Investigation of the noise-like structures of the total scattering cross-section of random media

Snow H. Tseng[1], Allen Taflove[1], Duncan Maitland[2], Vadim Backman[3], and Joseph T. Walsh, Jr.[3]

[1]*Department of Electrical and Computer Engineering, Northwestern University, Evanston, Illinois 60208*

[2] *Medical Physics and Biophysics Division, Lawrence Livermore National Laboratory, Livermore, California 94550*

[3] *Biomedical Engineering Department, Northwestern University, Evanston, Illinois 60208*

Abstract: The pseudospectral time-domain (PSTD) algorithm is implemented to numerically solve Maxwell's equations to obtain the optical properties of millimeter-scale random media consisting of hundreds of micron-scale dielectric scatterers. Our methodology accounts for near-field interactions and coherent interference effects that are not easily modeled using other techniques. In this paper, we show that the total scattering cross-section (TSCS) of a cluster of closely packed scatterers exhibits a high-frequency oscillation structure, similar to noise. Furthermore, the characteristics and origin of such *noise-like* oscillation structure have been analyzed and determined based on first-principles.

© 2005 Optical Society of America

OCIS codes: (290.4210) Multiple scattering; (030.1670) Coherent optical effects

References and links

1. A. Amelink and H. Sterenborg, "Measurement of the local optical properties of turbid media by differential path-length spectroscopy," Appl. Opt. **43**, 3048-3054 (2004).
2. B.E. Barrowes, C.O. Ao, F.L. Teixeira, J.A. Kong, and L. Tsang, "Monte Carlo simulation of electromagnetic wave propagation in dense random media with dielectric spheroids" IEICE T. Electron. **E83C**, 1797-1802 (2000).
3. T. Khan and H.B. Jiang, "A new diffusion approximation to the radiative transfer equation for scattering media with spatially varying refractive indices" J. Opt. A-Pure Appl. Op. **5**, 137-141 (2003).
4. A.M.K. Nilsson, P. Alsholm, A. Karlsson, and S. Andersson-Engels, "T-matrix computations of light scattering by red blood cells" Appl. Opt. **37,** 2735-2748 (1998).
5. J.J. Guo, L. Tsang, K.H. Ding, A.T.C. Chang, and C.T. Chen, "Frequency dependence of scattering by dense media of small particles based on Monte Carlo simulation of Maxwell equations" IEEE T. Geosci. Remote **40**, 153-161 (2002).
6. E. Amic, J.M. Luck, and T.M. Nieuwenhuizen, "Anisotropic multiple scattering in diffusive media" J. Phys. A: Math. Gen. **29**, 4915-4955 (1996).
7. J. Pearce and D.M. Mittleman, "Propagation of single-cycle terahertz pulses in random media" Opt. Lett. **26**, 2002-2004 (2001).
8. M. Haney and R. Snieder, "Breakdown of wave diffusion in 2D due to loops" Phys. Rev. Lett. **91** (2003).
9. L. Marti-Lopez, J. Bouza-Dominguez, J.C. Hebden, S.R. Arridge, and R.A. Martinez-Celorio, "Validity conditions for the radiative transfer equation" J. Opt. Soc. Am. A **20**, 2046-2056 (2003).
10. S.H. Tseng, J.H. Greene, A. Taflove, D. Maitland, V. Backman, and J.T. Walsh, "Exact solution of Maxwell's equations for optical interactions with a macroscopic random medium" Opt. Lett. **29**, 1393-1395 (2004); **30**, 56-57 (2005).
11. S.H. Tseng, A. Taflove, D. Maitland, V. Backman, and J.T. Walsh, "Extraction of Single-Scatterer Signatures from Multiple-Scattering of Closely Packed Random Media" (submitted to Appl. Opt.).

#7691 - $15.00 US Received 1 June 2005; revised 28 July 2005; accepted 28 July 2005

12. S.H. Tseng, A. Taflove, D. Maitland, V. Backman, and J.T. Walsh, "Simulation of enhanced backscattering of light by numerically solving Maxwell's equations without heuristic approximations" Opt. Express **13**, 3666-3672 (2005).
13. Q.H. Liu, "The PSTD algorithm: A time-domain method requiring only two cells per wavelength" Microw. Opt. Technol. Let. **15**, 158-165 (1997).

