Register Allocation, i

Overview & spilling

L1

```
    p ::= (label f ...)
    f ::= (label nat nat i ...)
    i ::= (w <- s)
        | (w <- (mem x n8))
        | ((mem x n8) <- s)
        | (w aop= t)
        | (w sop= sx)
        | (w sop= num)
        | (w <- t cmp t)
        | label
        | (goto label)
        | (cjump t cmp t label label)
        | (call u nat)
        | (call print 1)
        | (call allocate 2)
        | (call array-error 2)
        | (tail-call u nat0-6)
        | (return)

 aop= ::= += | -= | *= | &=
 sop= ::= <<= | >>=
  cmp ::= < | <= | =
    u ::= w | label
    t ::= x | num
    s ::= x | num | label
    x ::= w | rsp
    w ::= a | rax | rbx | rbp | r10 | r11 | r12 | r13 | r14 | r15
    a ::= rdi | rsi | rdx | sx | r8 | r9
   sx ::= rcx
label ::= sequence of chars matching #rx"^:[a-zA-Z_][a-zA-Z_0-9]*$"
```

L2

```
    p ::= (label f ...)
    f ::= (label nat nat i ...)
    i ::= (w <- s)
        | (w <- (mem x n8))
        | ((mem x n8) <- s)
        | (w aop= t)
        | (w sop= sx)
        | (w sop= num)
        | (w <- t cmp t)
        | label
        | (goto label)
        | (cjump t cmp t label label)
        | (call u nat)
        | (call print 1)
        | (call allocate 2)
        | (call array-error 2)
        | (tail-call u nat0-6)
        | (return)
        | (w <- (stack-arg n8))
 aop= ::= += | -= | *= | &=
 sop= ::= <<= | >>=
  cmp ::= < | <= | =
    u ::= w | label
    t ::= x | num
    s ::= x | num | label
    x ::= w | rsp
    w ::= a | rax | rbx | rbp | r10 | r11 | r12 | r13 | r14 | r15
    a ::= rdi | rsi | rdx | sx | r8 | r9
   sx ::= rcx | var
label ::= sequence of chars matching #rx"^:[a-zA-Z_][a-zA-Z_0-9]*$"
  var ::= sequence of chars matching #rx"^[a-zA-Z_][a-zA-Z_0-9-]*$"
```

L2 semantics: variables

L2 behaves just like L1, except that non-reg variables are function local, e.g.,

```
(define (f x)
  (+ (g x) 1))

(define (g x)
  (+ x 2))

(f 10)
```

⇒

```
(:m
 (:m 0 0
     (rdi <- 21)
     (tail-call :f 1))
 (:f 1 0
     (temp <- 2)
     ((mem rsp -8) <- :gret)
     (call :g 1) :gret
     (rax += temp)
     (return))
 (:g 1 0
     (temp <- 4)
     (rax <- rdi)
     (rax += temp)
     (return)))
```

The assignment to `temp` in `g` does not break `f`, but if `temp` were a register, it would.

L2 semantics: stack-arg

L2 has a convenience for accessing stack-based arguments: **`(stack-arg n8)`**. It is equivalent to **`(mem rsp ?)`** where the **`?`** is the **`n8`**, plus enough space for the spills. That is, **`(stack-arg 0)`** is always the last stack argument, **`(stack-arg 8)`** is always the second to last argument, etc.

This means that if the second natural number in the function header changes, then the **`stack-arg`** references don't have to change – they will still be referring to the same arguments

From L2 to L1

Register allocation, in three parts; for each function body we do:

- **Liveness analysis** ⇒ interference graph (nodes are variables; edges indicate "cannot be in the same register")
- **Graph coloring** ⇒ register assignments
- **Spilling:** coping with too few registers
- Bonus part, **coalescing** eliminating redundant `(x <- y)` instructions

