

# Inference rules

$$\frac{A \quad B}{C}$$

This is a *rule*, which says: If I know *A* and *B*, then I can conclude *C* (could be more than just two things above the bar)

Terminology: *A* and *B* are *premises* *C* is the *conclusion*

# Inference Rule Example: Logical Or

$$\frac{A}{A \vee B}$$

$$\frac{B}{A \vee B}$$

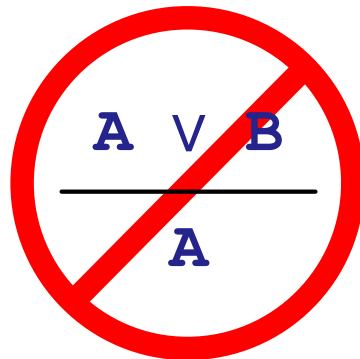
$$\frac{A \vee B}{A}$$

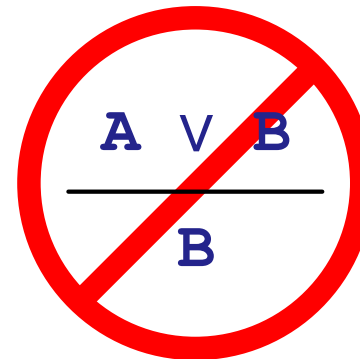
$$\frac{A \vee B}{B}$$

# Inference Rule Example: Logical Or

$$\frac{A}{A \vee B}$$

$$\frac{B}{A \vee B}$$


$$\frac{A \vee B}{A}$$


$$\frac{A \vee B}{B}$$

# Type Relation

We are defining a *type judgment*

$$\Gamma \vdash \mathbf{e} : \tau$$

Which means: with the type bindings in  $\Gamma$ , I can conclude that  $\mathbf{e}$  has the type  $\tau$

$\Gamma$  is the *type environment*: a map from  $\langle \mathbf{id} \rangle$  to  $\tau$  (type)

# Type Rules

$\Gamma \vdash \langle \text{num} \rangle : \text{num}$

$\Gamma \vdash \text{true} : \text{bool}$

$\Gamma \vdash \text{false} : \text{bool}$

$$\frac{\Gamma \vdash \mathbf{e}_1 : \text{num} \quad \Gamma \vdash \mathbf{e}_2 : \text{num}}{\Gamma \vdash \{+ \mathbf{e}_1 \ \mathbf{e}_2\} : \text{num}}$$

# Type Rules

$\Gamma \vdash \langle \text{num} \rangle : \text{num}$

$\Gamma \vdash \text{true} : \text{bool}$

$\Gamma \vdash \text{false} : \text{bool}$

$\Gamma \vdash \mathbf{e}_1 : \text{num} \quad \Gamma \vdash \mathbf{e}_2 : \text{num}$

---

$\Gamma \vdash \{+ \mathbf{e}_1 \ \mathbf{e}_2\} : \text{num}$

$1 : \text{num}$

$\text{true} : \text{bool}$

# Type Rules

$\Gamma \vdash \langle \text{num} \rangle : \text{num}$

$\Gamma \vdash \text{true} : \text{bool}$

$\Gamma \vdash \text{false} : \text{bool}$

$\Gamma \vdash e_1 : \text{num}$

$\Gamma \vdash e_2 : \text{num}$

---

$\Gamma \vdash \{+ e_1 e_2\} : \text{num}$

$1 : \text{num}$

$\text{true} : \text{bool}$

$1 : \text{num}$

$2 : \text{num}$

---

$\{+ 1 2\} : \text{num}$

# Type Rules

$\Gamma \vdash \langle \text{num} \rangle : \text{num}$

$\Gamma \vdash \text{true} : \text{bool}$

$\Gamma \vdash \text{false} : \text{bool}$

$\Gamma \vdash e_1 : \text{num}$

$\Gamma \vdash e_2 : \text{num}$

---

$\Gamma \vdash \{+ e_1 e_2\} : \text{num}$

$1 : \text{num}$

$\text{true} : \text{bool}$

$1 : \text{num}$

$2 : \text{num}$

---

$\{+ 1 2\} : \text{num}$

$1 : \text{num}$

$\text{false} : \text{bool}$

---

$\{+ 1 \text{false}\} : \text{no type}$



# Type Rules

$\Gamma \vdash \langle \text{num} \rangle : \text{num}$

$\Gamma \vdash \text{true} : \text{bool}$

$\Gamma \vdash \text{false} : \text{bool}$

$$\frac{\Gamma \vdash e_1 : \text{num} \quad \Gamma \vdash e_2 : \text{num}}{\Gamma \vdash \{+ e_1 e_2\} : \text{num}}$$
$$\frac{\frac{1 : \text{num} \quad 2 : \text{num}}{\{+ 1 2\} : \text{num}} \quad 3 : \text{num}}{\{+ \{+ 1 2\} 3\} : \text{num}}$$

