Active Set Identification via Parametric Linear Programming

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Active-Set Methods for NLP

How to move beyond SQP?

- Solve larger problems
- Identify optimal active set more quickly/cheaply

Look at S**E**QP approaches

- 1) estimate active set, then; 2) solve EQP
- Advantages:
  - can change many activities at once
  - can use iterative (projected CG) methods to solve EQPs

SEQP Methods for NLP

Gradient Projection:

- most effective method for bound constrained NLP
  - Lancelot, L-BFGS-B, Tron, etc.
- how to extend gradient projection to more general constraints?

SLQP:

- estimate active-set by solving an LP
- then solve EQP to generate step
  - Fletcher, Sainz de la Maza; Fletcher, Chin
  - Byrd, Gould, Nocedal, W.
- can we do better?

SLQP algorithm

- **Given**: x_k
- **Solve LP:**

$$\begin{aligned} \min \quad & \nabla f(x)^T d \\ \text{s.t} \quad & h(x) + \nabla h(x)^T d = 0 \\ & g(x) + \nabla g(x)^T d \geq 0 \\ & \| d \|_\infty \leq \Delta^{\text{LP}} \end{aligned}$$

⇒ Working set W

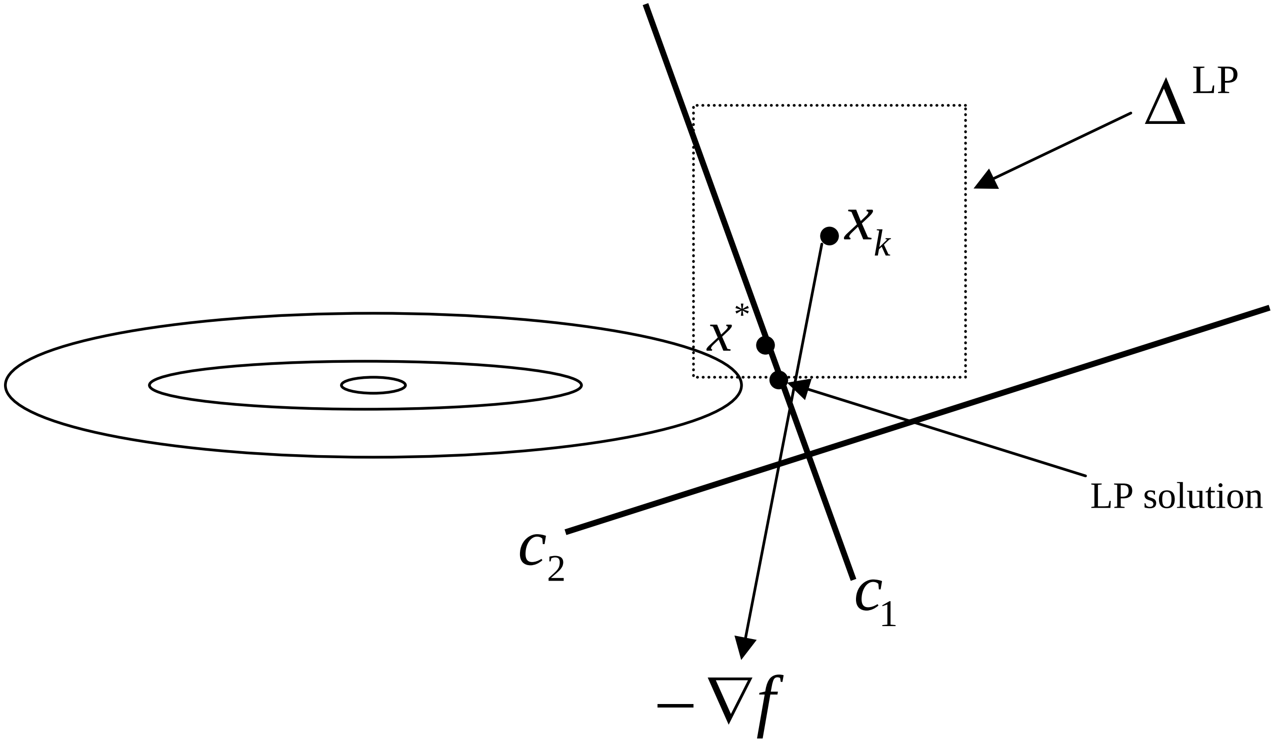
- **Compute a step, *d***, by solving the **EQP**:

$$\begin{aligned} \min_d \quad & \nabla f(x)^{\mathrm{T}} d + \frac{1}{2} d^{\mathrm{T}} H d \\ \text{subject to} \quad & h_i(x) + \nabla h_i(x)^{\mathrm{T}} d = 0, i \in E \\ & g_i(x) + \nabla g_i(x)^{\mathrm{T}} d = 0, i \in W \cap I \\ & \|d\| \leq \Delta \end{aligned}$$

- **Set**: $x_{k+1} = x_k + ad$

SLQP Active-Set Identification

Example



How to choose Δ^{LP} ???

Choosing DeltaLP

Thereom: C. Oberlin and S. Wright 2005

If MFCQ and close enough to the solution there exists

$$\Delta^{\mathrm{LP}} \in [\| x - x^* \|^{\sigma}, \overline{\Delta}]$$

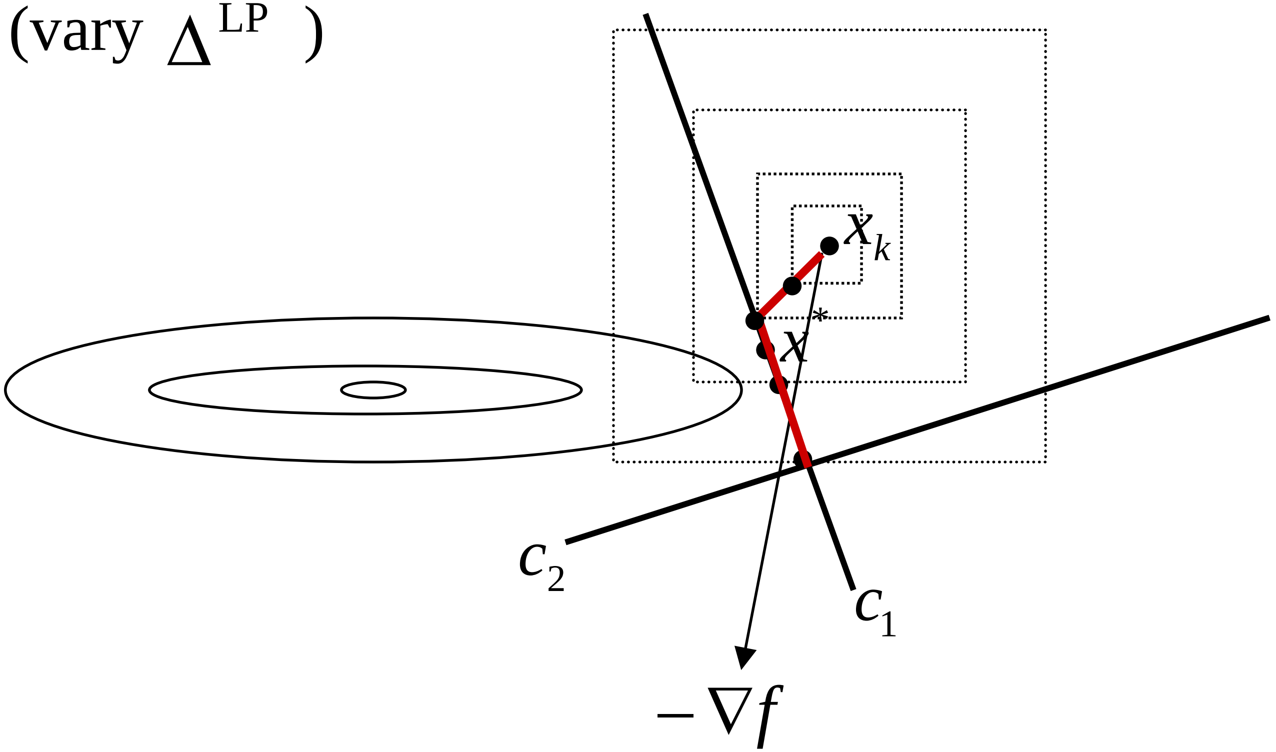
Giving $A \subset A^*$

How to choose Δ^{LP} in this range (especially if range is small)?