Example Function

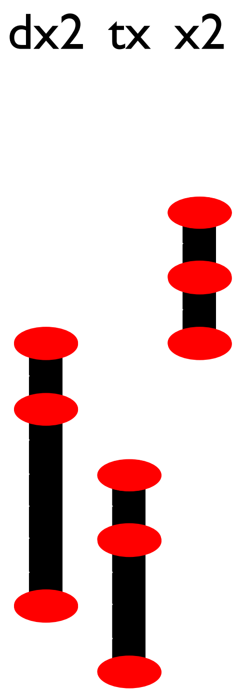
int f(int x) = $2x^2$ + 3x + 4

```
(:f
 1 0
 (x2 <- rdi)
 (x2 *= x2)
 (dx2 <- x2)
 (dx2 *= 2)
 (tx <- rdi)
 (tx *= 3)
 (rax <- dx2)
 (rax += tx)
 (rax += 4)
 (return))
```

Example Function: live ranges

```
int f(int x) = 2x² + 3x + 4
```

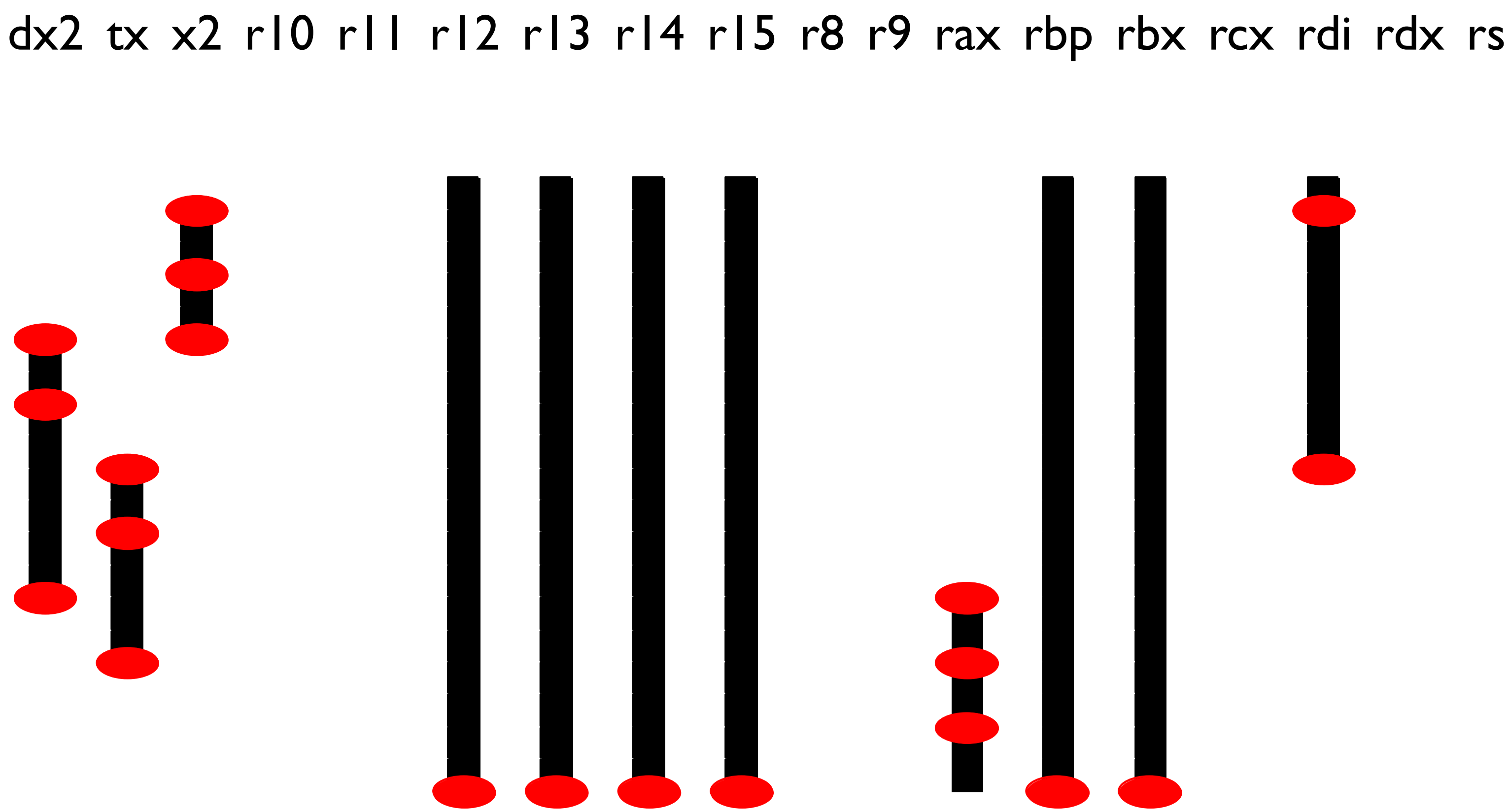
```
(:f
 1 0
 (x2 <- rdi)
 (x2 *= x2)
 (dx2 <- x2)
 (dx2 *= 2)
 (tx <- rdi)
 (tx *= 3)
 (rax <- dx2)
 (rax += tx)
 (rax += 4)
 (return))
```