# Types: Conditionals

```
{if true 1 2}
```

# Types: Conditionals

```
{if true 1 2}
```

*bool*

# Types: Conditionals

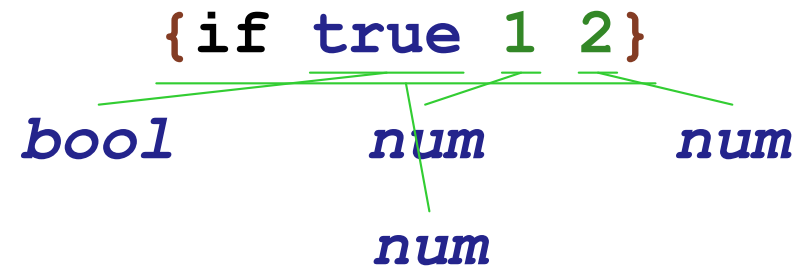
```
{if true 1 2}
```

*bool*                      *num*

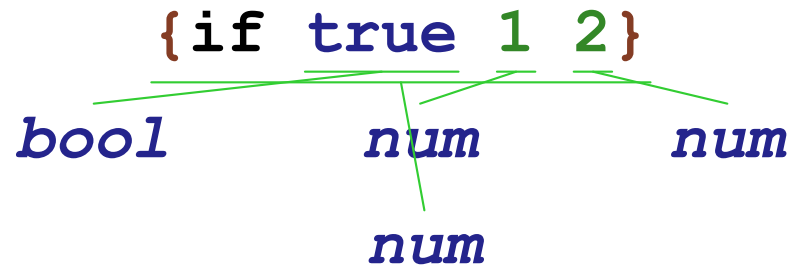
# Types: Conditionals



# Types: Conditionals



# Types: Conditionals



`{if {+ 1 2} 1 2}`

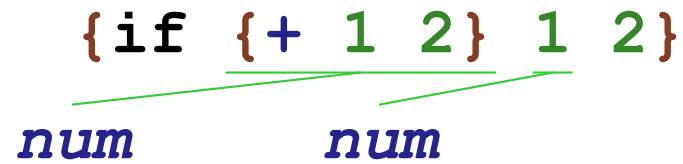
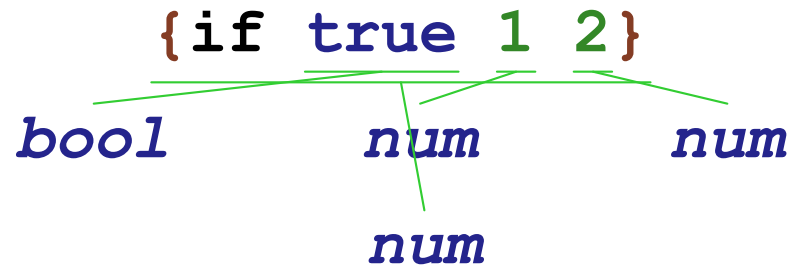
# Types: Conditionals

```
bool      num      num  
            
num  
{if true 1 2}
```

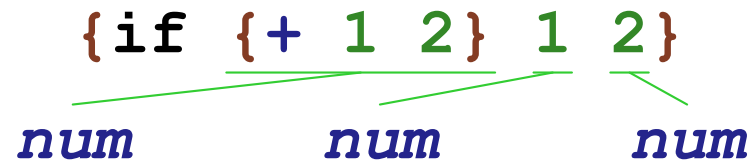
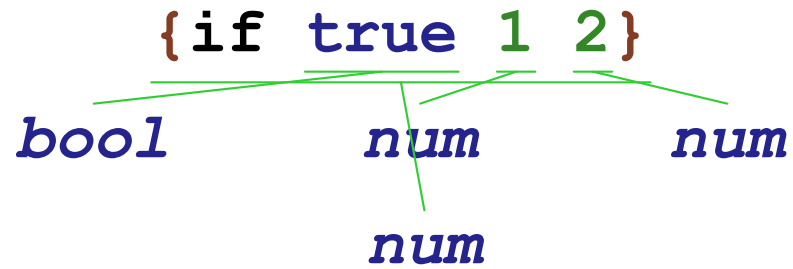
```
num  
            
{if {+ 1 2} 1 2}
```



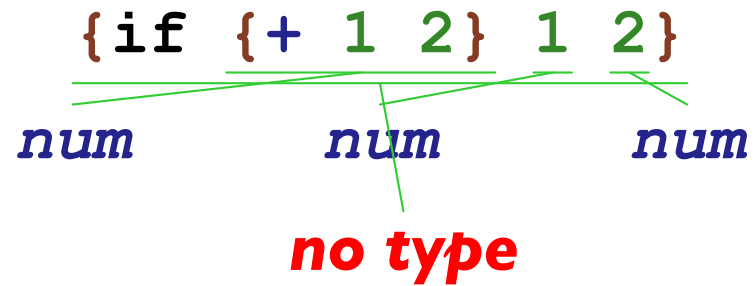
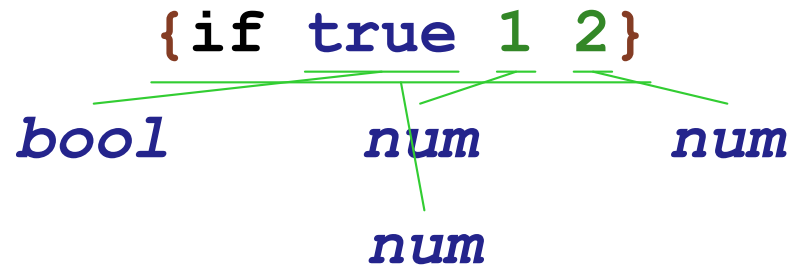
# Types: Conditionals



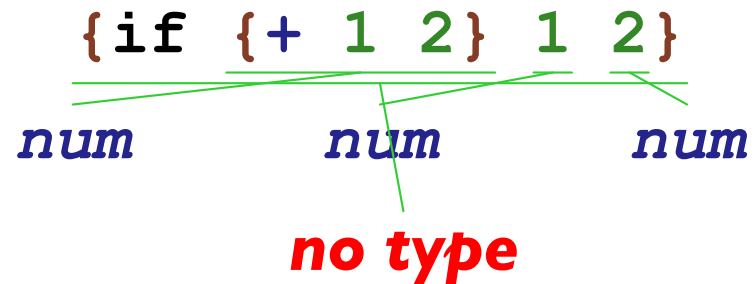
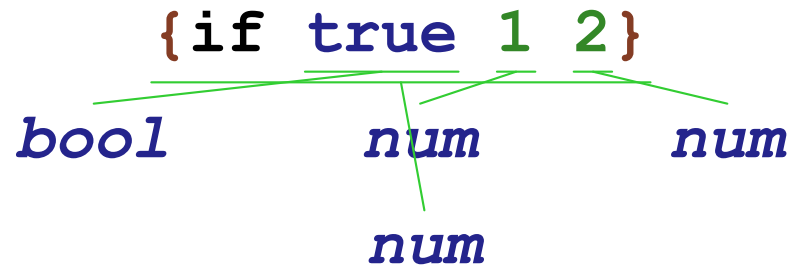
# Types: Conditionals