Parametric LP Path

Example revisited

(vary Δ^{LP})



Need $\Delta^{\mathrm{LP}} \in [2,4]$ to get optimal active set

(explore a range of values)

Parametric SLQP Algorithm

1. Given x_k

1. Generate path from solving sequence of LPs for

$$\Delta^{LP} \in [\Delta^{min}, \Delta^{max}]$$

1. Minimize quadratic model along this path to define a Cauchy point

1. Define working set to be the constraints active at the Cauchy point

1. Solve EQP based on current working set to generate a step

1. Update iterate

Parametric SLQP vs Gradient Projection

Parametric SLQP = Gradient Projection in infinity norm

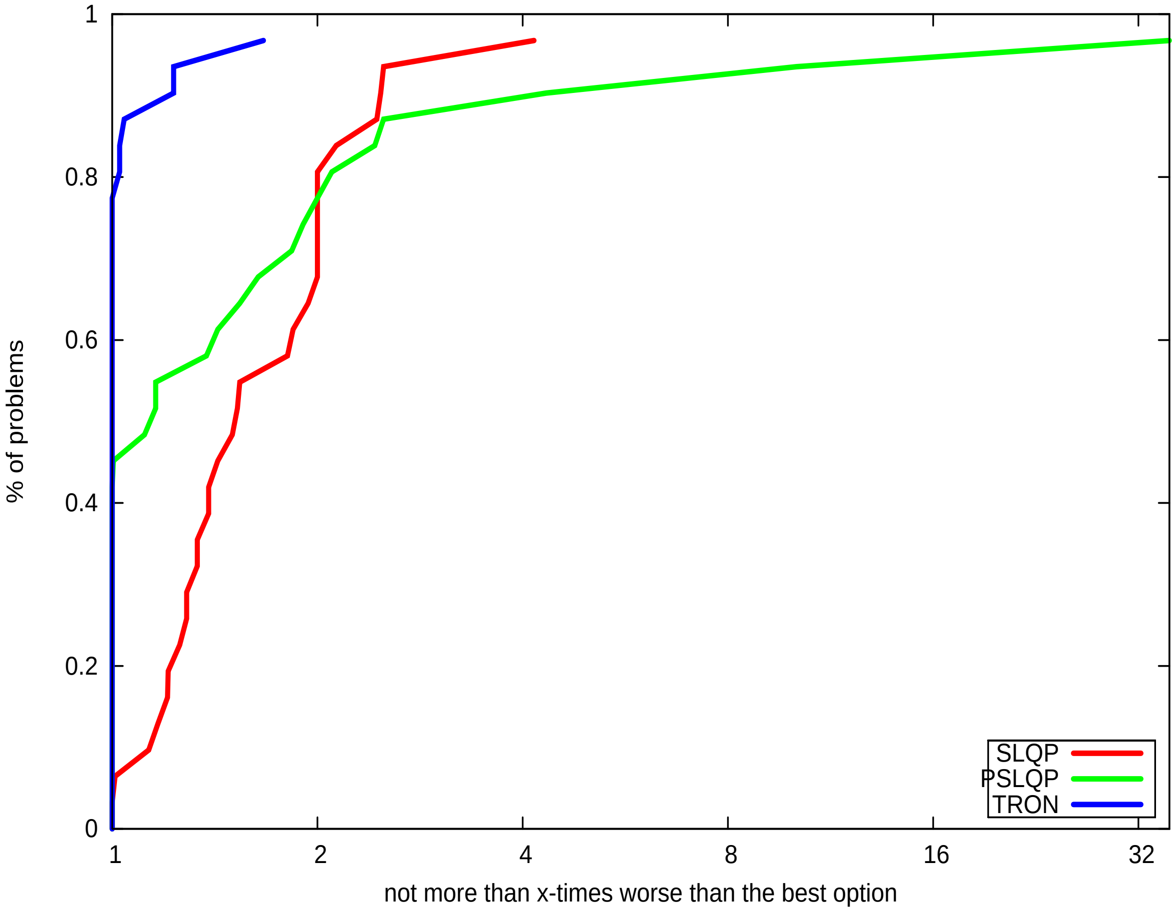
- on bound constrained problems
- defining steepest descent direction in the infinity norm as