Example Function: live ranges

int f(int x) = $2x^2 + 3x + 4$

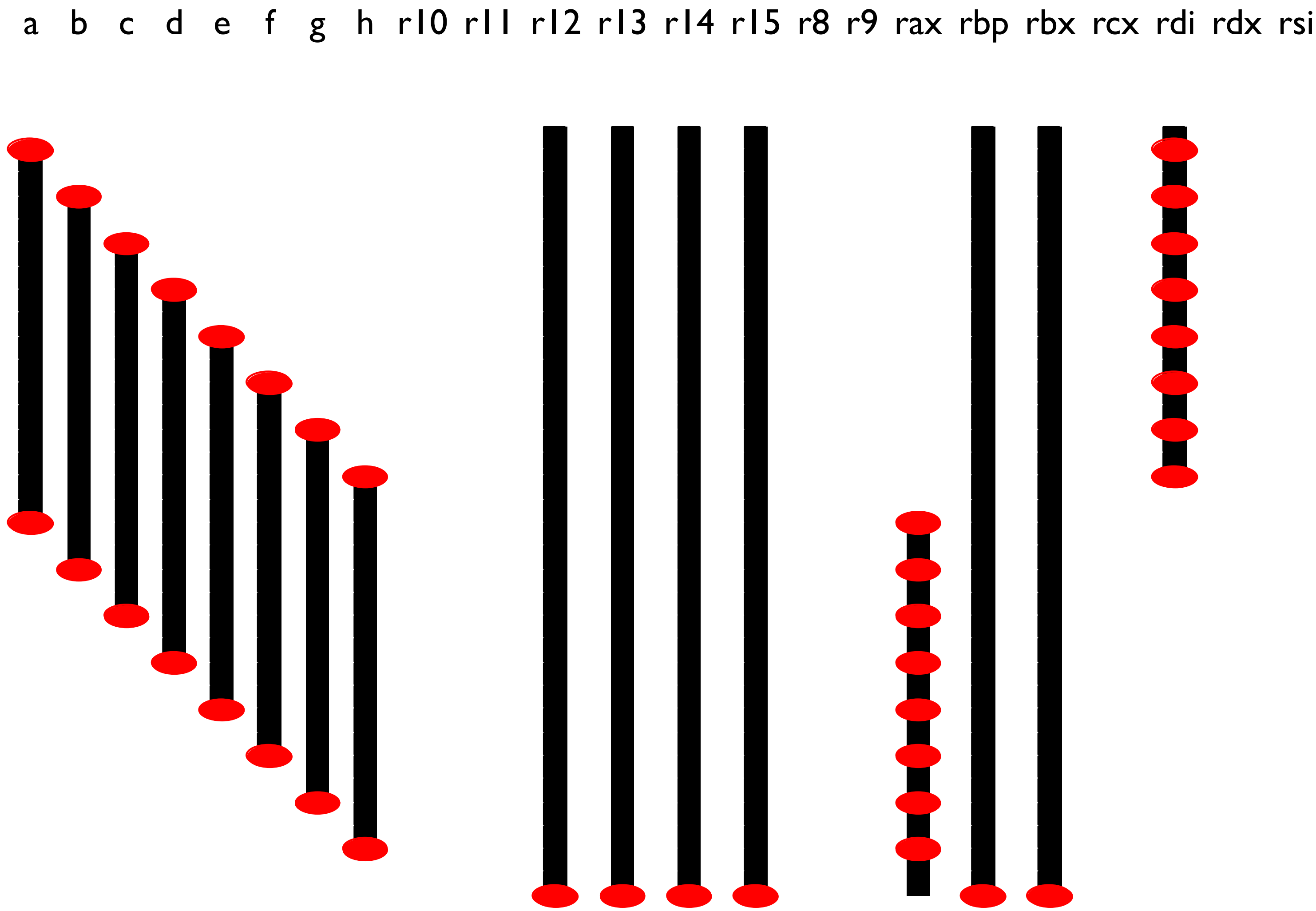
```
(:f
 1 0
 (x2 <- rdi)
 (x2 *= x2)
 (dx2 <- x2)
 (dx2 *= 2)
 (tx <- rdi)
 (tx *= 3)
 (rax <- dx2)
 (rax += tx)
 (rax += 4)
 (return))
```