1. Introduction

In this paper, we report the observation and analysis of the high-frequency oscillation of the total scattering cross-section (TSCS) spectrum of random media consisting of hundreds of micron-scale dielectric scatterers. In particular, this high-frequency oscillation resembles noise and can easily be overlooked in optical experiments. Based on Maxwell's equations, the pseudospectral time-domain (PSTD) algorithm is employed to calculate the optical properties of random media consisting of closely packed dielectric scatterers. The characteristics and origin of the *noise-like* TSCS oscillation structure are further analyzed and determined.

To date, most research attempts to determine the optical properties of macroscopic random media involve certain degree of heuristic approximations based on radiative transfer theory, including Monte Carlo simulations that assume independent scattering of point-like scatterers, and the diffusion approximation where light is treated as a diffusion problem [1-6]. The range of validity and accuracy for such approximations remain to be determined [7-9] —all approximation methods are fundamentally limited by the assumptions imposed. Furthermore, the near-field interactions and coherent interference effects such as speckle are not easily account for using conventional approximation methods. As a result, research in light scattering by random media has mostly centered on utilizing singly scattered light while suppressing multiply scattered light.

Recently, optical characteristics of macroscopic random media have been studied by numerically solving Maxwell's equations, rigorously accounting for multiply scattered light. As reported in [10], the TSCS of an optically thick, closely packed random medium is determined by the overall geometry and average refractive index, but insensitive to the microscopic geometrical structures. In addition, further study has shown that information concerning microscopic geometrical details of the random media can be extracted from the macroscopic scattered light [11]. No doubt that a rich amount of information is contained in the multiply scattered light, which begs the question: *What information can be further extracted from macroscopic scattered light?* To answer this question, an analysis based on fundamental electromagnetic theory that accounts for near-field interactions and coherent effects is indispensable.

2. Methods

The simulations reported in this paper are based on the methodology we have recently reported [10, 12]. By implementing the pseudospectral time-domain (PSTD) technique [13], combined with parallel computing technology, our methodology provides a numerical approach to the problem of light scatting by macroscopic random media, accounting for the near-field interactions and coherent interference effects.

To numerically solve the Maxwell's equations, the spatial derivatives of the electric and magnetic fields are obtained using the differentiation theorem for Fourier transforms:

$$\left\{ \left.\frac{\partial V}{\partial x}\right|_i \right\} = -\mathbf{F}^{-1}\left(j\tilde{k}_x \mathbf{F}\{V_i\} \right) \tag{1}$$

where $\mathbf{F}$ and $\mathbf{F}^{-1}$ denote, respectively, the forward and inverse discrete Fourier transforms, and $\tilde{k}_x$ is the Fourier transform variable representing the *x*-component of the numerical

Received 1 June 2005; revised 28 July 2005; accepted 28 July 2005
(C) 2005 OSA

wavevector. The spatial derivatives $\{(\partial V/\partial x)_i\}$ can be calculated in one step. In multiple dimensions, this process is repeated for each cut parallel to the major axes of the space lattice. PSTD techniques have been shown to possess spectral accuracy; that is, errors due to spatial sampling decrease exponentially as the meshing density increases beyond the Nyquist rate.

In this paper, the PSTD technique is employed to model transverse-magnetic (TM) scattering of light by a macroscopic cluster of N non-contacting, randomly positioned, infinitely long dielectric cylinders of diameter d in free space. The TSCS for a 2-D system is defined as the total scattering cross-section *per unit length* for a system consisting of infinite cylinders. In each simulation, the cluster of cylinders is illuminated by a plane wave with an incident angle of 90^o. With a spatial resolution of dx = 0.33 μm and temporal resolution of dt = 10^{-16} sec, the TSCS of the cluster of infinite dielectric cylinders is obtained for a broadband spectrum (λ_0 = 1 μm - 600 μm) in a single simulation. Without heuristic approximations, our methodology enables accurate simulation of the optical characteristics of macroscopic random media that are not easily modeled using other techniques.