$$d_i^{\infty} = -sign(\nabla f(x)_i)\|\nabla f(x)\|_{\infty}$$

$$d^{\mathrm{LP}}(\Delta^{\mathrm{LP}}) = d^{\mathrm{GP}}(\tau) = P(\tau d^{\infty})$$

Parametric SLQP can be viewed as a way to extend gradient projection to more general NLP!

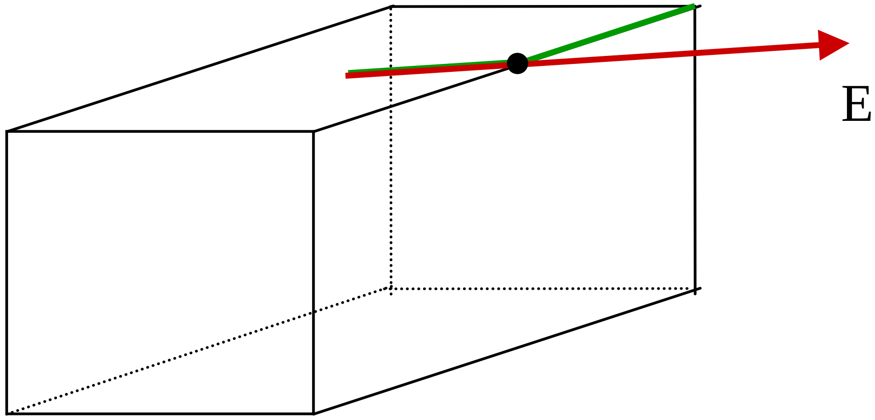
Results: Bound Constrained



