TFAE Grammar

```
<TFAE>  ::=  <num>
          |  {+ <TFAE> <TFAE>}
          |  {- <TFAE> <TFAE>}
          |  <id>
          |  {fun {<id> : <TE>} <TFAE>}
          |  {<TFAE> <TFAE>}

<TE>    ::=  num
          |  (<TE> -> <TE>)
```

TFAE Types

```
(define-type Type
  [numT]
  [arrowT (arg : Type)
          (result : Type)])

(define-type TypeEnv
  [mtEnv]
  [aBind (name : symbol)
         (type : Type)
         (rest : TypeEnv)])
```

TFAE Expressions

```
(define-type TFAE
  [num (n : number)]
  [add (l : TFAE)
       (r : TFAE)]
  [sub (l : TFAE)
       (r : TFAE)]
  [id (name : symbol)]
  [fun (name : symbol)
    (t : Type)
    (body : TFAE)]
  [app (rator : TFAE)
       (rand : TFAE)])
```

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      ...)))
```

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [num (n) ...])))
```

$\Gamma \vdash$ **<num>** : ***num***

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [num (n) (numT)])))
```

$\Gamma \vdash$ **<num>** : ***num***

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [add (l r)
           ... (typecheck l env) ...
           ... (typecheck r env) ...])))
```

$$\frac{\Gamma \vdash \mathbf{e}_1 \ : \ \mathit{num} \qquad \Gamma \vdash \mathbf{e}_2 \ : \ \mathit{num}}{\Gamma \vdash \{\mathbf{+} \ \mathbf{e}_1 \ \mathbf{e}_2\} \ : \ \mathit{num}}$$

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [add (l r)
           (type-case Type (typecheck l env)
             [numT ()
                   ... (typecheck r env) ...]
             [else (error 'typecheck
                          "add expects a num")])])))
```

$$\frac{\Gamma \vdash \mathbf{e}_1 : \mathit{num} \qquad \Gamma \vdash \mathbf{e}_2 : \mathit{num}}{\Gamma \vdash \{+\ \mathbf{e}_1\ \mathbf{e}_2\} : \mathit{num}}$$

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [add (l r)
           (type-case Type (typecheck l env)
             [numT ()
                   (type-case Type (typecheck r env)
                     [numT () (numT)]
                     [else (error 'typecheck
                                  "add expects a num")])]
             [else (error 'typecheck
                          "add expects a num")])])))
```

$$\frac{\Gamma \vdash \mathbf{e}_1 : \mathit{num} \qquad \Gamma \vdash \mathbf{e}_2 : \mathit{num}}{\Gamma \vdash \{+\ \mathbf{e}_1\ \mathbf{e}_2\} : \mathit{num}}$$

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [id (name) ...])))
```

[… <id>←τ …] ⊢ <id> : τ

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [id (name) (get-type name env)])))
```

[… <id>←τ …] ⊢ <id> : τ

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [fun (name arg-type body)
           ...])))
```

$$\frac{\Gamma[\,\texttt{<id>} \leftarrow \tau_1\,] \vdash \texttt{e} : \tau_2}{\Gamma \vdash \{\texttt{fun}\ \{\texttt{<id>} : \tau_1\}\ \texttt{e}\} : (\tau_1 \rightarrow \tau_2)}$$

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [fun (name arg-type body)
        ... (typecheck body (aBind name
                                   arg-type
                                   env)) ...])))
```

$$\frac{\Gamma[\,\texttt{<id>} \leftarrow \tau_1\,] \vdash \texttt{e} : \tau_2}{\Gamma \vdash \{\texttt{fun}\ \{\texttt{<id>} : \tau_1\}\ \texttt{e}\} : (\tau_1 \rightarrow \tau_2)}$$

