched}>$ can be solved analytically in the weakly disordered limit (i.e., $dn \ll n_0$) [7,9]. The mean value of the reflection coefficient and its standard deviation both have the same value, $<r_{matched}> = \sigma(r_{matched}) = L/\xi$, where the inverse of the localization length has the form $\xi^{-1} = 2k^2 <dn^2> \times l_c/[1+(2kl_c)^2]$. This is true for a weakly disordered sample where $\xi > L$, which can also be defined as a Born approximation limit of the scattering from the weakly disordered part of the refractive index.

3. Reflection statistics of mismatched disordered media

However, index mismatched weakly disordered samples are quite common for optical scattering experiments. For example, biological cells and tissues have refractive indices $n_0 \sim 1.3 - 1.5$ and $dn \sim 0.01 - 0.1$ with the outside air medium $n_{out} = 1$. In the case of weak refractive index fluctuations, the backscattering light transport properties of such biological cells can be decomposed into a multiple-transport 1D channel or a quasi-1D parallel multichannel problem [12]. It was recently shown that quasi-1D multichannel backscattering would provide sensitivity to changes in the nanoscale signal relative to a three-dimensional (3D) bulk for weakly disordered media such as biological cells. Furthermore, the quasi-1D analysis approach has been proven to be useful for early pre-cancer screening by detecting changes in the nanoscale refractive index fluctuations of cells related to the progress of carcinogenesis in different types of cancers [13–16].

To derive the form of Eq. (1), we start from a stochastic Langevin equation (here, stochasticity enters into the equation through the $dn\,(x)$ term) for the index mismatched case ($n_{out} \neq n_0$) which gives the reflection amplitude R_t. For simplicity, we will consider that the sample is kept in air, i.e., $n_{out} = 1$ and $n_0 > 1$. Substituting these terms (n_{out}, n_0, and dn) with color noise, the Langevin equation for the mismatched case can be derived following the invariant imbedding approach [7]:

$$\frac{dR_t(L)}{dL} = 2ikR_t(L) + \frac{ik}{2}[(n_0^2 - 1) + 2\,n_0 dn(L)] \times [1 + R_t(L)]^2. \tag{2}$$

The complex total reflection amplitude $R_t(L)$ from a weakly disordered medium can be considered as a combination of: (i) a deterministic sinusoidal oscillation component R_{slab} based on the pure background of a thin-film slab of length L without any stochastic $dn(x)$ terms, and (ii) a R component that contains $dn(x)$ terms. Therefore, we may write $R_t = R_{slab} + R$. Each term can then be easily derived from Eq. (2) as follows:

$$R_t(L) = R_{slab}(L) + R(L), \tag{3a}$$

$$\frac{dR_{slab}(L)}{dL} = 2ikR_{slab} + \frac{ik}{2}(n_0^2 - 1) \times [1 + R_{slab}]^2 , \tag{3b}$$

$$\frac{dR(L)}{dL} = 2ikR + \frac{ik}{2}(2n_0 dn(L)) \times [1 + R_{slab} + R]^2 + \frac{i}{2}k(n_0^2 - 1) \times [2R(1 + R_{slab}) + R^2]. \tag{3c}$$

In Eq. (3a-c), the perturbative contribution by the stochastic term $dn(x)$ has many cross-terms between R_{slab} and R. We will assume that R is in the first order in $dn(x)$. By performing a phase transformation as below in Eq. (3b-c), we can further simplify and assimilate the $R_{slab} - R$ cross-terms in the equation. For this, we introduce new variables, $Q(L)$ and $\alpha(L)$, which are derived from $R(L)$ by a phase transformation as follows:

$$R_{slab}(L) = Q_{slab}(L) \cdot e^{2ik\alpha(L)} , \tag{4a}$$

$$R(L) = Q(L) \cdot e^{2ik\alpha(L)}. \tag{4b}$$

This yields a new set of simplified equations for $Q(L)$ and $\alpha(L)$ which further simplifies to:

$$\frac{d\alpha(L)}{dL} = 1 + \frac{(n_0^2 - 1)}{2}(1 + R_{slab}), \tag{5a}$$

$$\frac{dQ}{dL} = \frac{i}{2}k(2n_0 dn(L))e^{-2ik\alpha}[1 + R_{slab} + Qe^{2ik\alpha}]^2 + \frac{i}{2}k(n_0^2 - 1)(Q)^2 e^{2ik\alpha}. \tag{5b}$$