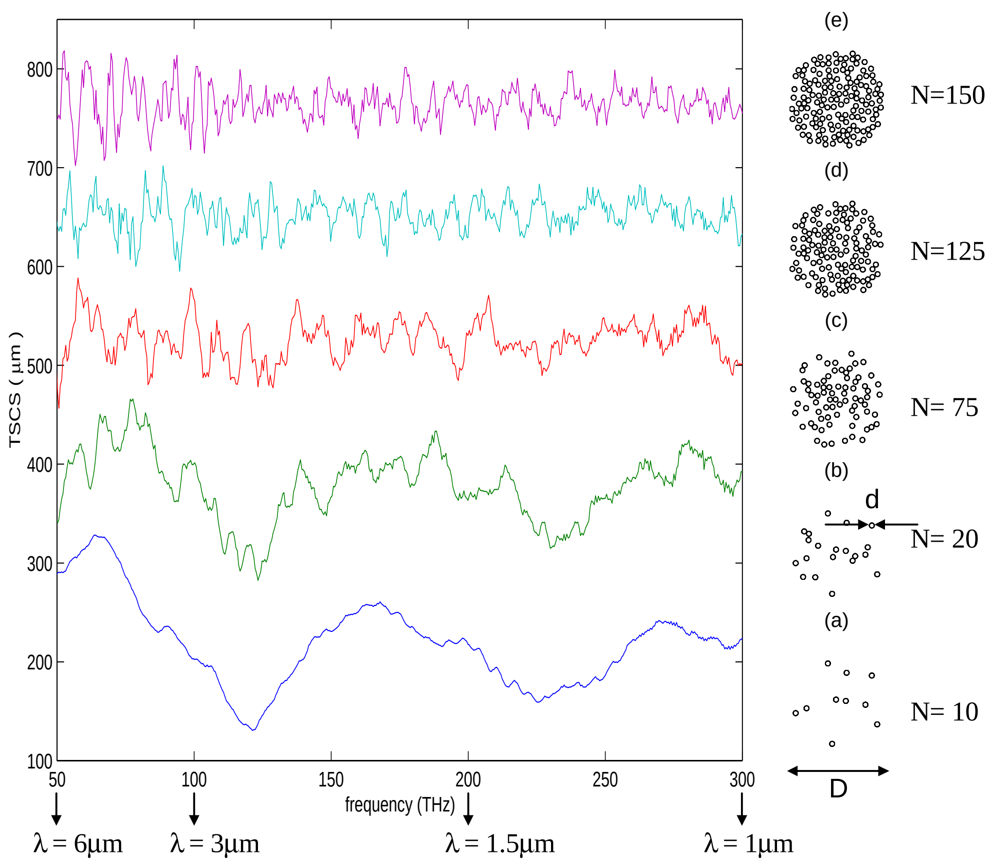

Fig. 1. Comparison of the total scattering cross-section (TSCS) of a cluster consisting of N dielectric cylinders. With an overall-diameter D = 280 μm, each cluster consists of randomly positioned, non-contacting, n = 1.2 dielectric cylinders of diameter d = 14 μm. Five cases are shown (a)-(e): N = 10, 20, 75, 125, and 150, respectively. (Each TSCS curve is offset on the vertical axis to facilitate comparison.) It is apparent that the high-frequency oscillation of the TSCS spectrum increases with increasing N.

3. PSTD Simulation

The PSTD-computed TSCS spectra of a cluster geometry consisting of various numbers of dielectric cylinders are shown in Fig. 1. With an overall diameter D = 280 μm, each cluster consists of randomly positioned, non-contacting, n = 1.2, dielectric cylinders of diameter d = 14 μm. Note that the *noise-like* oscillation of the TSCS spectrum becomes more pronounced as N increases, suggesting that it may be related to the microscopic geometrical details within the random media.