# Types: Conditionals

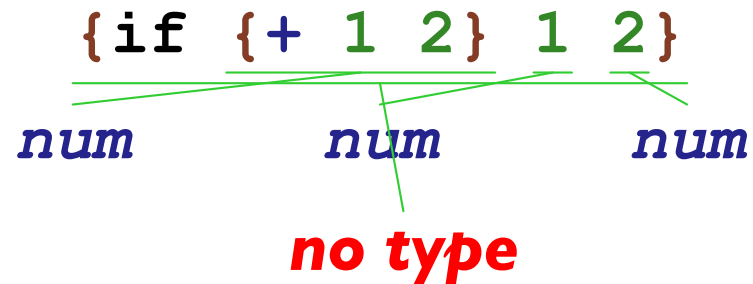
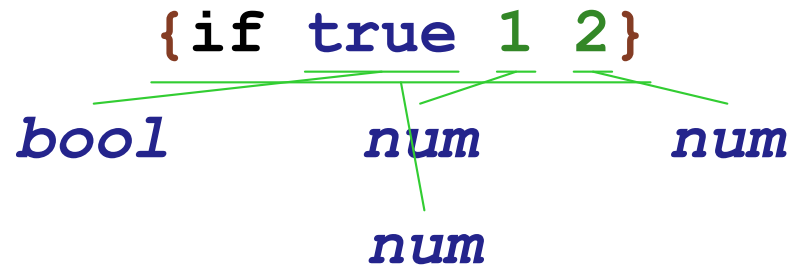


# Types: Conditionals

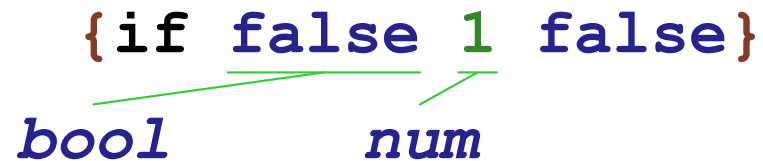
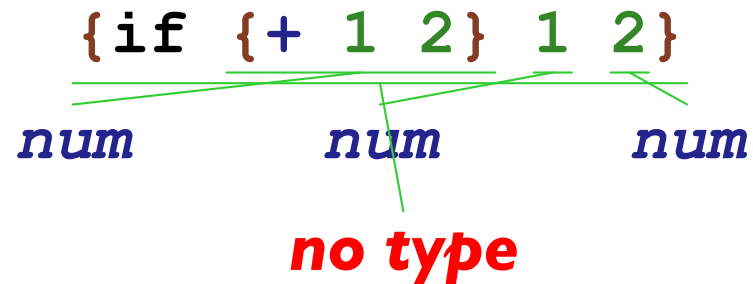
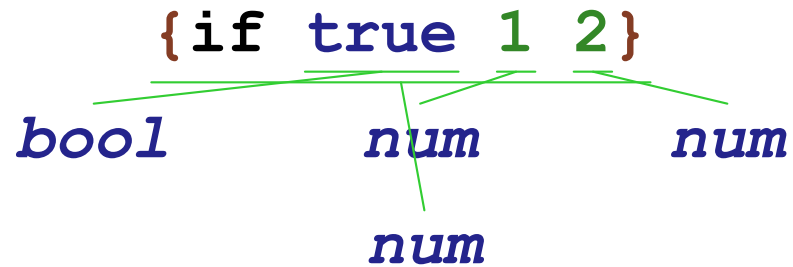


`{if false 1 false}`

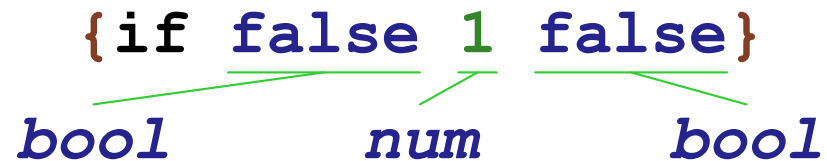
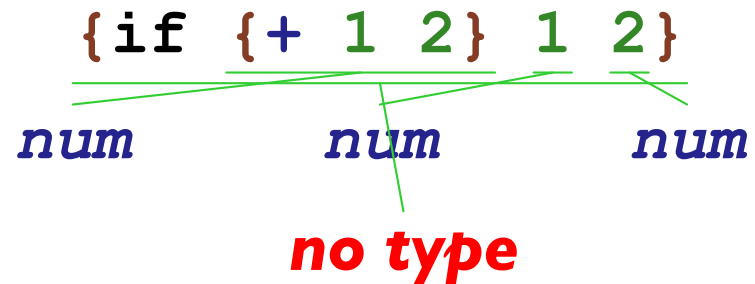
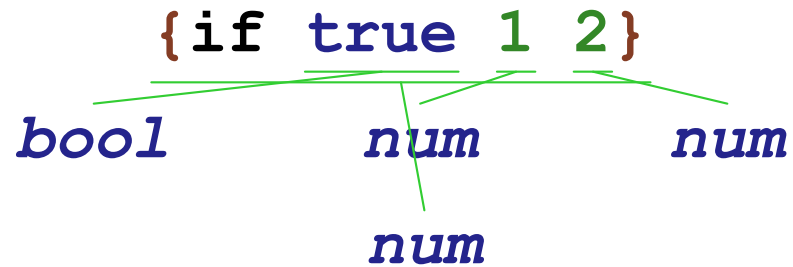
# Types: Conditionals