With the new representation above Eq. (5), the mean $<r_t>$ and the standard deviation $\sigma(r_t)$ of the reflectance for mismatched case $r_{mismatched} = r_t \equiv R_t R_t^*$ can be derived. By using Eq. (3a) and performing a realization averaging over the disordered samples, we obtain:

$$<r_t(L)> = <R_t R_t^*> = <(Q_{slab} + Q) \times (Q_{slab} + Q)^*> , \\ = r_{slab} + Q_{slab}^* <Q> + c.\,c. + <|Q|^2> , \tag{6a}$$

where $r_t = |R_t|^2$ and

$$r_{slab} = |R_{slab}|^2 = |Q_{slab}|^2 = \frac{(n_0^2 - 1)^2 \sin^2(n_0 kL)}{4n_0^2 + (n_0^2 - 1)^2 \sin^2(n_0 kL)}. \tag{6b}$$

In Eq. (6a), we have separated the slab's pure reflection/interference contribution, r_{slab}, (when $dn = 0$) and the disorder contribution. The pure slab solution, that is for $n_0 - 1 > 0$ and dn=0, is presented in Eq. (6b). This also confirms the validation of the invariant imbedding Langevin Eq. (2) with mismatched situation. In particular, the

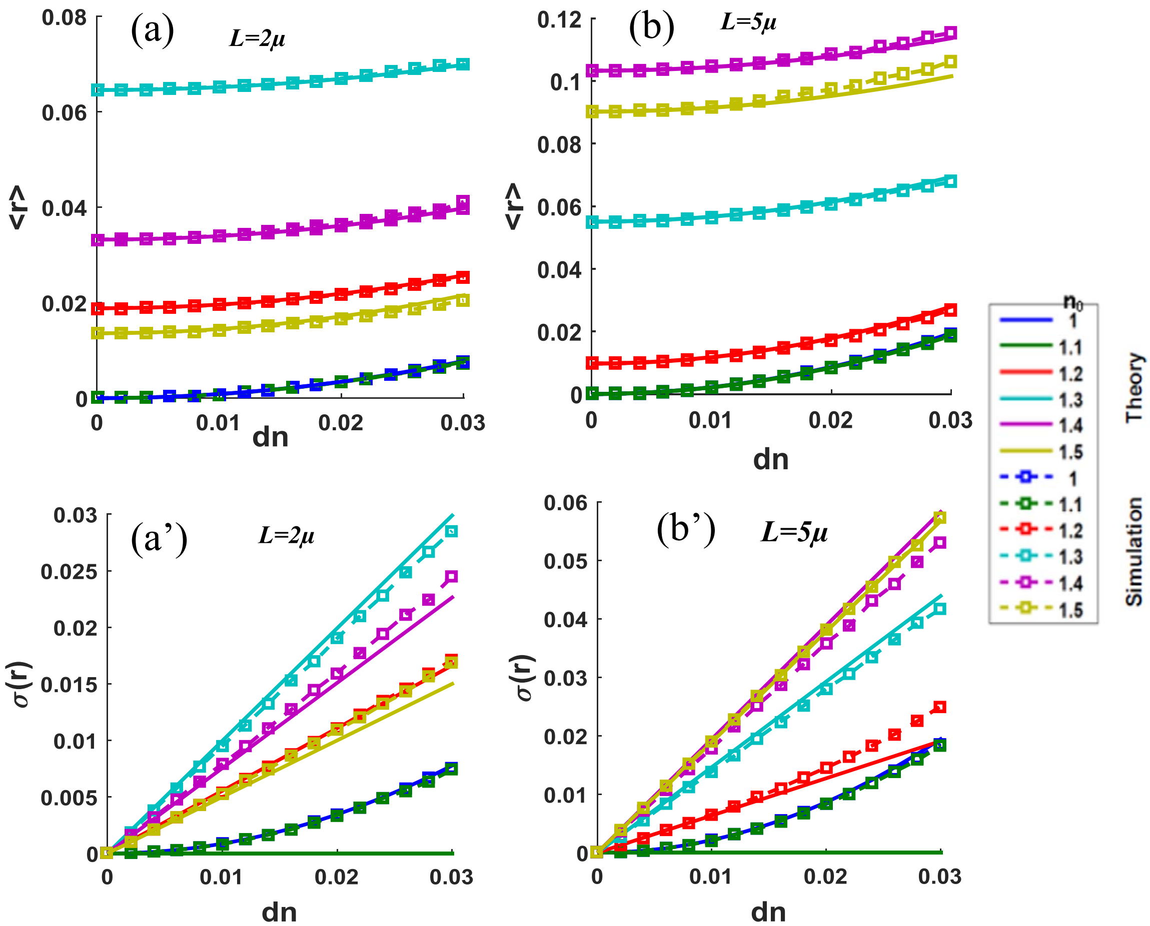

Fig. 2. (a)-(b) Plots of analytical calculations (solid lines) of Eq. (12) with the pure numerical integration of Eq. (11) for $<r>$ and numerical simulations (squares) based on Eq. (2) with lengths: (a) L =2μ and (b) L =5μ. (a')-(b') The plot of the semi-analytical calculations (solid lines) of Eq. (15) and numerical simulations (squares) for $\sigma(r)$ based on Eq. (2) with lengths: (a') L =2μ and (b') L =5μ. Refractive index of outside medium is taken as air n_{out} =1. The mean refractive