Secondly, by subtracting out the smoothed TSCS spectrum, the TSCS oscillation (in Fig. 1.) is extracted and shown in Fig. 2. The oscillation of the TSCS spectrum corresponding to various N is compared. It is evident that with more scatterers closely packed together, the TSCS oscillation becomes more pronounced with increased complexity.

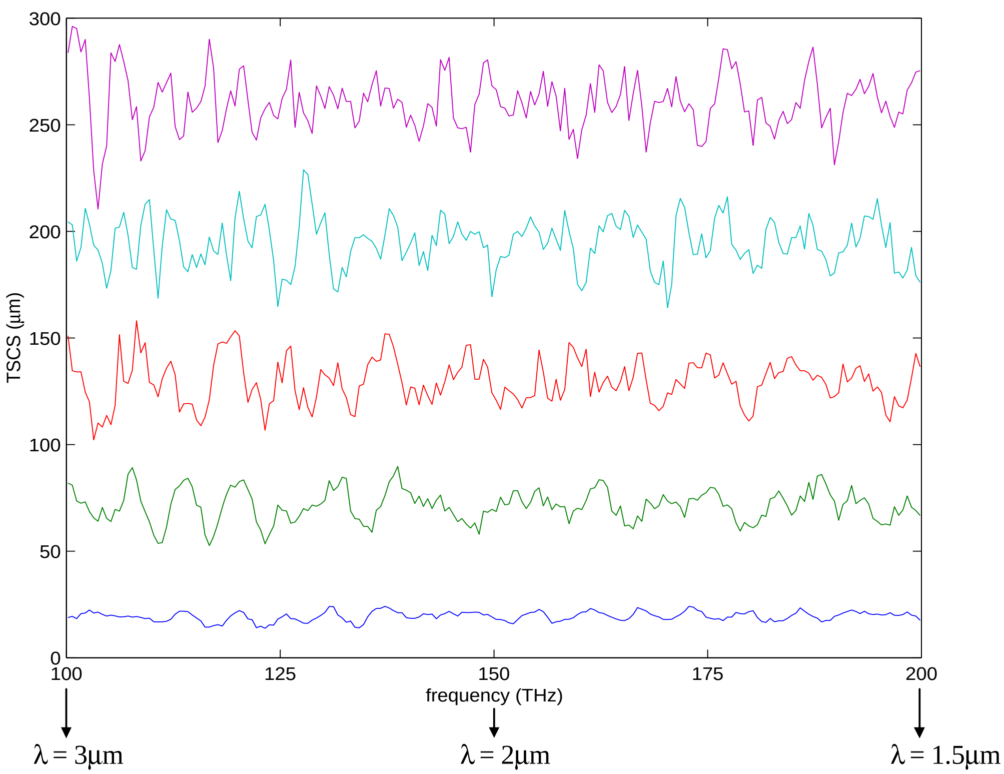

Fig. 2. Extracted oscillation structure of the TSCS spectra corresponding to various N. The high-frequency oscillations are extracted from the TSCS (as shown in Fig. 1) by subtracting out the smoothed TSCS curves. (The smoothed TSCS spectra are obtained by a running Gaussian-window average with FWHM of 20 THz.) Each curve is offset vertically with (from bottom to top) N = 10, 20, 50, 75, 100, 125, 150, 175, and 203, respectively. For larger N, the oscillation of the TSCS spectrum becomes more pronounced.

While fixing the cluster diameter D, the TSCS oscillation is analyzed with respect to N and shown in Fig. 3. Here we present an autocorrelation analysis of the TSCS oscillation. A relationship between the characteristic correlation interval $\delta\omega$ (characteristic correlation interval $\delta\omega \equiv$ the correlation frequency of the autocorrelation analysis of the TSCS oscillation as a function of frequency) and N is discovered—as the number of scatterers increases, the characteristic correlation interval $\delta\omega$ decreases monotonically.