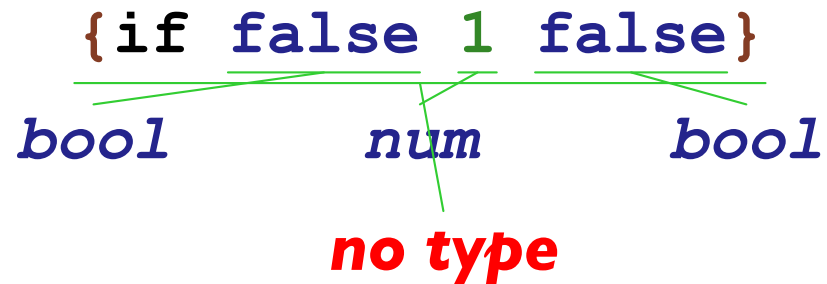
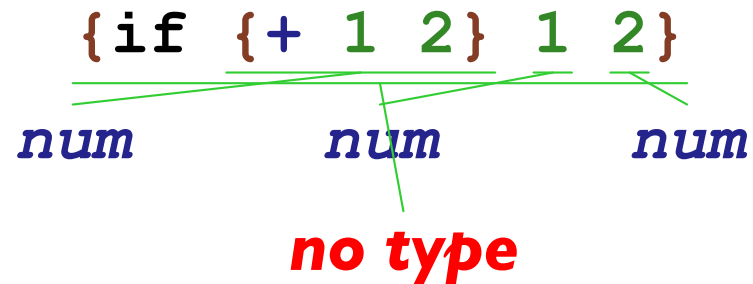
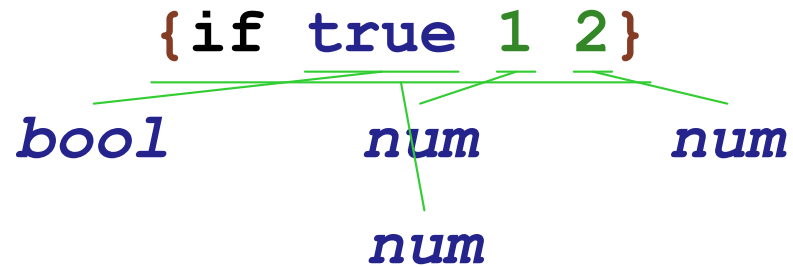
# Types: Conditionals



# Types: Conditionals



# Types: Conditionals





# Conditional Type Rules

$$\frac{\Gamma \vdash \mathbf{e}_1 : \mathit{bool} \quad \Gamma \vdash \mathbf{e}_2 : \tau_0 \quad \Gamma \vdash \mathbf{e}_3 : \tau_0}{\Gamma \vdash \{\mathbf{if} \ \mathbf{e}_1 \ \mathbf{e}_2 \ \mathbf{e}_3\} : \tau_0}$$

# Conditional Type Rules

$$\frac{\Gamma \vdash \mathbf{e}_1 : \mathit{bool} \quad \Gamma \vdash \mathbf{e}_2 : \tau_0 \quad \Gamma \vdash \mathbf{e}_3 : \tau_0}{\Gamma \vdash \{\mathbf{if} \ \mathbf{e}_1 \ \mathbf{e}_2 \ \mathbf{e}_3\} : \tau_0}$$

$$\frac{\mathit{true} : \mathit{bool} \quad 1 : \mathit{num} \quad 2 : \mathit{num}}{\{\mathbf{if} \ \mathit{true} \ 1 \ 2\} : \mathit{num}}$$

# Conditional Type Rules

$$\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \tau_0 \quad \Gamma \vdash e_3 : \tau_0$$

---

$$\Gamma \vdash \{\text{if } e_1 \ e_2 \ e_3\} : \tau_0$$
$$\text{true} : \text{bool} \quad 1 : \text{num} \quad 2 : \text{num}$$

---

$$\{\text{if } \text{true} \ 1 \ 2\} : \text{num}$$
$$\{+ \ 1 \ 2\} : \text{num} \quad 1 : \text{num} \quad 2 : \text{num}$$

---

$$\{\text{if } \{+ \ 1 \ 2\} \ 1 \ 2\} : \text{no type}$$

# Conditional Type Rules

$$\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \tau_0 \quad \Gamma \vdash e_3 : \tau_0$$

---

$$\Gamma \vdash \{\text{if } e_1 \ e_2 \ e_3\} : \tau_0$$
$$\text{true} : \text{bool} \quad 1 : \text{num} \quad 2 : \text{num}$$

---

$$\{\text{if true } 1 \ 2\} : \text{num}$$
$$\{+ \ 1 \ 2\} : \text{num} \quad 1 : \text{num} \quad 2 : \text{num}$$

---

$$\{\text{if } \{+ \ 1 \ 2\} \ 1 \ 2\} : \text{no type}$$
$$\text{false} : \text{bool} \quad 1 : \text{num} \quad \text{false} : \text{bool}$$