indexes of the samples are n_0 =1 (blue, matched case), 1.1(green), 1.2(red), 1.3(cyan), 1.4(purple), and 1.5(yellow). Refractive index fluctuations were varied such that dn =0 – 0.03. Correlation length of dn spatial fluctuations l_c =20 nm. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

solution of the invariant imbedding equation Eq. (2) converges to the reflection from a slab (of length L) solution for the finite mismatched situation (n_0-1 > 0) and no disorder (dn=0), as expected. Therefore, an addition of weak RI fluctuations $dn(x) \ll (n_0-1)$ to the sample will also provide the correct evolution of the mismatched imbedding equation of $R(L)$ with the length L, i.e., $dR(L)/d(L)$ in Eq. (2). Furthermore, we have shown later (in Fig. 2) that the theoretical solutions also match well with the pure stochastic numerical simulations, for each slab solutions at $dn=0$, supporting the validation of the Eq. (2) for mismatched solution .

To evaluate the averages in Eq. (6a) and the standard deviations (see below), we calculate the following terms in leading order of dn as:

$$<|Q|^2>,\ <(Q)^2>,\ <Q>,$$

which can be written explicitly as:

$$<|Q|^2> = -\frac{i}{2}kn_0\int_0^L dL'[e^{2ik\alpha^*}(1+R_{slab}^*)^2 \times <2dnQ> - c.c.], \quad (7a)$$

$$<(Q)^2> = ikn_0\int_0^L dL' e^{-2ik\alpha}(1+R_{slab})^2 \times <2dnQ>, \quad (7b)$$

$$<Q> = ik\int_0^L dL'[n_0(1+R_{slab}) <2dnQ> + \frac{1}{2}(n_0^2-1)e^{-2ik\alpha} <(Q)^2>]. \quad (7c)$$

