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**computational  
electrodynamics**

**THE FINITE-DIFFERENCE TIME-DOMAIN METHOD**

THIRD EDITION

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# Preface to the Third Edition

The first and second editions of this book were published in 1995 and 2000, respectively. We are gratified with their high level of use by both the university and industrial-research communities. The second edition is often the text in senior year and graduate electrical engineering courses in computational electrodynamics, and both editions are frequently cited in refereed journal papers as primary background references for FDTD methods and applications.

This new third edition is extensively revised and expanded. We have had two primary goals in this regard. First, we have worked to update the book's discussions of FDTD theory and applications to keep pace with the continuing, rapid changes in these areas since 2000. This allows the professional engineer or scientist to have a convenient single-source reference concerning the latest FDTD techniques and research problems. Second, we have worked to further enhance the educational content of the book from both a fundamental theoretical perspective, and from the standpoint of the course instructor's ease of use.

## **New Material: Advances in FDTD Theory and Numerical Algorithms**

Specifically, this third edition contains a large body of new material that discusses in great detail the following recent advances in FDTD theory and numerical algorithms:

- New invited Chapter 17, “Advances in PSTD Techniques,” by Qing Liu and Gang Zhao, who have pioneered the theory and application of pseudospectral time-domain computational solutions of Maxwell's equations;
- New invited Chapter 18, “Advances in Unconditionally Stable Techniques,” by Hans De Raedt, who, within the framework of the matrix-exponential technique, has pioneered the unification of existing algorithms, as well as the synthesis of completely novel algorithms, for unconditionally stable computational solutions of the time-dependent Maxwell's equations;
- New invited Chapter 19, “Advances in Hybrid FDTD-FE Techniques,” by Thomas Rylander, Fredrik Edelvik, Anders Bondeson, and Douglas Riley, who have pioneered the development and application of provably stable hybrids of FDTD and finite-element time-domain techniques;
- New invited Chapter 20, “Advances in Hardware Acceleration for FDTD,” by Ryan Schneider, Sean Krakiwsky, Laurence Turner, and Michal Okoniewski, who have led the development of computer hardware / software that promise one order-of-magnitude speedups of FDTD solutions implemented on normal laboratory computers;
- New invited Section 5.9 in Chapter 5, by John Schneider, describing his development of advanced numerical dispersion-compensation techniques for the total-field / scattered-field FDTD wave-source condition;



- New invited Sections 7.7 and 7.9 to 7.11 in Chapter 7 by Stephen Gedney, describing the theory, numerical implementation, and illustrative results of his convolutional PML absorbing boundary condition, the most effective such technique yet for terminating open-region FDTD computational modeling spaces;
- New invited Section 8.7 in Chapter 8 by Xu Li, who has solved the puzzle of why FDTD had previously not properly modeled the backscattering of certain weakly backscattering objects;
- New Sections 9.2.3 and 9.4.3 in Chapter 9 on Drude media, so important for FDTD modeling of metals at optical frequencies;
- New invited Section 9.5 in Chapter 9 by Wojciech Gwarek, who has pioneered the efficient circuit model of linear magnetized ferrites in FDTD simulations;
- New invited Sections 9.6.1 to 9.6.5 in Chapter 9 by Masafumi Fujii, who has led the development of improved FDTD algorithms for nonlinear dispersive media;
- New invited Section 9.8 in Chapter 9 by Shih-Hui Chang on advances in FDTD modeling of quantum-gain materials characterized by a four-level, two-electron atomic system constrained by the Pauli Exclusion Principle;
- New Sections 10.6.1 and 10.6.3 in Chapter 10 on the simple, robust, Yu-Mittra techniques for modeling curved surfaces comprised of either a perfect electric conductor or a dielectric material;
- New invited Section 10.8.3 in Chapter 10 by Malgorzata Celuch-Marcysiak on her simple, robust, ultrawideband equivalent-circuit model of the frequency-dependent skin effect;
- New invited Section 11.8 in Chapter 11 by Nicolas Chavannes on his robust subgridding technique that allows numerically stable local mesh refinement;
- New invited Section 15.7 in Chapter 15 by Wojciech Gwarek, who has pioneered efficient and accurate  $S$ -parameter extraction from FDTD models of general waveguides;
- New invited Section 15.9.6 in Chapter 15 by Tzong-Lin Wu, who has innovated an efficient FDTD subcell model of the arbitrary two-terminal linear lumped network;
- New invited Sections 16.10 to 16.16 in Chapter 16 by Geoffrey Burr, who has helped to lead the development of FDTD techniques for modeling photonic crystals.