---

$$\{\text{if false } 1 \ \text{false}\} : \text{no type}$$

# Types: Variables and Functions

**x** : **no type**

# Types: Variables and Functions

`x : no type`

`{fun {x : bool} x}`

# Types: Variables and Functions

**x** : **no type**

```
{fun {x : bool} x}
      bool
```

# Types: Variables and Functions

**x : no type**

`{fun {x : bool} x}`

---

`bool`

`(bool → bool)`



# Types: Variables and Functions

**x : no type**

`{fun {x : bool} x}`

---

`bool`

`(bool → bool)`

`{fun {x : bool} {if x 1 2}}`

# Types: Variables and Functions

**x : no type**

```
{fun {x : bool} x}
```

---

*bool*

*(bool → bool)*

```
{fun {x : bool} {if x 1 2}}
```

---

*bool*

# Types: Variables and Functions

**x : no type**

`{fun {x : bool} x}`

*bool*

$(bool \rightarrow bool)$

`{fun {x : bool} {if x 1 2}}`

*bool*

*num*

# Types: Variables and Functions

**x : no type**

`{fun {x : bool} x}`

*bool*

$(bool \rightarrow bool)$

`{fun {x : bool} {if x 1 2}}`

*bool*

*num*

*num*

# Types: Variables and Functions

**x : no type**

```
{fun {x : bool} x}
```

*bool*

*(bool → bool)*

```
{fun {x : bool} {if x 1 2}}
```

*bool*

*num*

*num*

*num*

# Types: Variables and Functions

**x : no type**

`{fun {x : bool} x}`

*bool*

*(bool → bool)*

`{fun {x : bool} {if x 1 2}}`

*bool*

*num*

*num*

*num*

*(bool → num)*

# Variable and Function Type Rules

$$[ \dots \langle \text{id} \rangle \leftarrow \tau \dots ] \vdash \langle \text{id} \rangle : \tau$$
$$\Gamma [ \langle \text{id} \rangle \leftarrow \tau_1 ] \vdash \mathbf{e} : \tau_2$$

---

$$\Gamma \vdash \{ \text{fun } \{ \langle \text{id} \rangle : \tau_1 \} \mathbf{e} \} : (\tau_1 \rightarrow \tau_2)$$
$$\emptyset \vdash \mathbf{x} : \text{no type}$$

# Variable and Function Type Rules

$$[ \dots \langle \text{id} \rangle \leftarrow \tau \dots ] \vdash \langle \text{id} \rangle : \tau$$

$$\Gamma [ \langle \text{id} \rangle \leftarrow \tau_1 ] \vdash \mathbf{e} : \tau_2$$

---

$$\Gamma \vdash \{ \mathbf{fun} \{ \langle \text{id} \rangle : \tau_1 \} \mathbf{e} \} : (\tau_1 \rightarrow \tau_2)$$

$$\emptyset \vdash \mathbf{x} : \mathbf{no\ type}$$

$$[ \mathbf{x} \leftarrow \mathbf{bool} ] \vdash \mathbf{x} : \mathbf{bool}$$

---

$$\emptyset \vdash \{ \mathbf{fun} \{ \mathbf{x} : \mathbf{bool} \} \mathbf{x} \} : (\mathbf{bool} \rightarrow \mathbf{bool})$$



# Variable and Function Type Rules

$$[ \dots \langle \text{id} \rangle \leftarrow \tau \dots ] \vdash \langle \text{id} \rangle : \tau$$
$$\Gamma [ \langle \text{id} \rangle \leftarrow \tau_1 ] \vdash \mathbf{e} : \tau_2$$

---

$$\Gamma \vdash \{ \text{fun } \{ \langle \text{id} \rangle : \tau_1 \} \mathbf{e} \} : (\tau_1 \rightarrow \tau_2)$$
$$\emptyset \vdash \mathbf{x} : \text{no type}$$
$$[ \mathbf{x} \leftarrow \text{bool} ] \vdash \mathbf{x} : \text{bool}$$

---

$$\emptyset \vdash \{ \text{fun } \{ \mathbf{x} : \text{bool} \} \mathbf{x} \} : (\text{bool} \rightarrow \text{bool})$$
$$[ \mathbf{x} \leftarrow \text{bool} ] \vdash \mathbf{x} : \text{bool} \quad [ \mathbf{x} \leftarrow \text{bool} ] \vdash \mathbf{1} : \text{num} \quad [ \mathbf{x} \leftarrow \text{bool} ] \vdash \mathbf{2} : \text{num}$$

---

$$[ \mathbf{x} \leftarrow \text{bool} ] \vdash \{ \text{if } \mathbf{x} \ \mathbf{1} \ \mathbf{2} \} : \text{num}$$

---

$$\emptyset \vdash \{ \text{fun } \{ \mathbf{x} : \text{bool} \} \{ \text{if } \mathbf{x} \ \mathbf{1} \ \mathbf{2} \} \} : (\text{bool} \rightarrow \text{num})$$

# Types: Function Calls

```
{{fun {x : bool} {if x 1 2}} true}
```

# Types: Function Calls

```
{{fun {x : bool} {if x 1 2}} true}  
  (bool → num)
```

# Types: Function Calls

{{fun {x : bool} {if x 1 2}} true}  
*(bool → num)*      *bool*

# Types: Function Calls

$\frac{\{\{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} \ \text{true}\}}{\text{num}}$

$(\text{bool} \rightarrow \text{num})$        $\text{bool}$

$\text{num}$

# Types: Function Calls

$\frac{\{\{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} \ \text{true}\}}{\text{num}}$

$(\text{bool} \rightarrow \text{num})$        $\text{bool}$

$\text{num}$

$\{\{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} \ 5\}$

# Types: Function Calls

{{fun {x : bool} {if x 1 2}} true}  
*(bool → num)      bool*  
*num*

{{fun {x : bool} {if x 1 2}} 5}  
*(bool → num)*

# Types: Function Calls

$\frac{\{\{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} \ \text{true}\}}{(\text{bool} \rightarrow \text{num}) \quad \text{bool}}$   
 $\text{num}$

$\frac{\{\{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} \ 5\}}{(\text{bool} \rightarrow \text{num}) \quad \text{num}}$



# Types: Function Calls

$\frac{\{\{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} \ \text{true}\}}{(\text{bool} \rightarrow \text{num}) \quad \text{bool}}$   
*num*

$\frac{\{\{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} \ 5\}}{(\text{bool} \rightarrow \text{num}) \quad \text{num}}$   
**no type**