By using the Ornestein-Uhlenbeck stochastic process and Novikov theorem [7], the averaging of the above Eq. (7a-c) were performed. Later been shown that the analytical solutions match well with the numerical simulations, validating the analytical steps. For example, the disorder average of the product term $<2dn(L)R>$ is:

$$<2dn(L)R> = \frac{i}{2}kn_0(\frac{g}{2}) \times [1 - \frac{\partial}{\partial(L/l_c)}]e^{-2ik\alpha}(1+R_{slab})^2 + O(dn^2 \cdot l_c^3) + O(dn^4 \cdot l_c^2), \quad (8)$$

where the value of g and α can be expressed as:

$$g = 8 <dn^2> l_c \quad \text{(disorder strength)}, \quad (9a)$$

$$\alpha = \int_0^L ik[2 + (n_0^2-1) \times (1+R_{slab})]\ dL'. \quad (9b)$$

Finally, by using Eqs. (7–9) and averaging over the ensemble space, that is, performing ensemble averaging, we can write the average $<r_t>$ in terms of $<dn^2>$ and l_c as:

$$<r_{mismatched}> = <r_t> = r_{slab} + <dn^2> l_c \cdot k^2[Q_{slab}^*(n_0^2 I_1 + kn_0^2(n_0^2-1)I_2) + c.c. + n_0^2 I_3 + c.c.] + O(dn^4), \quad (10)$$

where we have defined I_1, I_2, and I_3 in the above deterministic equations as:

$$I_1 = -2\int_0^L dL'(1+R_{slab}) \times [1 - \frac{\partial}{\partial(L'/l_c)}]e^{-2ik\alpha}(1+R_{slab})^2, \quad (11a)$$

$$I_2 = -i \int_0^L dL' e^{-2ik\alpha(L')} \int_0^{L'} dL'' e^{-2ik\alpha(L'')} (1 + R_{slab}(L''))^2 \times [1 - \frac{\partial}{\partial(L''/l_c)}] e^{-2ik\alpha(L'')} (1 + R_{slab}(L''))^2, \tag{11b}$$

$$I_3 = \int_0^L dL' e^{-2ik\alpha^*} (1 + R^*_{slab})^2 \times [1 - \frac{\partial}{\partial(L'/l_c)}] e^{-2ik\alpha} (1 + R_{slab})^2. \tag{11c}$$

Eq. (10) can now be rewritten in terms of a pure slab's reflection term (without $dn(x)$) and the average of the fluctuation terms. For weak disorder with short range correlation $2kl_c < 1$, the quantities I_1, I_2, and I_3 are approximately independent of l_c. In this case, we obtain. (12b)

$$\begin{aligned} < r_t > &= < r_{mismatched} > = \\ &= r_{slab} + \frac{1}{2} < dn^2 > l_c \cdot k^2 L \times [1 + F(k, dn, l_c, n_0, L)] + O[dn^4 l_c^2] \\ &= r_{slab} + \frac{L}{\xi} \times [1 + F(k, dn, l_c, n_0, L)]. \end{aligned} \tag{12a, 12b}$$

Where we have defined:

$$r_{matched} = \frac{L}{\xi} = \frac{1}{2} < dn^2 > l_c \cdot k^2 L$$

Here, ξ is the localization length, which is readily derived [7]. Further, we have defined

$$F(k, n_0, L, r_{slab}) = (2/L).\ [Q^*_{slab}(n_0^2 I_1 + k n_0^2 (n_0^2 - 1) I_2) + c.\ c. + n_0^2 I_3 + c.\ c.\ ] - 1. \tag{12c}$$

Thus, we can write a relationship between the matched and mismatched cases from Eq. (12):

$$< r_{mismatched} > = < r_t > = r_{slab} + < r_{matched} > \times\ [1 + F(k, dn, l_c, n_0, L)]. \tag{13a}$$