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [fun (name arg-type body)
        (arrowT arg-type
                (typecheck body (aBind name
                                       arg-type
                                       env)))])))
```

$$\frac{\Gamma[\,\texttt{<id>} \leftarrow \tau_1\,] \vdash \texttt{e} : \tau_2}{\Gamma \vdash \{\texttt{fun}\ \{\texttt{<id>} : \tau_1\}\ \texttt{e}\} : (\tau_1 \rightarrow \tau_2)}$$

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [app (fn arg)
           ...])))
```

$$\frac{\Gamma \vdash \mathbf{e}_1 \ : \ (\tau_2 \rightarrow \tau_3) \qquad \Gamma \vdash \mathbf{e}_2 \ : \ \tau_2}{\Gamma \vdash \{\mathbf{e}_1 \ \mathbf{e}_2\} \ : \ \tau_3}$$

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [app (fn arg)
           ... (typecheck fn env) ...
           ... (typecheck arg env) ...])))
```

$$\frac{\Gamma \vdash \mathbf{e}_1 : (\tau_2 \rightarrow \tau_3) \qquad \Gamma \vdash \mathbf{e}_2 : \tau_2}{\Gamma \vdash \{\mathbf{e}_1\ \mathbf{e}_2\} : \tau_3}$$

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [app (fn arg)
           (type-case Type (typecheck fn env)
             [arrowT (arg-type result-type)
                     ... (typecheck arg env) ...]
             [else (error 'typecheck
                          "app expects a function")])])))
```

$$\frac{\Gamma \vdash \mathbf{e}_1 : (\tau_2 \rightarrow \tau_3) \qquad \Gamma \vdash \mathbf{e}_2 : \tau_2}{\Gamma \vdash \{\mathbf{e}_1\ \mathbf{e}_2\} : \tau_3}$$

TFAE Type Checker

```
(define typecheck : (TFAE TypeEnv -> Type)
  (lambda (fae env)
    (type-case TFAE fae
      [app (fn arg)
           (type-case Type (typecheck fn env)
             [arrowT (arg-type result-type)
                     (if (equal? arg-type
                                 (typecheck arg env))
                         result-type
                         (error
                          'typecheck
                          "arg mismatch"))]
             [else (error 'typecheck
                          "app expects a function")])])))
```

$$\frac{\Gamma \vdash \mathbf{e}_1 : (\tau_2 \rightarrow \tau_3) \qquad \Gamma \vdash \mathbf{e}_2 : \tau_2}{\Gamma \vdash \{\mathbf{e}_1\ \mathbf{e}_2\} : \tau_3}$$

TFAE Type Checker

```
(define get-type : (symbol TypeEnv -> Type)
  (lambda (name env)
    (type-case TypeEnv env
      [mtEnv () (error 'typecheck
                       "unbound identifier")]
      [aBind (name2 type rest)
             (if (equal? name name2)
                 type
                 (get-type name rest))])))
```

Pairs

```
{with {pair : (num -> (num -> (num -> num)))
              {fun {x : num}
                   {fun {y : num}
                        {fun {s : num}
                             {if0 s x y}}}}}
  {with {fst : ((num -> num) -> num)
              {fun {p : (num -> num)}
                   {p 0}}}
    {with {snd : ((num -> num) -> num)
                {fun {p : (num -> num)}
                     {p 1}}}
      {snd {{pair 1} 2}}}}}
```

Pairs

```
{with {pair : (bool -> (bool -> (num -> bool)))
              {fun {x : bool}
                   {fun {y : bool}
                        {fun {s : num}
                             {if0 s x y}}}}}
  {with {fst : ((num -> bool) -> bool)
              {fun {p : (num -> bool)}
                   {p 0}}}
    {with {snd : ((num -> bool) -> bool)
                {fun {p : (num -> bool)}
                     {p 1}}}
      {snd {{pair true} false}}}}}
```

Pairs

```
{with {pair : (num -> (bool -> (num -> …)))
              {fun {x : num}
                   {fun {y : bool}
                        {fun {s : num}
                             {if0 s x y}}}}}
  {with {fst : ((num -> …) -> …)
              {fun {p : (num -> …)}
                   {p 0}}}
    {with {snd : ((num -> …) -> …)
                {fun {p : (num -> …)}
                     {p 1}}}
      {snd {{pair 1} false}}}}}
```