# Types: Function Calls

{{fun {x : bool} {if x 1 2}} true}  
*(bool → num)*      *bool*  
*num*

{{fun {x : bool} {if x 1 2}} 5}  
*(bool → num)*      *num*  
**no type**

{7 5}

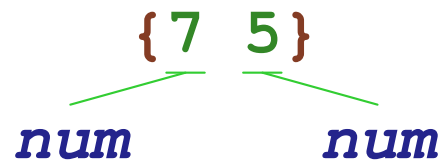
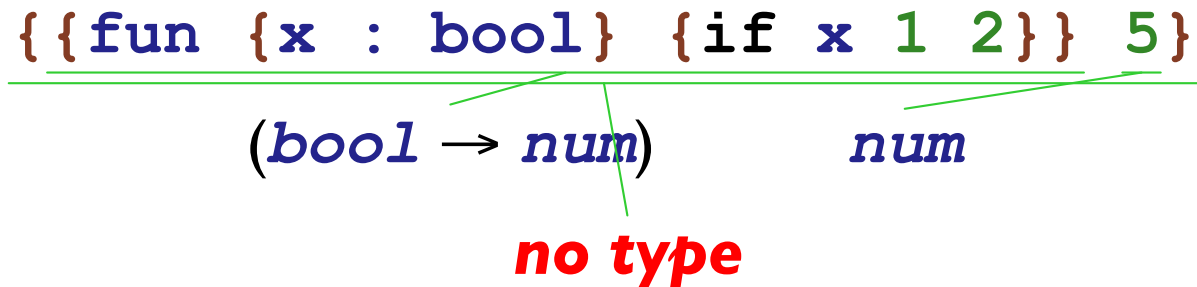
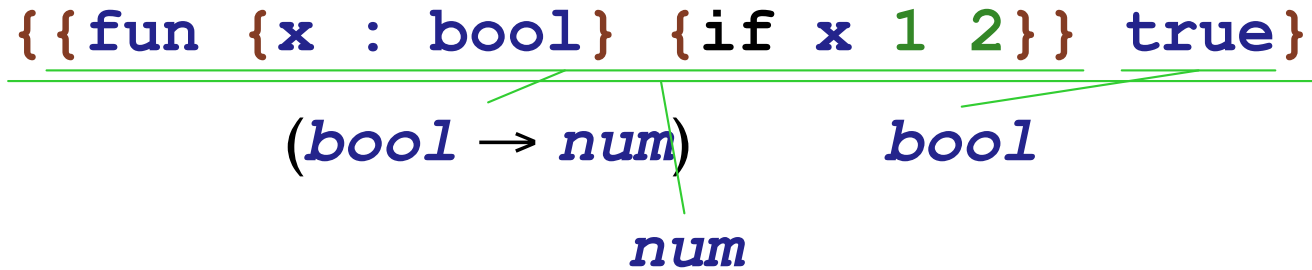
# Types: Function Calls

{{fun {x : bool} {if x 1 2}} true}  
*(bool → num)*      *bool*  
*num*

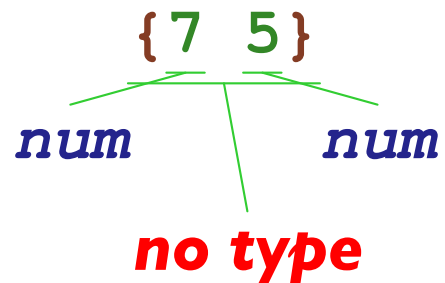
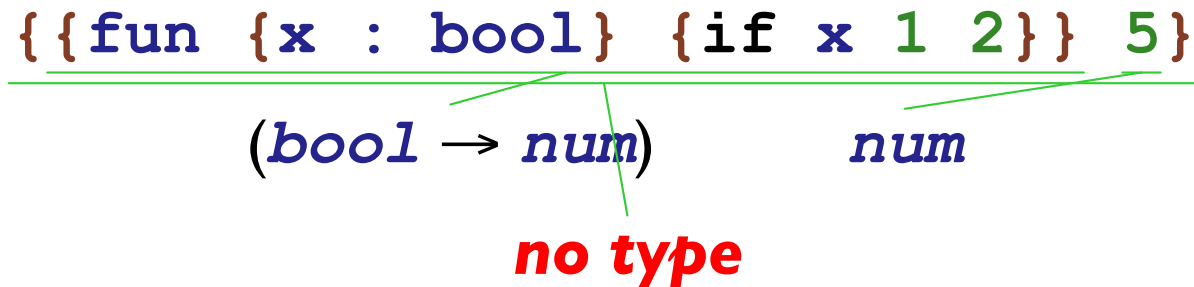
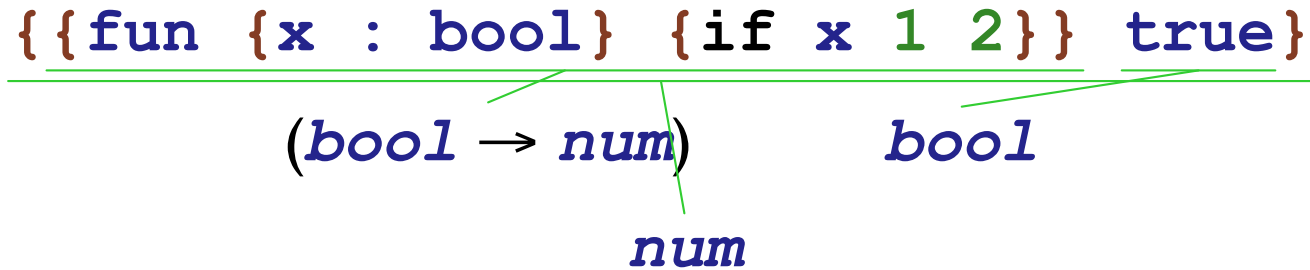
{{fun {x : bool} {if x 1 2}} 5}  
*(bool → num)*      *num*  
**no type**