### **New Material: Advances in FDTD Modeling Applications**

In addition to theoretical advances, the third edition contains significant new material that discusses in detail the following recent advances in FDTD modeling applications:

- New invited Section 3.8 in Chapter 3 by Jamesina Simpson, who describes her FDTD models of around-the-world electromagnetic wave propagation at extremely low frequencies;
- New invited Section 14.9 in Chapter 14 by Nicolas Chavannes, who describes his detailed FDTD modeling case study of the electromagnetic wave performance characteristics of the Motorola T250 tri-band cellphone;
- New Section 14.10.4 in Chapter 14, which describes work by Susan Hagness' group on the application of FDTD modeling of ultrawideband radar techniques for the early-stage detection of breast cancer;
- New invited Section 15.12 in Chapter 15 by Jamesina Simpson, who describes her FDTD models of potential hyperspeed digital interconnects in circuit boards realized by defect-mode waveguides in electromagnetic bandgap structures;
- New Sections 16.17 to 16.21 in Chapter 16, which describes work by Susan Hagness' group in PSTD modeling of frequency conversion in second-order nonlinear optical materials, including photonic crystals;
- New invited Sections 16.22 to 16.24 in Chapter 16 by Geoffrey Burr, who describes recent FDTD modeling applications in nanoplasmonics;
- New Sections 16.25 to 16.27 in Chapter 16, which describe applications of FDTD and PSTD modeling in biophotonics, especially in advancing the detection of early stage cervical and colon cancer.

These new examples serve not only to illustrate the power and beauty of FDTD modeling, but also to inform and excite the reader about the *integral role* that electromagnetic wave phenomena play in the design and operation of our society's most advanced electronics and photonics technologies.

#### **New Feature: A Web Site Dedicated to This Book**

To supplement this third edition, we and the publisher have created a Web site where, with the proper personal identification number (PIN), instructors can download solutions to the homework problems. This PIN also enables downloads of color graphics, videos, and text updates / errata, as generated by the authors. For details, visit [www.artechhouse.com](http://www.artechhouse.com) or contact the publisher via email at [artech@artechhouse.com](mailto:artech@artechhouse.com). While subject to copyright protection, the color graphics, videos, and text updates / errata downloaded in this manner can be freely distributed by the course instructor to his or her students. We believe that this feature greatly enhances the usefulness of this book as an instructional tool.

#### **Structuring University Courses Around This Book**

In our respective teaching experiences at Northwestern University and the University of Wisconsin-Madison, we have found that the material covered in this third edition is most appropriate for senior-year undergraduate students who have already taken at least one course in

electromagnetics. Having said that, our personal experience is that even students without the first course in electromagnetics can access most of the material in the first seven chapters, assuming that they have a strong background in vector calculus and computer programming. However, such students would require supplemental assistance to understand the basis of Maxwell's equations.

When used in a semester-length, senior year undergraduate course (i.e., UW-Madison), there is sufficient time to cover the first ten chapters. This includes time for the students to write working FDTD codes in one and two dimensions, with absorbing boundary conditions and total-field / scattered-field grid zoning. When used in a quarter-length, senior year undergraduate course (i.e., Northwestern), there is sufficient time to cover the first seven chapters.

We recommend that the final ten chapters be covered in a second semester or quarter at the graduate level. Some of the advanced new material in Chapters 17, 18, and 19 is appropriate for a special-topics course at the graduate level.

### **Acknowledgments**

In accomplishing this major rewrite and update of the second edition, we gratefully acknowledge all of our contributing chapter authors and coauthors. Their biographical sketches appear in the About the Authors section.

Finally, we acknowledge our respective family members who exhibited great patience and kept their good spirits while we worked long hours on this book. The first author appreciates the understanding and forbearance of his wife, Sylvia, during this past year of hyperintensive effort. The second author thanks her husband, Tim, for his love, friendship, and support. We may try their patience yet one more time in about five years, when fast-moving advances in FDTD theory and applications may indicate the need for a *fourth edition*.

*Allen Taflove, Evanston, Illinois  
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June 2005*