For a small length scale, $kl_c < 1$, the above equation Eq. (13a) variation with the parameters k, dn, lc, n_O, and L is given. From the above equation, it can be seen that, in the limit of the small L and small dn, there will be a negligible contribution from the interference term, that is $F(k, dn, l_c, n_O, L) \sim 0$ in the Eq. (13a), as expected, will simply change to:

$$< r_{mismatched} >_{dn \sim 0} \approx\ < r_t > = r_{slab} + < r_{matched} > . \tag{13b}$$

However, in case of larger L and higher dn values the $F(k, dn, l_c, n_O, L)$ has finite, significant contribution. In that case we need to calculate the value of $F(k, dn, l_c, n_O, L)$ numerically using Eq. (12c).

Finally, the mean-square fluctuation of r, $\sigma^2(r)$, can be evaluated as:

$$\begin{aligned} \sigma^2(r_{mismatched}) &= < r^2_{mismatched} > - (< r_{mismatched} >)^2 \\ &= < (r_{slab} + 2\,\mathrm{Re}(Q^*_{slab} Q) + |Q|^2)^2 > \\ &\quad - (< r_{slab} + 2\,\mathrm{Re}(Q^*_{slab} Q) + |Q|^2 >)^2 . \end{aligned} \tag{14a}$$

The above equation can now be further modified by applying Eqs. (4) and (7):

$$\sigma^2(r_{mismatched}) = Q^*_{slab} < Q > + c.\ c. + 2 r_{slab} < |Q|^2 > + O(dn^4 l_c^2). \tag{14b}$$

Taking the average over the disorder, we obtain a deterministic expression of the above equation:

$$\sigma^2(r_{mismatched}) = < dn^2 > l_c \cdot k^2 n_0^2 [Q^*_{slab} I_4 + c.\ c. + 2 r_{slab} I_3 + c.\ c.\ ] + O(dn^4 l_c^2)\ . \tag{14c}$$

For the mismatched case, it should be noted that the mean-square fluctuation of r has a barrier or mismatch-induced leading order: $< dn^2 > .l_c$. In the matched case, the leading order is higher, that is, $< dn^2 >^2 .l_c^2$. Therefore, we obtain the expression for the standard deviation $\sigma(r)$ as follows:

$$\sigma(r_{mismatched}) = < dn^2 >^{1/2} l_c^{1/2} \cdot k [Q^*_{slab} I_4 + c.\ c. + 2 r_{slab} I_3 + c.\ c.]^{1/2} + O(dn^2), \tag{15}$$

where we have defined:

$$I_4 = -2 \int_0^L dL' e^{-2ik\alpha} (1 + R_{slab})^2 [1 - \frac{\partial}{\partial(L'/l_c)}] e^{-2ik\alpha} (1 + R_{slab})^2. \tag{16}$$

Eq. (15) can be further written to the leading order as (following [7]):

$$\sigma(r_{mismatched}) = < dn^2 >^{1/2} l_c^{1/2} \cdot k L^{1/2} [G(k, dn, l_c, n_0, L) - 1]. \tag{17a}$$

where $G(k, dn, l_c, n_O, L)$ is a function without the $< dn^2 >$ term, defined as:

$$G(k, dn, n_0, l_c, L) = (2/L)^{1/2} [Q^*_{slab} I_4 + c.\ c. + 2 r_{slab} I_3 + c.\ c.\ ]^{1/2} + 1. \tag{17b}$$

It can be noted that in the matched case ($n_O=1$), the value of G is 1 since Q_{slab} and r_{slab} have the multiplicative factor $(n_O^2 - 1)=0$ as $n_O=1$. In this case, in $\sigma(r)$ the first order term in $< dn^2 >^{1/2}$ vanishes and the second order term $< dn^2 >$ is the leading term, recovering the matched case.