Example Function 2

`int f(int x) = x+x+x+x+x+x+x+x` (in a stupid compiler)

```
(:f
 1 0
 (a <- rdi)
 (b <- rdi)
 (c <- rdi)
 (d <- rdi)
 (e <- rdi)
 (f <- rdi)
 (g <- rdi)
 (h <- rdi)
 (rax <- a)
 (rax += b)
 (rax += c)
 (rax += d)
 (rax += e)
 (rax += f)
 (rax += g)
 (rax += h)
 (return))
```



No way to get all of **`a`**, **`b`**, **`c`**, **`d`**, **`e`**, **`f`**, **`g`**, and **`h`** into their own registers; so we need to *spill* one of them.

Spilling

Spilling is a program rewrite to make it easier to allocate registers

- Pick a variable
- Make a new location on the stack (increment the second `nat` in the function definition)
- Replace all writes to the variable with writes to the new stack location
- Replace all reads from the variable with reads from the new stack location

Sometimes that means introducing new temporaries

Spilling Example

Say we want to spill `a` to first location on the stack, `(mem rsp 0)`; two easy cases:

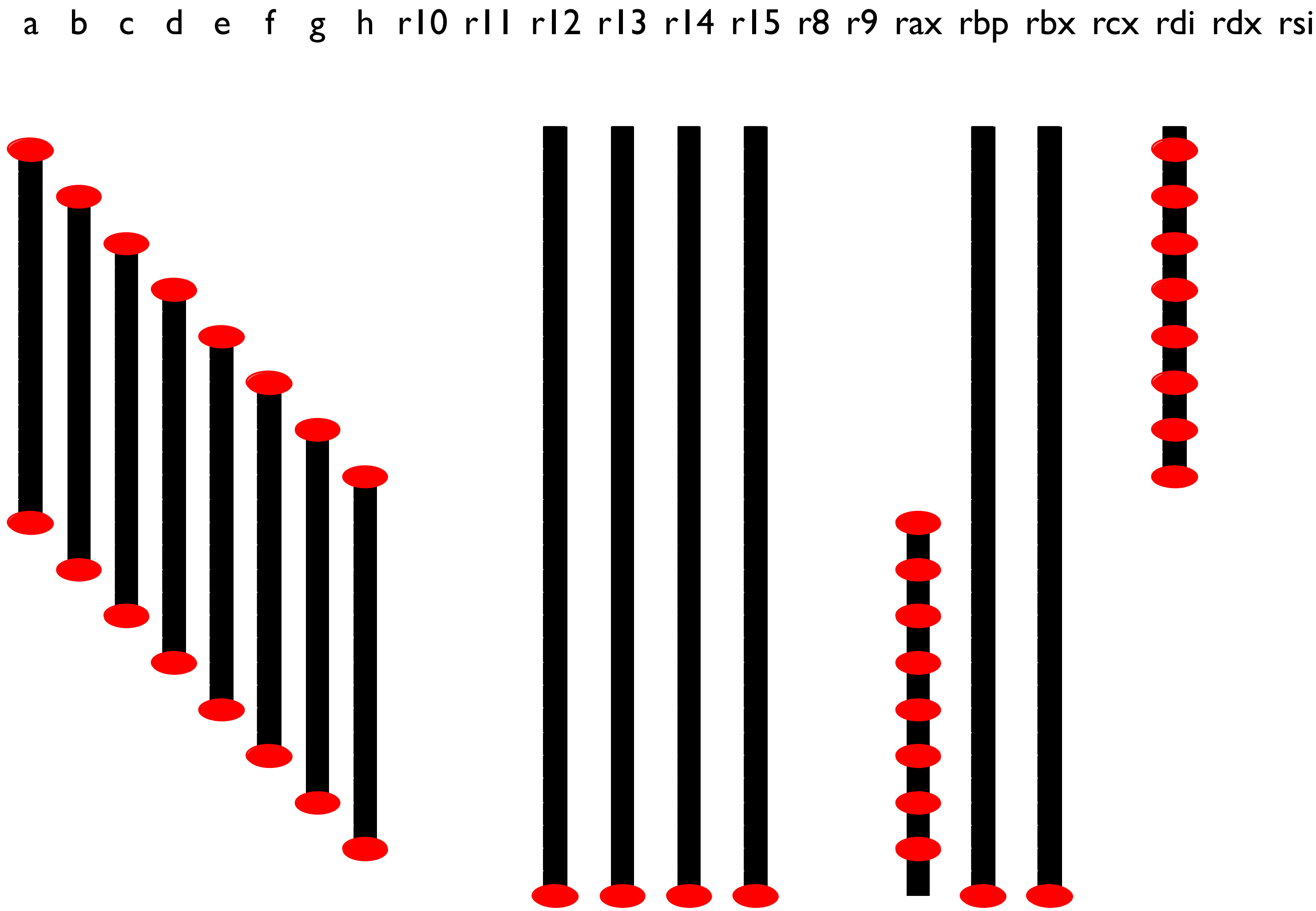
```
(a <- 1)  ⇒  ((mem rsp 0) <- 1)

(x <- a)  ⇒  (x <- (mem rsp 0))
```

