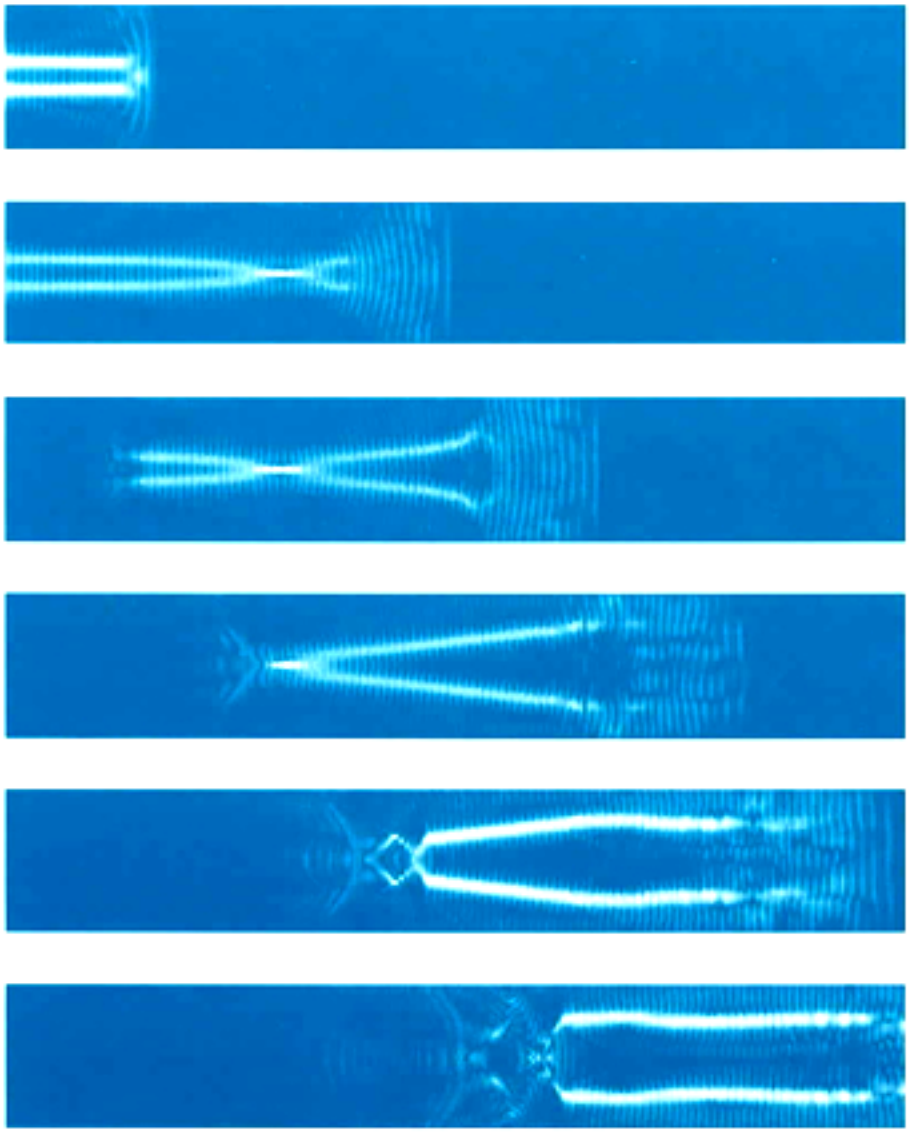
COMPUTATIONAL ELECTRODYNAMICS

The Finite-Difference Time-Domain Method



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Preface

Almost exactly twenty years ago, I submitted the first two journal papers of my research career to *IEEE Transactions on Microwave Theory and Techniques* (*MTT*). These papers described my initial explorations of what later became known as the *finite-difference time-domain* (FD-TD) method for Maxwell's equations. The two papers, and the Ph.D. dissertation research that they reported, grew from a graduate seminar course at Northwestern University in bioelectromagnetic hazards that Prof. Morris Brodwin had conducted in 1972. During my independent study for this seminar, I sought to obtain a model for UHF and microwave penetration into the human eye to better understand the formation of "microwave cataracts," which had been observed in a number of radar technicians during World War II. At first, there appeared to be no viable means to solve Maxwell's equations for the complex, three-dimensional biological tissue geometry represented by the eye, its surrounding muscle and fat tissues, and its embedding within the bony orbit of the skull. Analytical models were available for the small set of generic shapes for which the Helmholtz equation could be solved using the separation of variables technique. However, these simple shapes, including the half space, the layered half space, the sphere, the layered sphere, and the infinite cone, were very inadequate for modeling the tissue geometry of the eye. Further, a back-of-the-envelope estimate of the implications of the required tissue and wavelength space resolution ruled out the frequency-domain method of moments, then capable of solving for only a few hundred field unknowns. The eye geometry required the solution of almost 100,000 field unknowns, *two to three orders of magnitude* beyond anything published in the moment method community at that time.

Almost having given up on Prof. Brodwin's seminar project, I found myself randomly leafing through back issues of *IEEE Transactions on Antennas and Propagation* in the dark, claustrophobic stacks of the old Tech library (replaced years later by a large, bright, open building). It was then that I stumbled upon Kane Yee's 1966 paper. Six years had gone by since its publication, with very sparse references to it recorded in *Science Citation Index*. And yet the paper was the *Grail*. I sensed that the Yee algorithm had tremendous promise. It could handle material inhomogeneities and did

not require matrix inversion, meaning that I could use the University's Control Data CDC 6400 computer to crunch the problem to its conclusion. Of course, a few "minor" details had to be solved, such as sourcing a plane wave, obtaining a rudimentary absorbing boundary condition, understanding the algorithm's numerical stability properties, and progressing from one to two to three dimensions in my code development.