For a small length scale, $kl_c < 1$, the Eq. (17a) variation with dn is given. From the equation, it can be seen that in the limit of the small L ($\ll\xi$) and small dn, there will be a negligible interference contribution, that is the term $G(k, dn, l_c, n_O, L) \sim 1$. There will not be a contribution from the first order term in the standard deviation of the refractive index, or from the dn^2 terms but the contribution will come from the second order correction or $\sim dn^4$ terms, and (17a) will be replaced by:

$$\sigma(r_{mismatched})_{dn \to 0} \sim \sigma(r_{matched}). \tag{17c}$$

However, in case of larger L and dn, $G(k, dn, l_c, n_O, L)$ term is more than 1. In that case, we need to calculate the value of $G(k, dn, l_c, n_O, L)$ numerically using Eq. (17b).

Here we emphasize again that the value of $\sigma(r)$ calculated here with the assumption that the reflection from the fluctuating part is less than the reflection amplitude from the slab.

3.1. Numerical simulations of stochastic equation and analytical semi-integral equation results

Semi-integral Eq. (10) for $< r >_{mismatched}$ and (15) $\sigma_{mismatched}$ are evaluated and plotted in Fig. 2(a) and (a′) respectively for a sample length L =2 μ; similarly, and in Fig. 2(b) and (b′) for L =5 μ. In the plot for a constant length, the varying parameters are dn and for different mismatched parameters. The other parameters remain the same: wavelength =500 nm and l_c =20 nm. We also performed direct stochastic simulation of Eq. (2) and then performed realization averages numerically as shown by the dotted lines in Fig. 2(a)-(a′) and (b)-(b′), respectively. It can be seen that the disorder averaged analytical semi-integral results and the corresponding numerical results agree well for all the parameters, thereby validating our semi-integral analytical equation form. For the case $2kl_c > 1$, detailed calculations will be reported in a separate paper.

3.2. Extension of calculation to a wavelength spectra

For the simplicity of the calculation and for the proof of the concept, we have taken into consideration only one wave vector/wavelength here, however, the formalism is true for any wave vector/wavelength. In case of a wavelength spectra, the equation for a single wavelength equation can be added up systematically. In particular, one can sum up the $< r(k) >$ and $\sigma(r(k))$ equation for single wave contributions from the developed formalism, to obtain the result of a full spectra Δk: .

$$< r >_{\Delta k} = \sum_{k}^{k+\Delta k} \langle r(k) \rangle \tag{18a}$$

$$\sigma(r)_{\Delta k} = \sum_{k}^{k+\Delta k} \sigma(r(k)) \tag{18b}$$

4. Conclusions and discussions

In conclusion, our results show that the average reflection coefficient, for the mismatched case with weak disorder and short range correlation, is linearly dependent on that of the matched case; the average reflectance $<r>_{mismatched}$ is proportional to $<dn^2>l_c$, with a shift due to the slab reflection. However, the value of the standard deviation of the reflection coefficient has a different form (Eq. (15)). This is because, as seen in Eq. (14(b)), the index mismatched parameter ($n_0^2 - 1$) contributes (through Q_{slab} and r_{slab}) and this changes the leading term of the mean-square fluctuations from $(<dn^2>.l_c)^2$ to $<dn^2>.l_c$. Therefore, the RMS fluctuations, or STD for the mismatched case, $\sigma(r(dn, l_c)_{mismatched})$ is proportional to $<dn^2>^{1/2}l_c^{1/2}$. Furthermore, the relative fluctuation $\sigma(r_{mismatched})/<r_{mismatched}>$ decreases (<1) with the increase of the mismatched parameter ($n_0^2 - 1$). However, the relative $\sigma(r_{mismatched})$ value for the mismatched case, compared to the matched case, increases. The decrease by a square root factor in the standard deviation $\sigma(r)$, from the matched to mismatched case, will drastically increase the value as $\sigma(r)^{1/2} >> \sigma(r)$, for $\sigma(r) \ll 1$.