Example Function 2, need to spill

`int f(int x) = x+x+x+x+x+x+x+x` (in a stupid compiler)

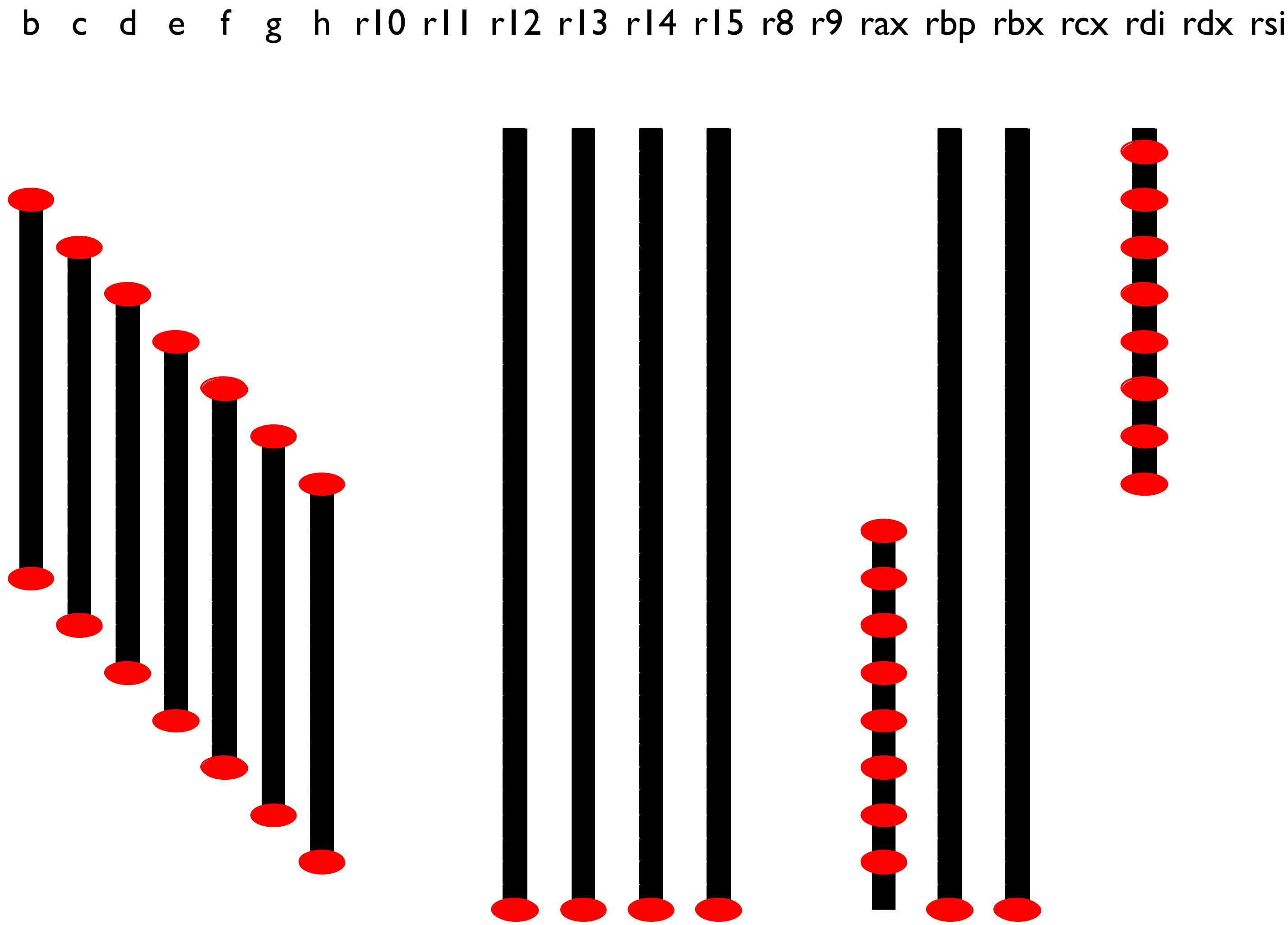
```
(:f
 1 0
 (a <- rdi)
 (b <- rdi)
 (c <- rdi)
 (d <- rdi)
 (e <- rdi)
 (f <- rdi)
 (g <- rdi)
 (h <- rdi)
 (rax <- a)
 (rax += b)
 (rax += c)
 (rax += d)
 (rax += e)
 (rax += f)
 (rax += g)
 (rax += h)
 (return))
```



Example Function 2, spilling a

`int f(int x) = x+x+x+x+x+x+x+x` (in a stupid compiler)

```
(:f
 1 1
 ((mem rsp 0) <- rdi)
 (b <- rdi)
 (c <- rdi)
 (d <- rdi)
 (e <- rdi)
 (f <- rdi)
 (g <- rdi)
 (h <- rdi)
 (rax <- (mem rsp 0))
 (rax += b)
 (rax += c)
 (rax += d)
 (rax += e)
 (rax += f)
 (rax += g)
 (rax += h)
 (return))
```



Spilling Example

A trickier case:

```
(a *= a)  ⇒  (a_new <- (mem rsp 0))
             (a_new *= a_new)
             ((mem rsp 0) <- a_new)
```

