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ON THE CONCEPTS OF PHOTONS AND PHOTODETECTION

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The concepts of photons and photodetection are examined in a unified manner by a careful construction of localized-photon creation and annihilation operators. Photons are shown to be localizable in their direction of propagation. An ideal but realistic photodetector is shown to sense the localized photons. It is found that a photodetector with a finite bandwidth may give a small but non-zero mean photocurrent in the vacuum.

1. INTRODUCTION

The concept of photons is often debated in the literature. For example, there are questions as to whether photons can be exactly located in free space.[1] On the other hand, the concept of photodetection has been developed in quantum optics,[2] showing that the quantization of the electromagnetic field becomes important when dealing with detectors with near-unity quantum efficiency. The quantization of the electromagnetic field and the concept of photons should be related. In fact, it has been implicitly assumed in quantum optics that the number of photoelectron emissions in a detector is approximately equal to the number of photons entering the detector if the detector has a unity quantum efficiency. However, as far as I know, such a viewpoint has not been shown in any exact manner. In this short paper, I give a summary for a theory that will provide some answers to the above questions by building a concept of photons based on a quantum measurement theory of an ideal detector.

I approach the problems by constructing operators that can be interpreted as the creation and annihilation operators of photons in some localized regions of space. These operators will be referred to as the localized-photon creation and annihilation operators (LPCAOs). I specialize the construction to one spatial dimension, defined by the direction of photon propagation. Such a specialization is not a loss of generality, as it will enable us to describe the realistic situation where photodetection is achieved by having a beam of photons propagating into a detection region. For simplicity, plane-wave modes will be used in this construction.

2. CONSTRUCTION AND PROPERTIES OF LPCAOs

Let us begin by quantizing the electromagnetic field with Coulomb gauge in a box of volume V_Q with periodic boundary conditions in the usual manner, and denote the corresponding Heisenberg annihilation operators by $\{\hat{a}_n(t)\}$. The spatial coordinates are given in terms of a Cartesian coordinate situated at the center of V_Q. The coordinate unit vectors will be denoted by $\bar{e}_x$, $\bar{e}_y$, and $\bar{e}_z$, respectively. Let $V_Q = l_Q^3$ and divide V_Q into 2N+1 small slabs of equal lengths l_L along the $\pm\bar{e}_z$ directions, where N is a positive integer. The centers of the 2N+1 slabs, namely slab $\{0, ..., \pm N\}$, are at $z = \{R_0, ..., R_{\pm N}\}$, respectively, where $R_p \equiv p\ l_L$. A localized-photon annihilation operator (LPAO) with *positive* wave vector, $\bar{k}_{Lm} = k_{Lm}\bar{e}_z$ ($k_{Lm} > 0$), is defined as follows:

$$\hat{b}_{l_{Lm}}(R_p, t) \equiv \int dz \left|\frac{k_{Lm}}{l_L}\right|^{1/2} exp(-ik_{Lm}z)\hat{\mathcal{A}}(z, t, +), \qquad (1)$$

where $\hat{\mathcal{A}}(z, t, +)$ is an operator related to the $+z$ propagating part of the vector-potential operator,[3] and is given by:

$$l_Q^{-1/2} \sum_{\substack{n \\ k_n>0}} |k_n|^{-1/2}\{\hat{a}_n(t)exp(ik_nz) + \hat{a}_n^\dagger(t)exp(-ik_nz)\}. \qquad (2)$$

The spatial integration in (1) is carried out only over the length of the slab located at R_p, defined by the interval: $[-l_L/2 + R_p,\ l_L/2 + R_p)$. The sum in (2) is carried out only over positive k_n. One can also define a $-z$ propagating vector-potential-like operator, $\hat{\mathcal{A}}(z, t, -)$, using an equation similar to (2), but with a sum over negative k_n. The localized-photon annihilation operators with *negative* k_{Lm} can then be defined in a similar manner as (1) but with $\hat{\mathcal{A}}(z, t, +)$ replaced by $\hat{\mathcal{A}}(z, t, -)$. The k-vector k_n is quantized in the volume V_Q of length l_Q so that $k_n \equiv 2n\pi/l_Q$, where $n \in \{0, \pm 1, \pm 2, ...\}$. However, the k-vector k_{Lm} is quantized in each slab of length