#7691 - $15.00 US
(C) 2005 OSA
Received 1 June 2005; revised 28 July 2005; accepted 28 July 2005

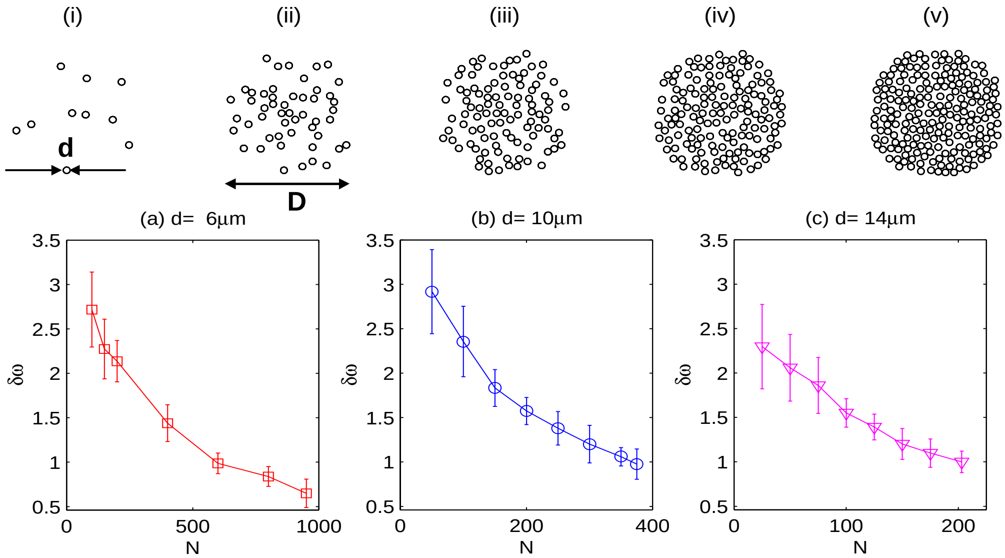

Fig. 3. Characteristic correlation interval $\delta\omega$ of the TSCS high-frequency oscillation of clusters consisting of various numbers of cylinders. With an overall diameter D = 280 μm, each cluster consists of N dielectric cylinders of diameter d. An example of the geometry is shown in (i)-(v), depicting the geometry of a cluster consisting of diameter d = 14 μm dielectric cylinders (n = 1.2), with various numbers of cylinders within each cluster [(i)-(v): N = 10, 50, 100, 150, and 203, respectively.] The characteristic correlation interval $\delta\omega$ is shown in (a)-(c), corresponding to clusters consisting of d-μm-diameter cylinders: (a) d = 6 μm, (b) d = 10 μm, (c) d = 14 μm. It is apparent that the characteristic correlation interval $\delta\omega$ decreases monotonically as the number of scatterers increases.

By varying D while fixing N, the TSCS oscillation is further analyzed with respect to the minimum spacing s between scatterers as shown in Fig. 4. It is readily seen that the characteristic correlation interval $\delta\omega$ does not depend significantly on the overall cluster diameter D or the minimum spacing s, suggesting that the TSCS oscillation is insensitive to the overall dimensions of the random media, or the spacing between scatterers.

From Fig. 3 alone, it is arguable that the decrease of $\delta\omega$ could also be due to the decrease of spacing between cylinders, since both N and the spacing between cylinders are varied simultaneously. Yet, it is shown in Fig. 4 that $\delta\omega$ is insensitive to the spacing between scatterers. Therefore, Figs. 3 and 4 together show that the decrease in $\delta\omega$ is related to the change of N, rather than the change of the spacing between scatterers.

In summary, by numerically solving Maxwell's equations, the research findings presented in this paper provide insight to the study of optical characteristics of macroscopic random media. It is shown that the *noise-like* TSCS oscillation indeed contains microscopic geometrical information of the random media. Furthermore, we have determined that the characteristic correlation interval $\delta\omega$ of the TSCS oscillation structure is directly related to the number of scatterers within the random media, but insensitive of the overall geometry of the random media, or the spacing between scatterers.