*{7 5}*  
*num*

# Types: Function Calls



# Types: Function Calls



# Function Call Type Rule

$$\frac{\Gamma \vdash \mathbf{e}_1 : (\tau_2 \rightarrow \tau_3) \quad \Gamma \vdash \mathbf{e}_2 : \tau_2}{\Gamma \vdash \{\mathbf{e}_1 \ \mathbf{e}_2\} : \tau_3}$$

# Function Call Type Rule

$$\frac{\Gamma \vdash \mathbf{e}_1 : (\tau_2 \rightarrow \tau_3) \quad \Gamma \vdash \mathbf{e}_2 : \tau_2}{\Gamma \vdash \{\mathbf{e}_1 \ \mathbf{e}_2\} : \tau_3}$$

$$\frac{\emptyset \vdash \{\text{fun } \{\mathbf{x} : \text{bool}\} \{\text{if } \mathbf{x} \ 1 \ 2\}\} : (\text{bool} \rightarrow \text{num}) \quad \emptyset \vdash \text{true} : \text{bool}}{\emptyset \vdash \{\{\text{fun } \{\mathbf{x} : \text{bool}\} \{\text{if } \mathbf{x} \ 1 \ 2\}\} \ \text{true}\} : \text{num}}$$

# Function Call Type Rule

$$\frac{\Gamma \vdash e_1 : (\tau_2 \rightarrow \tau_3) \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash \{e_1 \ e_2\} : \tau_3}$$

$$\frac{\emptyset \vdash \{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} : (\text{bool} \rightarrow \text{num}) \quad \emptyset \vdash \text{true} : \text{bool}}{\emptyset \vdash \{\{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} \ \text{true}\} : \text{num}}$$

$$\frac{\emptyset \vdash \{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} : (\text{bool} \rightarrow \text{num}) \quad \emptyset \vdash 5 : \text{num}}{\emptyset \vdash \{\{\text{fun } \{x : \text{bool}\} \{\text{if } x \ 1 \ 2\}\} \ 5\} : \text{no type}}$$



# Function Call Type Rule

$$\frac{\Gamma \vdash \mathbf{e}_1 : (\tau_2 \rightarrow \tau_3) \quad \Gamma \vdash \mathbf{e}_2 : \tau_2}{\Gamma \vdash \{\mathbf{e}_1 \ \mathbf{e}_2\} : \tau_3}$$

$$\frac{\emptyset \vdash \{\mathbf{fun} \ \{\mathbf{x} : \mathbf{bool}\} \ \{\mathbf{if} \ \mathbf{x} \ \mathbf{1} \ \mathbf{2}\}\} : (\mathbf{bool} \rightarrow \mathbf{num}) \quad \emptyset \vdash \mathbf{true} : \mathbf{bool}}{\emptyset \vdash \{\{\mathbf{fun} \ \{\mathbf{x} : \mathbf{bool}\} \ \{\mathbf{if} \ \mathbf{x} \ \mathbf{1} \ \mathbf{2}\}\} \ \mathbf{true}\} : \mathbf{num}}$$

$$\frac{\emptyset \vdash \{\mathbf{fun} \ \{\mathbf{x} : \mathbf{bool}\} \ \{\mathbf{if} \ \mathbf{x} \ \mathbf{1} \ \mathbf{2}\}\} : (\mathbf{bool} \rightarrow \mathbf{num}) \quad \emptyset \vdash \mathbf{5} : \mathbf{num}}{\emptyset \vdash \{\{\mathbf{fun} \ \{\mathbf{x} : \mathbf{bool}\} \ \{\mathbf{if} \ \mathbf{x} \ \mathbf{1} \ \mathbf{2}\}\} \ \mathbf{5}\} : \mathbf{no \ type}}$$

$$\frac{\emptyset \vdash \mathbf{7} : \mathbf{num} \quad \emptyset \vdash \mathbf{5} : \mathbf{num}}$$