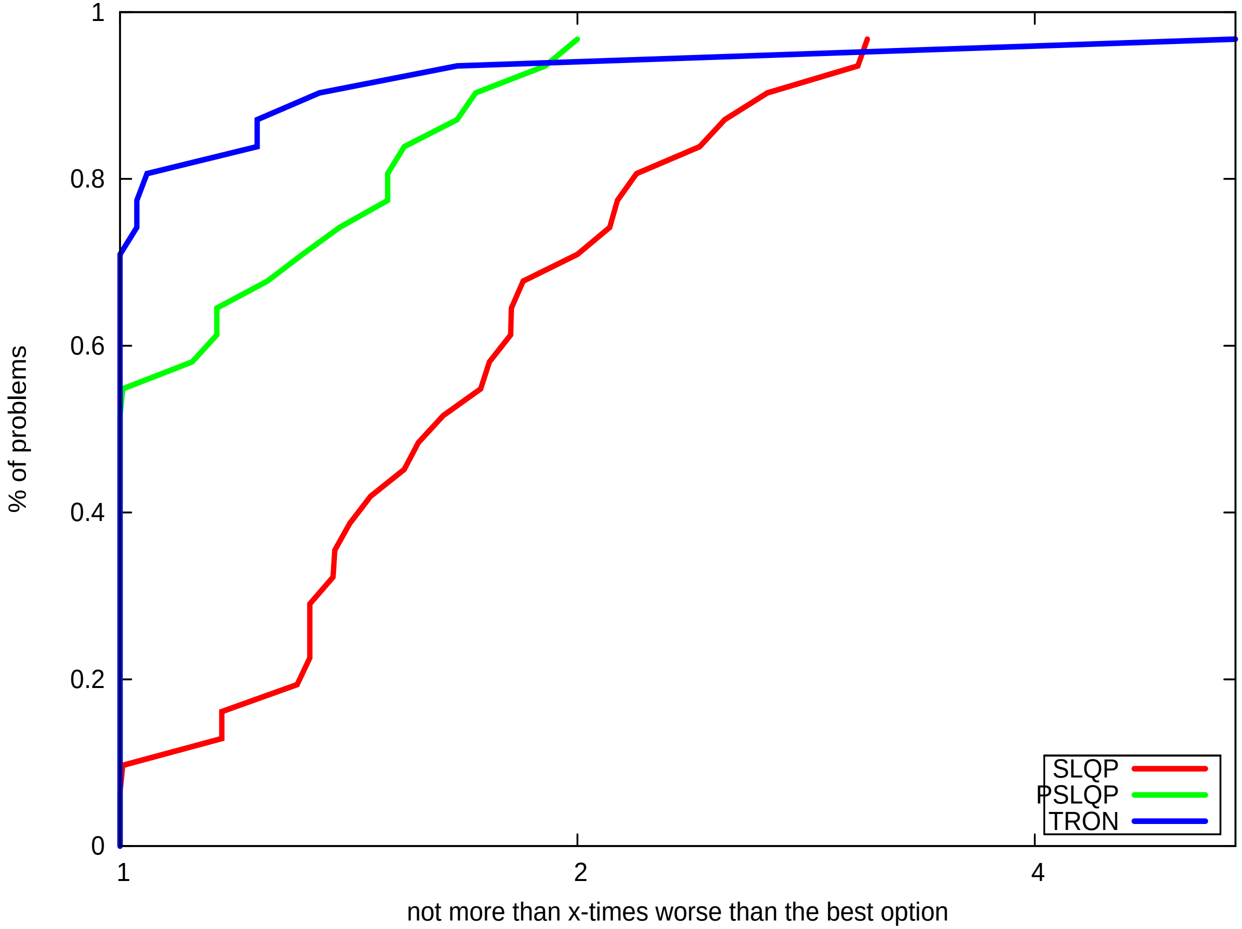
Constraints violated during EQP

What to do if inactive constraints are violated during EQP? (How to recover from poor estimates?)

- Truncate step
- Project step back onto feasible space and do a (piece-wise) line search
- Add activities during the EQP and resolve (how?)