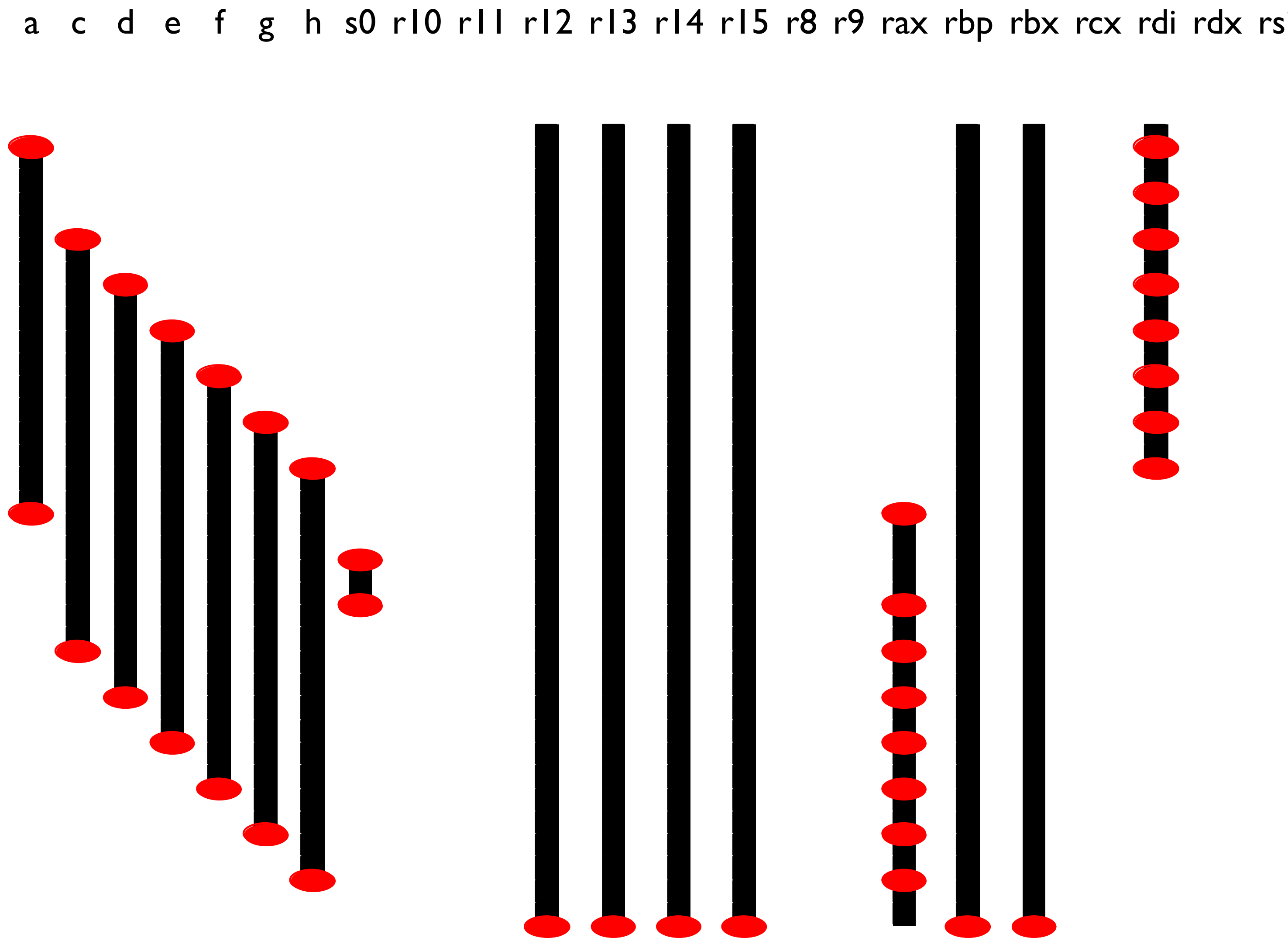
In general, make up a new temporary for each instruction that uses the variable to be spilled

This makes for very short live ranges.

Example Function 2, spilling b

`int f(int x) = x+x+x+x+x+x+x+x` (in a stupid compiler)

```
(:f
 1 1
 (a <- rdi)
 ((mem rsp 0) <- rdi)
 (c <- rdi)
 (d <- rdi)
 (e <- rdi)
 (f <- rdi)
 (g <- rdi)
 (h <- rdi)
 (rax <- a)
 (s0 <- (mem rsp 0))
 (rax += s0)
 (rax += c)
 (rax += d)
 (rax += e)
 (rax += f)
 (rax += g)
 (rax += h)
 (return))
```



Example Function 2, spilling b

Even though we still have eight temporaries, we can still allocate them to our seven unused registers because the live ranges of `s0` and `a` don't overlap and so they can go into the same register.

Your job

Implement:

```
spill : (label nat nat i ...) ;; original func
        var                   ;; to spill
        var                   ;; prefix for temporaries
     -> (label nat nat i ...) ;; spilled func
```

Here’s how to two example spilled functions from the earlier slides would look like as calls to spill:

```
(spill «the original program»
       'a
       's)

(spill «the original program»
       'b
       's)
```

See the assignment handout for more details on the precise spec for test cases and your spill function’s interface