#7691 - $15.00 US Received 1 June 2005; revised 28 July 2005; accepted 28 July 2005
(C) 2005 OSA

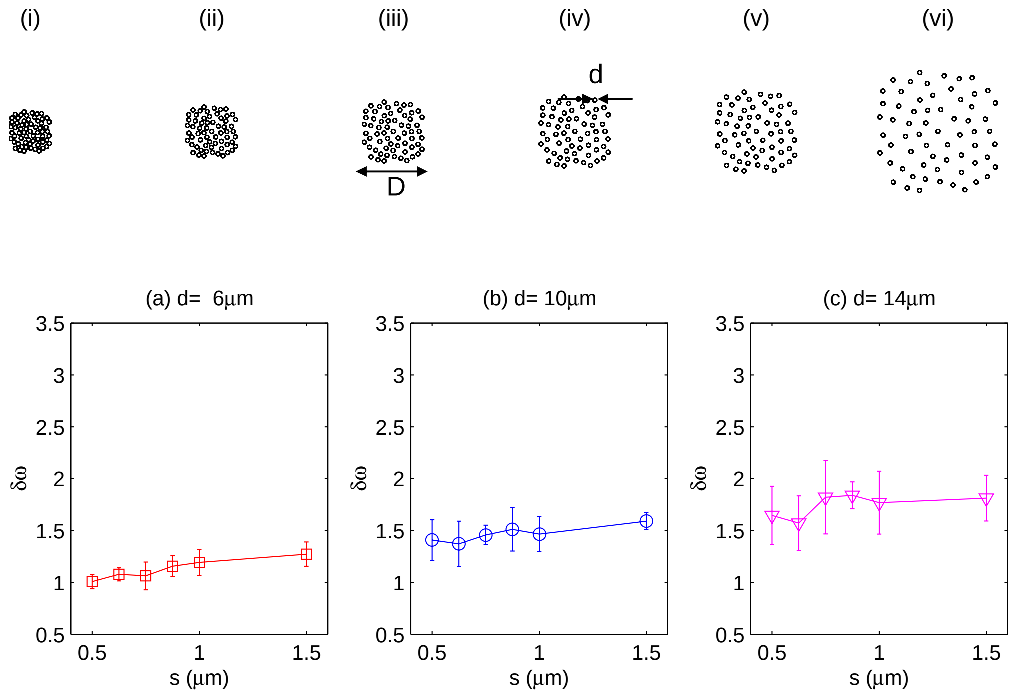

Fig. 4. Characteristic correlation interval $\delta\omega$ of the TSCS oscillation for a cluster of fixed number of cylinders (N = 64), with various cluster diameter D. (i)-(vi): depicts the geometry of a cluster consisting of diameter d = 14 µm dielectric cylinders (n = 1.2), with various cluster diameters: D = 160 µm, 200 µm, 240 µm, 280 µm, 320 µm, and 480 µm, respectively. The characteristic correlation interval $\delta\omega$ is shown in (a)-(c), corresponding to clusters consisting of d-µm-diameter cylinders: (a) d = 6 µm, (b) d = 10 µm, (c) d = 14 µm. From (a)-(c) it is readily shown that the correlation length does not depend significantly on the overall cluster diameter D, or the spacing s between scatterers.

The results reported in this paper point toward the emerging feasibility of direct, exact Maxwell's equations modeling of light scattering through millimeter-volume random media consisting of closely packed scatterers. More generally, our results have a wider implication: the near-field interactions and coherent interference effects of closely packed random media can be accurately investigated by numerically solving Maxwell's equations, revealing optical signatures indicative of microscopic structural information concerning the random media which has not been shown before. It should be noted that the methodology reported in this paper can be readily applied to many important systems, including biological tissues structures, quasi-crystal arrangements of particles, to investigate the near-field interactions and coherent interference effects. Further study is currently in progress and will be reported in subsequent publications.

Acknowledgments

The authors thank the National Institutes of Health National Cancer Institute Contract Grants 5R01-CA085991 and 5R01-HD044015, NSF Grant BES-0238903, and the NSF TeraGrid Grant No. MCB040062N for their support of this research. This study was performed under the auspices of the U.S. Department of Energy under Contract No. W-7405-ENG-48. Snow H. Tseng's email address is snow@ece.northwestern.edu.