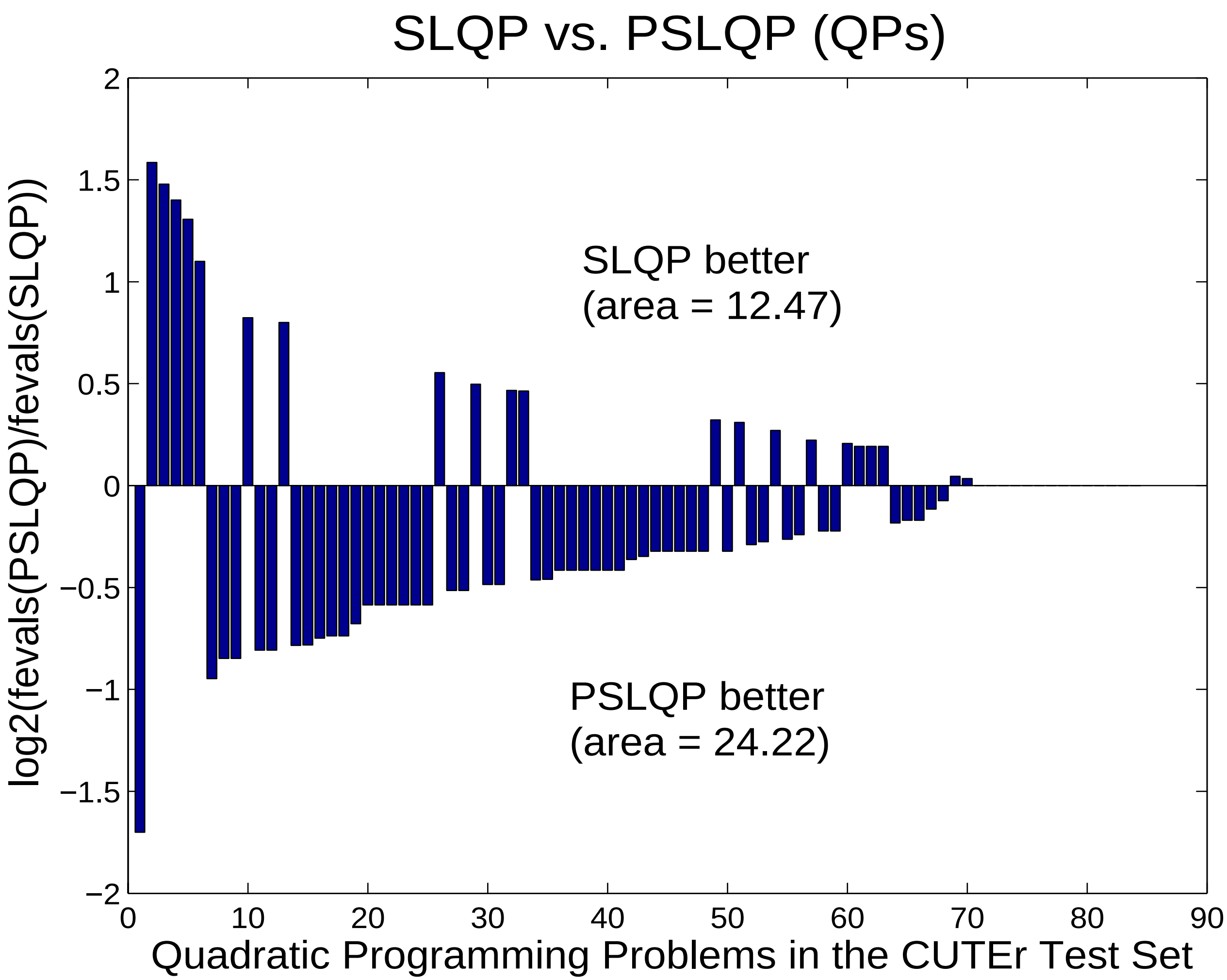
Results: Bound Constrained (revised)



Results: Bound Constrained (revised)

PROBLEM	PSLQP	TRON
biggsb1	503	501
bqpgauss	53	206
chenhark	269	204
cvxbqp1	2	2
gridgena	6	10
jnlbrng1	29	26
jnlbrng2	16	16
jnlbrnga	24	25
jnlbrngb	11	11
mccormck	10	7
minsurfo	12	7
ncvxbqp1	4	2
ncvxbqp2	15	10
ncvxbqp3	15	10
nobndtor	17	23
nonscomp	10	8
obstclae	27	27
obstclbm	21	21
pentdi	2	2
torsion1	39	40
torsion2	40	21
torsion3	21	21
torsion4	22	19
torsion5	12	12
torsion6	13	16
torsiona	39	40
torsionb	40	26
torsionc	21	21
torsiond	22	19
torsione	12	12
torsionf	13	16