Pairs

```
{with {pair : (num -> (bool -> (num -> …)))
              {fun {x : num}
                   {fun {y : bool}
                        {fun {s : num}
                             {if0 s x y}}}}}
  {with {fst : ((num -> …) -> …)
              {fun {p : (num -> …)}
                   {p 0}}}
    {with {snd : ((num -> …) -> …)
                {fun {p : (num -> …)}
                     {p 1}}}
      {snd {{pair 1} false}}}}}
```

No possible type for …

TPFAE Grammar

```
<TPFAE>  ::=  <num>
           |  {+ <TPFAE> <TPFAE>}
           |  {- <TPFAE> <TPFAE>}
           |  <id>
           |  {fun {<id> : <TE>} <TPFAE>}
           |  {<TPFAE> <TPFAE>}
           |  {pair <TPFAE> <TPFAE>}        NEW
           |  {fst <TPFAE>}                 NEW
           |  {snd <TPFAE>}                 NEW

<TE>     ::=  num
           |  bool
           |  (<TE> -> <TE>)
           |  (<TE> * <TE>)                 NEW
```

TPFAE Grammar

```
<TPFAE>  ::=  <num>
           |  {+ <TPFAE> <TPFAE>}
           |  {- <TPFAE> <TPFAE>}
           |  <id>
           |  {fun {<id> : <TE>} <TPFAE>}
           |  {<TPFAE> <TPFAE>}
           |  {pair <TPFAE> <TPFAE>}        NEW
           |  {fst <TPFAE>}                 NEW
           |  {snd <TPFAE>}                 NEW

<TE>     ::=  num
           |  bool
           |  (<TE> -> <TE>)
           |  (<TE> * <TE>)                 NEW
```

$$\frac{\Gamma \vdash \mathbf{e}_1 : \tau_1 \qquad \Gamma \vdash \mathbf{e}_2 : \tau_2}{\Gamma \vdash \{\mathbf{pair}\ \mathbf{e}_1\ \mathbf{e}_2\} : (\tau_1 \times \tau_2)}$$

TPFAE Grammar

```
<TPFAE>  ::=  <num>
           |  {+ <TPFAE> <TPFAE>}
           |  {- <TPFAE> <TPFAE>}
           |  <id>
           |  {fun {<id> : <TE>} <TPFAE>}
           |  {<TPFAE> <TPFAE>}
           |  {pair <TPFAE> <TPFAE>}        NEW
           |  {fst <TPFAE>}                 NEW
           |  {snd <TPFAE>}                 NEW

<TE>     ::=  num
           |  bool
           |  (<TE> -> <TE>)
           |  (<TE> * <TE>)                 NEW
```

$$\frac{\Gamma \vdash \mathbf{e} : (\tau_1 \times \tau_2)}{\Gamma \vdash \{\mathbf{fst}\ \mathbf{e}\} : \tau_1}$$

TPFAE Grammar

```
<TPFAE>  ::=  <num>
           |  {+ <TPFAE> <TPFAE>}
           |  {- <TPFAE> <TPFAE>}
           |  <id>
           |  {fun {<id> : <TE>} <TPFAE>}
           |  {<TPFAE> <TPFAE>}
           |  {pair <TPFAE> <TPFAE>}        NEW
           |  {fst <TPFAE>}                 NEW
           |  {snd <TPFAE>}                 NEW

<TE>     ::=  num
           |  bool
           |  (<TE> -> <TE>)
           |  (<TE> * <TE>)                 NEW
```

$$\frac{\Gamma \vdash \mathbf{e} : (\tau_1 \times \tau_2)}{\Gamma \vdash \{\mathbf{snd}\ \mathbf{e}\} : \tau_2}$$