The phenomena of the mismatched-induced enhancement can be useful for enhancing a weakly reflective signal from the weak refractive index fluctuations ($<dn^2>^{1/2}$) imbedded in a strong uniform refractive index background (n_0), such as in biological cells. Our results show that the backscattering signal from the fluctuation part of the refractive index can be enhanced by increasing the mean background refractive index. This is due to the multiple reflections of the wave within the background allowing more (i.e. longer) interaction times between the wave within the higher refractive index background boundaries and the imbedded refractive index fluctuations. Therefore, the developed method has potential applications for enhancing scattering from biological cells where the refractive index fluctuations ($<dn^2>^{1/2}$ varies from 0.001 to ~.02) are imbedded within the cell's background refractive index n_0~1.38–1.5, with mismatched refractive index ~0.38–0.5, with respect to the air medium index 1. Recently it has been shown that nanoscale fluctuations in a cell increase with the progress of early carcinogenesis due to the rearrangements of the cell's building blocks (DNA, RANA, Lipids *etc.*). Thus, detecting these small changes in the refractive index fluctuations in cancer/pre-cancer cells, relative to the normal cells, we can detect the progression of carcinogenesis [13,14]. Therefore, enhancing the nanoscale signal from biological cells by simply changing the background (i.e., slab) refractive index will significantly increase cancer detection sensitivity by optical experiments. For example, one can easily enhance the reflection signal from the fluctuating part by simply increasing the background refractive index, such as by dipping or treating the cell in a nonreactive liquid having a higher refractive index than the cell (i.e., >1.38). This is important for practical applications, as just a simple cell treatment/preparation could enhance the diagnostic or sensitivity/specificity, and will have potential clinical applications in improving cancer detection. Our results further suggest the best way to obtain a stronger signal from the fluctuating parts in a cell is keeping the cell in a refractive index high-contrast situation. For example, a stronger signal from nano-fluctuations can be obtained when the cells are kept on a slide and exposed to air interface, producing a high-contrast situation, than when the cells are kept on a slide covered with a coverslip, producing a low-contrast situation. Furthermore, the present work is significant as well as for other types of weakly embedded disordered media such as polymers and dielectric thin films.

Although we have considered here the one-dimensional back reflection case, the fact of signal enhancement from embedded fluctuation in a background will have similar physical reasons for higher dimensions, especially for back reflection statistics. For a thinner sample (relative to the probing wavelength λ), such as inner cheek cells or buccal cells (width ~500 nm), the back reflection can be treated as a bunch of quasi-one-dimensional parallel back-propagating interacting channels (area of a channel ~λ^2). It has been shown that the average value of the reflection from a channel in a multi-channel propagation decreases inversely with the number of channels [12]. Therefore, the solution of a 1D channel provides us with an understanding of the signal enhancement and framework for more practical three-dimensional signal enhancement. Other than biological media, artificial layered media could be made easily [17] for active (e.g., with lasing or absorption) and passive cases, for 1D or quasi-1D systems for different applications. Therefore, we believe the present work will have many applications, varying from biological cells to varieties of passive/active layered random media.

Recently there are reported studies [18] of the effect of boundary refractive index mismatching on photon diffusive reflectance from the bulk disordered media, and the results show that the matching significantly inproves the spatial resolution of the spatial photon sensitivity profile. This is in the diffusive case where photons mainly transported by random walks and wave interferences are negligible. The presence of a mismatched interface plays an important role, as it effects the Lambertian diffusive reflectance properties of the mismatched surface. Our results in one dimensional disordered mismatched media with strong interference effects here show a significant enhancement in signal due to the multiple reflections from the confined boundaries due to the mismatched condition and in the presence of scatteres, due to the interference effects. All these results demand further study of the effects of the boundary mismatched parameters in higher dimensions and its different practical applications.

Acknowledgements

This work was partly supported by NIH grants (Nos. R01EB003682, R01EB016983, R01CA128641 and U54CA143869), a NSF grant no CBET-0937987, a FedEx Institute of Technology Grant, and a Faculty Research Grant from the University of Memphis.

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