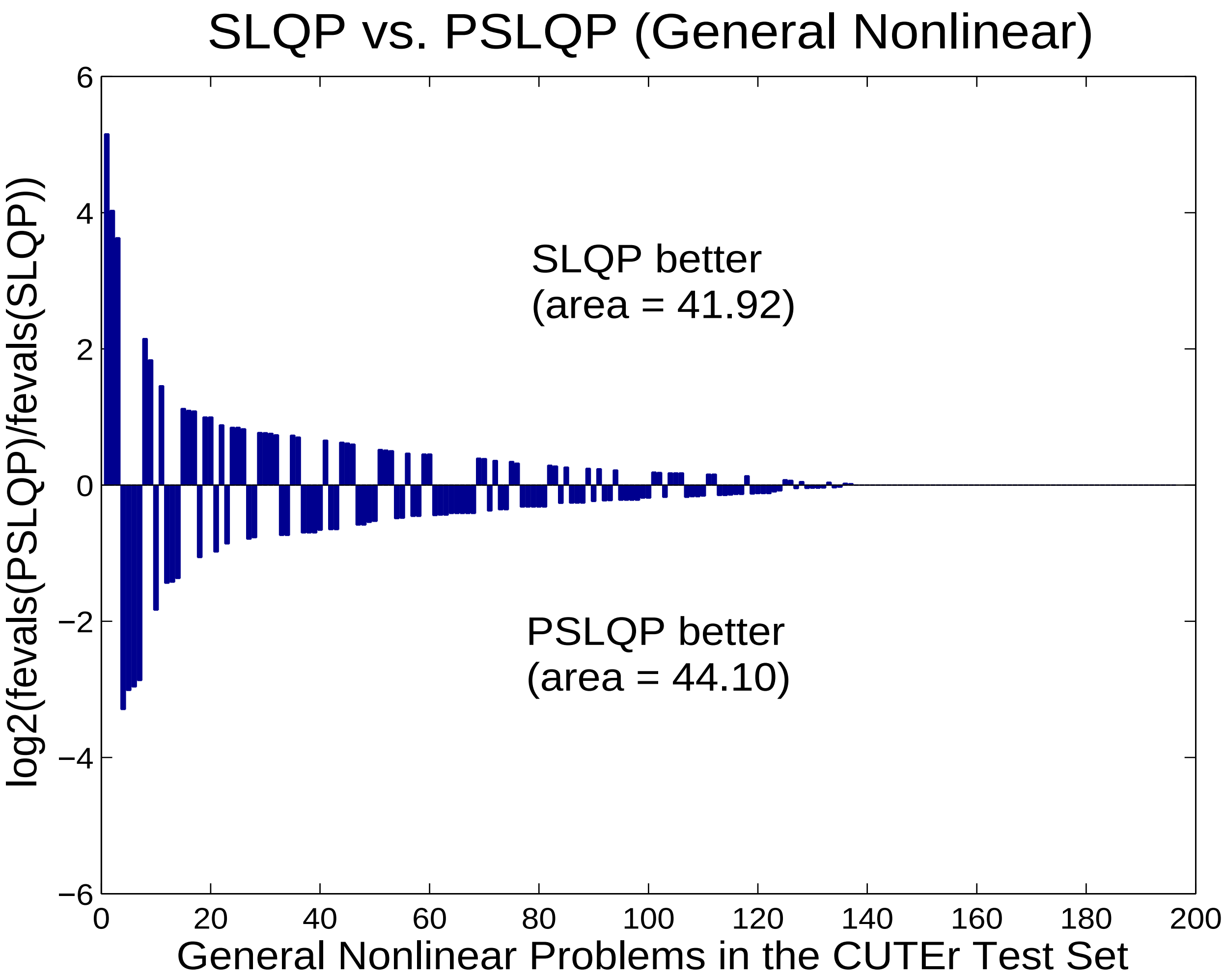
Results: Quadratic Programs



Morales plot

Robustness:
SLQP: 86/91
PSLQP: 85/91

Results: General NLPs



Robustness:
SLQP: 213/246
PSLQP: 211/246

Efficiency Issues

Classical Active Set SQP method

- **one** general QP with **lots** of subspace minimizations per iteration for large problems

SLQP method

- **one** LP (or parametric LP) + **one** EQP per iteration

How to effectively warm start the LPs???

Efficiency Issues

Warm starting the LP

$$
\begin{aligned}
\min \quad & \nabla f(x)^T d \\
\text{s.t} \quad & h(x) + \nabla h(x)^T d = 0 \\
& g(x) + \nabla g(x)^T d \geq 0 \\
& \| d \|_\infty \leq \Delta^{\mathrm{LP}}
\end{aligned}
$$

- Often many trust-region constraints active
- Problem constraints settle down near solution

…but trust region constraints don't!

Other Issues

Theory:

- Prove identification of optimal active set

Algorithmic:

- Dealing with degeneracy effectively
- Cheapen the LP solves

Thank You