So, my 1975 papers in *MTT* were published. And landed with a thud. Being eager, brash, and absolutely naive, I had expected the electromagnetics community to seize upon the marvelous Yee algorithm and apply it to *everything*. However, with the exception of the few industrial research firms and U.S. Government agencies active in the electromagnetic pulse area, FD-TD remained essentially unused for more than a decade.

Now, after much hard work and the welcome help of a rapidly growing user community, FD-TD is being used worldwide. And for just about *everything*, as I had foreseen in 1975.

It is my profound hope that the readers of this book, whether university students or professionals, can use and enjoy its material at a number of different levels. There is sufficient tutorial exposition for a class in computational electromagnetics at the senior undergraduate or first-year graduate level, at which the students have the traditional core electromagnetics background. I have taught such a course from drafts of this book for six years at Northwestern, and similar courses have been taught for the past two years at the University of Colorado at Boulder by my colleague, Melinda Piket-May. Over a ten-week academic quarter, it is possible for students to assimilate the first eight chapters, write their own working FD-TD codes in two dimensions with wave source and absorbing boundary conditions, and then use their codes to implement radiation and scattering models of their own design. Over an academic semester, it is possible to augment these basics with two or three of the remaining chapters, as selected by the instructor, to specialize in advanced topics involving unstructured meshing, electronics modeling, antennas, or parallel-processing software.

For the professional, this book is intended to be comprehensive and self-teaching. There is exposure to virtually all of the latest topics in FD-TD theory and practice, and a quite exhaustive list of references and bibliographic materials.

I gratefully acknowledge the contributors of the invited chapters in this book: Stephen Gedney and Faiza Lansing, Thomas Jurgens and Gregory Saewert, Melinda Piket-May, Eric Thiele, and Stephen Barnard. Also acknowledged are the contributions of my graduate students, who did the really hard work. A special thanks goes to my steadfast friend, Evans Harrigan, who has *believed* in FD-TD modeling ever since we met, and saw to it that my students had *all* of the time on Cray's marvelous supercomputers that they needed to do their work. Finally, I acknowledge my wife, Sylvia, and sons, Mike and Nate, who somehow were able to keep their composure while sharing a home with a very driven person who was utterly thrilled with the highs and quite agonized with the lows of FD-TD developments over twenty years.

This book is the culmination of a major phase in my life. However, the FD-TD story is only beginning. Let's move on to develop detailed FD-TD electromagnetics models of microchips, microlasers, and microcells, and bring the power of Maxwell's

equations to bear upon society's needs in ultrahigh-speed communications technology. In this manner, electromagnetic wave specialists can augment their current role in enabling people to freely communicate with each other worldwide, at the speed of light.

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