$$\emptyset \vdash \{\mathbf{7} \ \mathbf{5}\} : \mathbf{no \ type}$$

# Types: Multiple Arguments

```
{ fun {x : num y : num} {+ x y} }
```

# Types: Multiple Arguments

```
{ fun {x : num y : num} {+ x y}}
```

*num*



# Types: Multiple Arguments

```
{ fun {x : num y : num} {+ x y}}
```

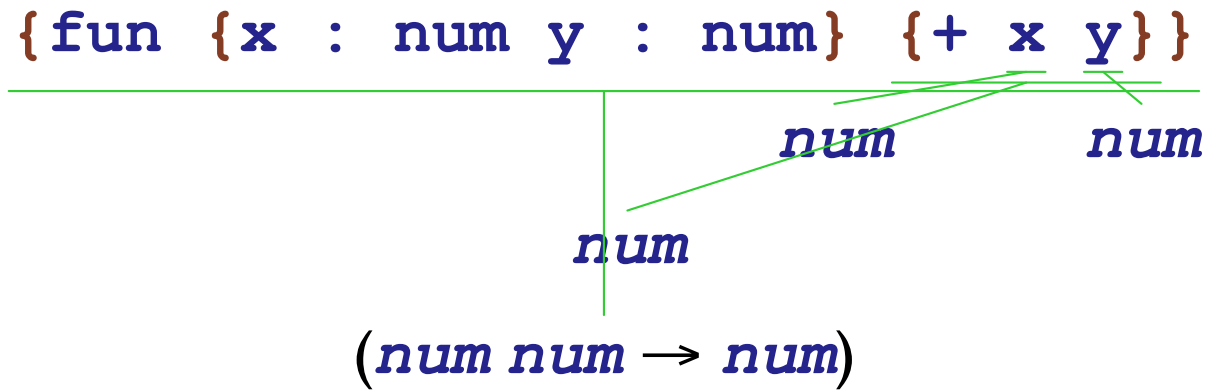
*num*                      *num*

# Types: Multiple Arguments

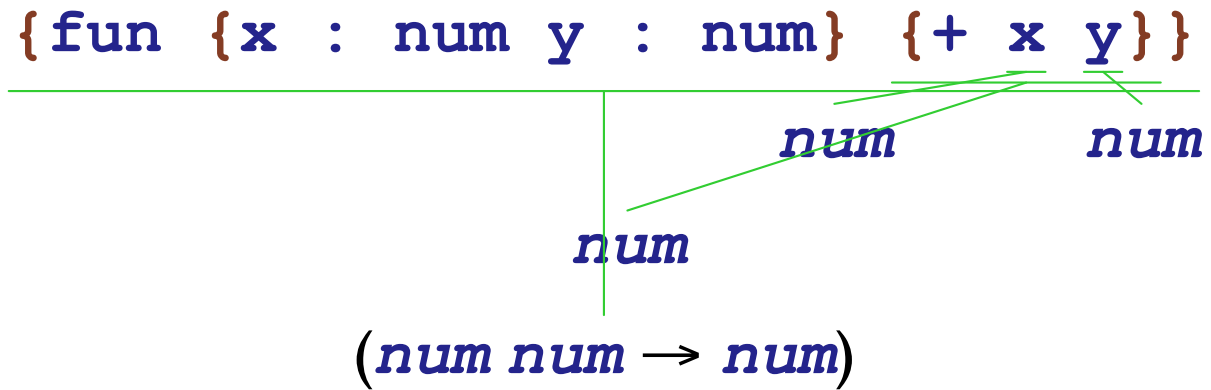
```
{ fun {x : num y : num} {+ x y}}
```

*num*                      *num*

# Types: Multiple Arguments



# Types: Multiple Arguments



`{{fun {x : num y : num} {+ x y}} 5 6}`

# Types: Multiple Arguments

$\frac{\text{\{fun \{x : num y : num\} \{+ x y\}\}}{\text{\{num num \rightarrow num\}}$

$\frac{\text{\{\{fun \{x : num y : num\} \{+ x y\}\} 5 6\}}{\text{\{num num \rightarrow num\}}$



# Types: Multiple Arguments

$\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\}$

---

$\text{num}$        $\text{num}$

$\text{num}$

$(\text{num } \text{num} \rightarrow \text{num})$

$\{\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\} 5 6\}$

---

$(\text{num } \text{num} \rightarrow \text{num})$        $\text{num}$

# Types: Multiple Arguments

$\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\}$

---

$\text{num}$        $\text{num}$        $\text{num}$

$\text{num}$

$(\text{num } \text{num} \rightarrow \text{num})$

$\{\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\} 5 6\}$

---

$(\text{num } \text{num} \rightarrow \text{num})$        $\text{num}$        $\text{num}$

# Types: Multiple Arguments

$\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\}$

---

$\text{num}$        $\text{num}$        $\text{num}$

$\text{num}$

$(\text{num } \text{num} \rightarrow \text{num})$

$\{\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\} 5 6\}$

---

$(\text{num } \text{num} \rightarrow \text{num})$        $\text{num}$        $\text{num}$

$\text{num}$

# Types: Multiple Arguments

`{fun {x : num y : num} {+ x y}}`

---

*num*      *num*

*num*

$(num\ num \rightarrow num)$

`{{fun {x : num y : num} {+ x y}} 5 6}`

---

$(num\ num \rightarrow num)$       *num*      *num*

*num*

`{{fun {x : num y : num} {+ x y}} 5}`

# Types: Multiple Arguments

$\frac{\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\}}{\text{num} \quad \text{num} \quad \text{num}}$   
 $(\text{num num} \rightarrow \text{num})$

$\frac{\{\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\} 5 6\}}{(\text{num num} \rightarrow \text{num}) \quad \text{num} \quad \text{num}}$   
 $\text{num}$

$\frac{\{\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\} 5\}}{(\text{num num} \rightarrow \text{num})}$

# Types: Multiple Arguments

$\frac{\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\}}{\text{num} \quad \text{num}} \text{num}$   
 $(\text{num num} \rightarrow \text{num})$

$\frac{\{\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\} 5 6\}}{(\text{num num} \rightarrow \text{num}) \quad \text{num} \quad \text{num}} \text{num}$

$\frac{\{\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\} 5\}}{(\text{num num} \rightarrow \text{num}) \quad \text{num}}$

# Types: Multiple Arguments

$\frac{\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\}}{\text{num} \quad \text{num}} \text{num}$   
 $(\text{num num} \rightarrow \text{num})$

$\frac{\{\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\} 5 6\}}{(\text{num num} \rightarrow \text{num}) \quad \text{num} \quad \text{num}} \text{num}$

$\frac{\{\{\text{fun } \{x : \text{num } y : \text{num}\} \{+ x y\}\} 5\}}{(\text{num num} \rightarrow \text{num}) \quad \text{num}}$   
**no type**

# Revised Function and Call Rules

$$\frac{\Gamma[\langle \text{id} \rangle_1 \leftarrow \tau_1 \dots \langle \text{id} \rangle_n \leftarrow \tau_n] \vdash \mathbf{e} : \tau_0}{\Gamma \vdash \{\text{fun } \{\langle \text{id} \rangle_1 : \tau_1 \dots \langle \text{id} \rangle_n : \tau_n\} \mathbf{e}\} : (\tau_1 \dots \tau_n \rightarrow \tau_0)}$$

$$\frac{\Gamma \vdash \mathbf{e}_0 : (\tau_1 \dots \tau_n \rightarrow \tau_0) \quad \Gamma \vdash \mathbf{e}_1 : \tau_1 \quad \dots \quad \Gamma \vdash \mathbf{e}_n : \tau_n}{\Gamma \vdash \{\mathbf{e}_0 \ \mathbf{e}_1 \ \dots \ \mathbf{e}_n\} : \tau_0}$$