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l_L so that $k_{Lm} \equiv 2m\pi/l_L$. The index m also takes on integer values, except that it is necessary to split the $m = 0$ case into two cases, namely $m = \pm 0$, defined by the limiting process $\pm 0 \equiv \lim_{\epsilon \to 0} \pm|\epsilon|$, respectively. In any case, it turns out that the $m = \pm 0$ modes do not carry the dynamics of real photons and do not participate in causing atomic transitions. Hence, I will not discuss them in this paper. The index m below shall then be taken as excluding the special cases of $m = \pm 0$.

It is also essential to take the limit $l_Q \to \infty$ in (2). Taking this limit resolves the question as to whether one should include $n = 0$ terms in (2), as the question then becomes immaterial. More importantly, the commutation relations for $\hat{A}(z,t,+)$ and $\hat{A}(z,t,-)$ are sensitive to the length l_Q, and the formulation below requires $l_Q \to \infty$.

The localized-photon creation operators are obtained by taking the Hermitian conjugate of the localized-photon annihilation operators. The LPCAOs at a general location r can be defined by replacing R_p in (1) by r.

One can derive the commutation relations: $[\hat{b}_{l_L m}(R_p,t), \hat{b}^\dagger_{l_L m'}(R_{p'},t)] = \delta_{mm'}\delta_{pp'}$, and $[\hat{b}_{l_L m}(R_p,t), \hat{b}_{l_L m'}(R_{p'},t)] = 0$, which are like the usual raising and lowering operator commutation relations. However, the delta function $\delta_{pp'}$ tells us that any two such operators from different slabs commute with each other. Such commuting property qualifies $\hat{b}^\dagger_{l_L m}(R_p,t)$ and $\hat{b}_{l_L m}(R_p,t)$ as creation and annihilation operators that are localized in slab p.

One can build a Fock space for slab p in the usual manner by defining the localized vacuum $|0>_{l_L R_p}$ to be such that $\hat{b}_{l_L m}(R_p,0)|0>_{l_L R_p} = 0$ for all m. This shall be called the localized Fock space, and the corresponding Fock states in the space shall be called the localized-photon number states. Note that the usual vacuum for the entire box of quantization $|0>$ is not annihilated by $\hat{b}_{l_L m\sigma}(R_p,0)$ because of the $\{\hat{a}^\dagger_n(t)\}$ terms in (2). Hence, $|0>_{l_L R_p}$ is not the same as $|0>$ but is related to the usual Fock states via a Bogoliubov transformation. One can also define a localized-photon number operator for slab p by: $\hat{N}_{l_L m}(r,t) \equiv \hat{b}^\dagger_{l_L m}(r,t)\,\hat{b}_{l_L m}(r,t)$.

Using the usual non-relativistic minimal interaction Hamiltonian applied to a collection of single-electron atoms, one can derive the following equation of motion for $\hat{a}_n(t)$:

$$\frac{\partial}{\partial t}\hat{a}_n(t) = -i\omega_n\hat{a}_n(t) - i(\frac{g_n}{\hbar})\sum_j \frac{\partial\hat{\mu}^{(j)}(t)}{\partial t} exp(-ik_n r_z^{(j)}), \quad (3)$$

where, $\omega_n \equiv |k_n|c$, $g_n \equiv (\hbar/2\epsilon_o\omega_n l_Q^3)^{1/2}$, $\hat{\mu}^{(j)}(t)$ is the dipole-moment operator for the j^{th} atom, and $r_z^{(j)}$ is the z coordinate of the j^{th} atom. In equation (3), the dipole approximation has been made. Using this equation of motion, one can derive the equation of motion for $\hat{b}_{l_L m}(r,t)$:

$$[\frac{\partial}{\partial t} + S_{Lm}c\frac{\partial}{\partial r}]\hat{b}_{l_L m}(r,t) = -i\omega_{Lm}\hat{b}_{l_L m}(r,t) - i(\frac{g_{Lm}}{\hbar})\sum_{\substack{j\ in\\ slab}}$$

$$\frac{\partial\hat{\mu}^{(j)}(t)}{\partial t}[exp(-ik_{Lm}r_z^{(j)}) - cos(\frac{k_{Lm}l_L}{2})exp(-ik_{Lm}r)], \quad (4)$$

where $S_{Lm} \equiv Sign(k_{Lm})$, $\omega_{Lm} \equiv |k_{Lm}|c$, and $g_{Lm} \equiv (\hbar/2\epsilon_o\omega_{Lm}l_Q^2 l_L)^{1/2}$. The sum in (4) includes only those atoms in the slab at r. Thus, $\hat{b}_{l_L m}(r,t)$ only interacts with the atoms in its volume of localization. In free space, $\hat{b}_{l_L m}(r,t)$ can be readily solved giving: $\hat{b}_{l_L m}(r+S_m ct,t) = \hat{b}_{l_L m}(r,0)\,exp(-i\omega_{Lm}t)$, which clearly shows a sense of photon propagation to the $\pm\bar{e}_z$ directions for $m \gtrless 0$, respectively. In fact it can be shown that the *classical limit* for an eigenstate of $\hat{b}_{l_L m}(r,0)$ is just a sinusoidal light pulse of length l_L.

To understand how photon energy causes atomic transitions, let us consider the simple case where a group of atoms is situated in the z=0 plane. Let us denote the electron occupation number operator for energy level l of the j^{th} atom by $\hat{n}_l^{(j)}(t)$. By inserting complete sets of atomic wave functions, the atomic operators in the Hamiltonian can be put into a second-quantized form. Using the second-quantized form of the Hamiltonian, one can conveniently derive the equation of motion for $\hat{n}_l^{(j)}(t)$. With the help of (4), one can also obtain an equation of motion for the localized-photon number operator $\hat{N}_{l_L m}(r,t)$. By integrating the equations of motion for both $\hat{n}_l^{(j)}(t)$ and $\hat{N}_{l_L m}(r,t)$ from $t = 0$ to $t = l_L/c \equiv T$, the following identity can be shown:

$$\sum_m \hbar\omega_{Lm}[\hat{N}_{l_L m}(-S_{Lm}\frac{l_L}{2},0) - \hat{N}_{l_L m}(S_{Lm}\frac{l_L}{2},T)] =$$

$$\sum_{jl} E_l[\hat{n}_l^{(j)}(T) - \hat{n}_l^{(j)}(0)] + \Delta\hat{E}_{int}, \quad (5)$$

where $\Delta\hat{E}_{int}$ is an operator term expressing the difference between the values of the atom-field interaction energy at $t = T$ and $t = 0$, respectively. Equation (5) shows the conservation of energy, that a decrease in the total local photon energy after the photons pass through the atoms is equal to an increase in the total atomic energy plus an increase in

the atom-field interaction energy. The left-hand side of (5) accounts for both the $+z$ and $-z$ propagating photons.

3. AN IDEAL PHOTODETECTOR

A photodetector can be constructed with a high density of ground-state atoms distributed in the $z=0$ plane. An ionization chamber is set up in such a way that, when triggered, it will ionize all electrons that are not in their ground states. This means that those electrons not ionized will be projected back to their ground states after the ionization. The triggering of the ionization is designed to occur periodically with time intervals $T \equiv l_L/c$. As this happens, the ionization current will be made up of bursts of electrons. The number of excited electrons at the end of each period is again governed by (5). However, the atom-field energy term is negligible. This follows because, before and after each period, the atoms are in the energy eigenstates, which have zero mean dipole moments. If we further assume that all the photon energy is absorbed at *unity-quantum efficiency*, then we have:

$$\sum_{jl} E_l \, \hat{n}_l^{(j)}(T) = \sum_m \hbar\omega_{Lm} \, \hat{N}_{l_L m}(-S_{Lm}\frac{l_L}{2}, 0). \qquad (6)$$

Equation (6) shows that such a detector does not, in general, detect photons. It only responds to photon energy. This is because of the non-zero probability for multi-photon transitions. However, such probability may be negligible at resonance. It can be shown that at a prominent resonant frequency ω_a such that $\omega_a = \omega_{Lm}$, (6) can be reduced to: $\sum_j \hat{n}_a^{(j)}(T) = \sum_{k_{Lm}c=\pm\omega_a} \hat{N}_{l_L m}(-S_{Lm} l_L/2, 0)$, where $\hat{n}_a^{(j)}(t)$ is the upper-level occupation number operator. The derivation of (6) is based on the identity (5), which suffers from the problem that it cannot tell us whether the above assumption at unity quantum efficiency is appropriate or not. However, $\hat{n}_l^{(j)}(T)$ can also be solved perturbatively in terms of operators at t=0. The perturbative approach supports the above-mentioned assumption and will be described elsewhere.

Because of the existence of $\{\hat{a}_n^\dagger\}$ terms in $\hat{b}_{l_L m}$, the vacuum expectation value of $\hat{N}_{l_L m}(0)$: $< 0|\hat{N}_{l_L m}(0)|0 >$, is not zero and is about 0.075 for $m = 1$. It is approximately inversely proportional to l_L/c at a fixed ω_{Lm}. It can be shown that the atom-field interaction time T basically limits the response bandwidth of the detector to the radio-frequency (RF) modulations on an optical beam. Its inverse: $1/T$, shall be referred to as the intrinsic RF bandwidth (IRFBW). The results here imply that a detector with a finite IRFBW may give a small but non-zero mean photocurrent in the vacuum.

The LPAO given by (1) relates the photon annihilation operators to a local spatial integration over the $+z$ and $-z$ propagating parts of the vector-potential operator. This is perhaps more satisfying[4] than the usual way of relating the photon annihilation operators to the negative-frequency part of the field. As the negative-frequency part can be isolated only through infinite-time integration, it is physically absurd to expect any finite-time measuring process to be able to "see" the negative-frequency part.

The concept of photons as particles in this formulation is best seen not as an absolute entity but as dependent on the IRFBW of the detector that is used to detect them. In this formulation, the photons seen by the detector are well defined mathematically, and both the photons and their energy can be localized exactly in the same region of space.

This formulation gives a theoretical framework to describe localization of photons along their direction of propagation. The use of plane-wave modes is, however, not necessary. For example, one can use modes that have finite lateral dimensions, which will enable one to describe photons that are localized laterally. This formulation clearly shows that, as time progresses, a photon wave packet propagating in free space will not spread out along its direction of propagation. One would, however, expect it to spread out laterally, in a way according to the law of diffraction.

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REFERENCES

1. For a series of references, see D. Han, Y. S. Kim and M. E. Noz, Phys. Rev. A 35, 1682 (1987).

2. For references, see R. S. Bondurant, Phys. Rev. A 32, 2797 (1985).

3. It is not possible to use $\pm z$ propagating *field* operators as they will give an unbounded $< 0|\hat{N}_{l_L m}(0)|0 >$.

4. Actually, $\hat{A}(z,t,\pm)$ has a component that interacts non-causally with the atoms. However, $\hat{b}_{l_L m}$ $(m \gtrless 0)$ only depends on the component of $\hat{A}(z,t,\pm)$ that is causal (within the coarse-grain-time l_L/c).