Cost of Substitution

```
(interp {with {x 1}
          {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}})
```

Cost of Substitution

```
(interp {with {x 1}
          {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}})
```

⇒

```
(interp {with {y 2}
          {+ 100 {+ 99 {+ 98 ... {+ y 1}}}}})
```

Cost of Substitution

```
(interp {with {x 1}
          {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}})
```

⇒

```
(interp {with {y 2}
          {+ 100 {+ 99 {+ 98 ... {+ y 1}}}}})
```

⇒

```
(interp {+ 100 {+ 99 {+ 98 ... {+ 2 1}}}})
```

Cost of Substitution

```
(interp {with {x 1}
          {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}})
```

⇒

```
(interp {with {y 2}
          {+ 100 {+ 99 {+ 98 ... {+ y 1}}}}})
```

⇒

```
(interp {+ 100 {+ 99 {+ 98 ... {+ 2 1}}}})
```

With **n** variables, evaluation will take O(**n**^{2}) time!

Deferring Substitution

```
(interp {with {x 1}
          {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}})
```

Deferring Substitution

```
(interp {with {x 1}
          {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}})
```

⇒

x = 1

```
(interp {with {y 2}
          {+ 100 {+ 99 {+ 98 ... {+ y x}}}}})
```

Deferring Substitution

```
(interp {with {x 1}
          {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}})
```

⇒

x = 1

```
(interp {with {y 2}
          {+ 100 {+ 99 {+ 98 ... {+ y x}}}}})
```

⇒

y = 2   x = 1

```
(interp {+ 100 {+ 99 {+ 98 ... {+ y x}}}})
```

Deferring Substitution

```
(interp {with {x 1}
          {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}})
```

⇒

```
                                          x = 1
(interp {with {y 2}
          {+ 100 {+ 99 {+ 98 ... {+ y x}}}}})
```

⇒

```
                                    y = 2   x = 1
(interp {+ 100 {+ 99 {+ 98 ... {+ y x}}}})
```

⇒ . . . ⇒

```
          y = 2   x = 1
(interp y)
```

Deferring Substitution with the Same Identifier

```
(interp {with {x 1}
          {with {x 2}
            x}}       )
```

Deferring Substitution with the Same Identifier

```
(interp {with {x 1}
           {with {x 2}
             x}}          )
```

⇒

x = 1

```
(interp {with {x 2}
           x}        )
```

Deferring Substitution with the Same Identifier

```
(interp {with {x 1}
           {with {x 2}
             x}})
```

⇒

```
x = 1
```

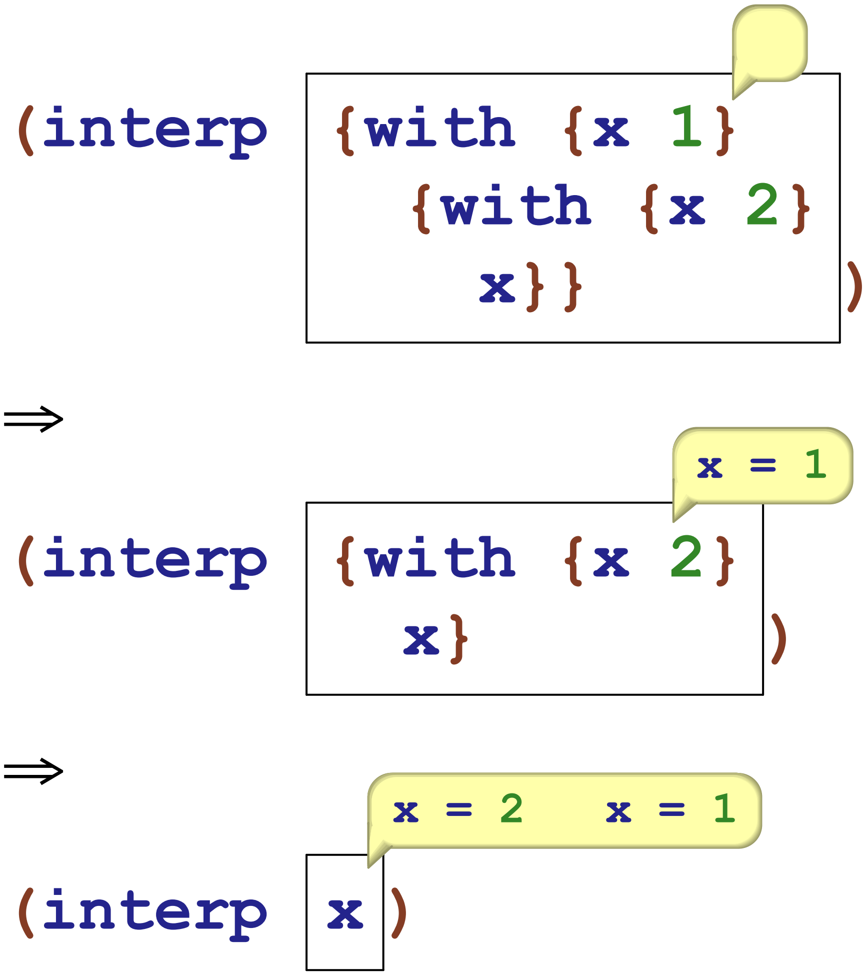
```
(interp {with {x 2}
           x})
```

⇒

```
x = 2    x = 1
```

```
(interp x)
```

Deferring Substitution with the Same Identifier



```
(interp {with {x 1}
          {with {x 2}
            x}})
```

⇒

`x = 1`

```
(interp {with {x 2}
          x})
```

⇒

`x = 2    x = 1`

```
(interp x)
```

Always add to start, then always check from start

Deferring Substitution with the Same Identifier

```
(interp  {with {x 1}
           {+ {with {x 2}
                    x}
              x}}         )
```

Deferring Substitution with the Same Identifier

```
(interp {with {x 1}
          {+ {with {x 2}
                   x}
             x}}            )
```

x = 1

```
(interp {+ {with {x 2} x}
             x}              )
```

Deferring Substitution with the Same Identifier

```
(interp {with {x 1}
          {+ {with {x 2}
                   x}
             x}})
```

x = 1

```
(interp {+ {with {x 2} x}
           x})
```

x = 1   x = 1

```
(+ (interp {with {x 2} x}) (interp x))
```

Deferring Substitution with the Same Identifier

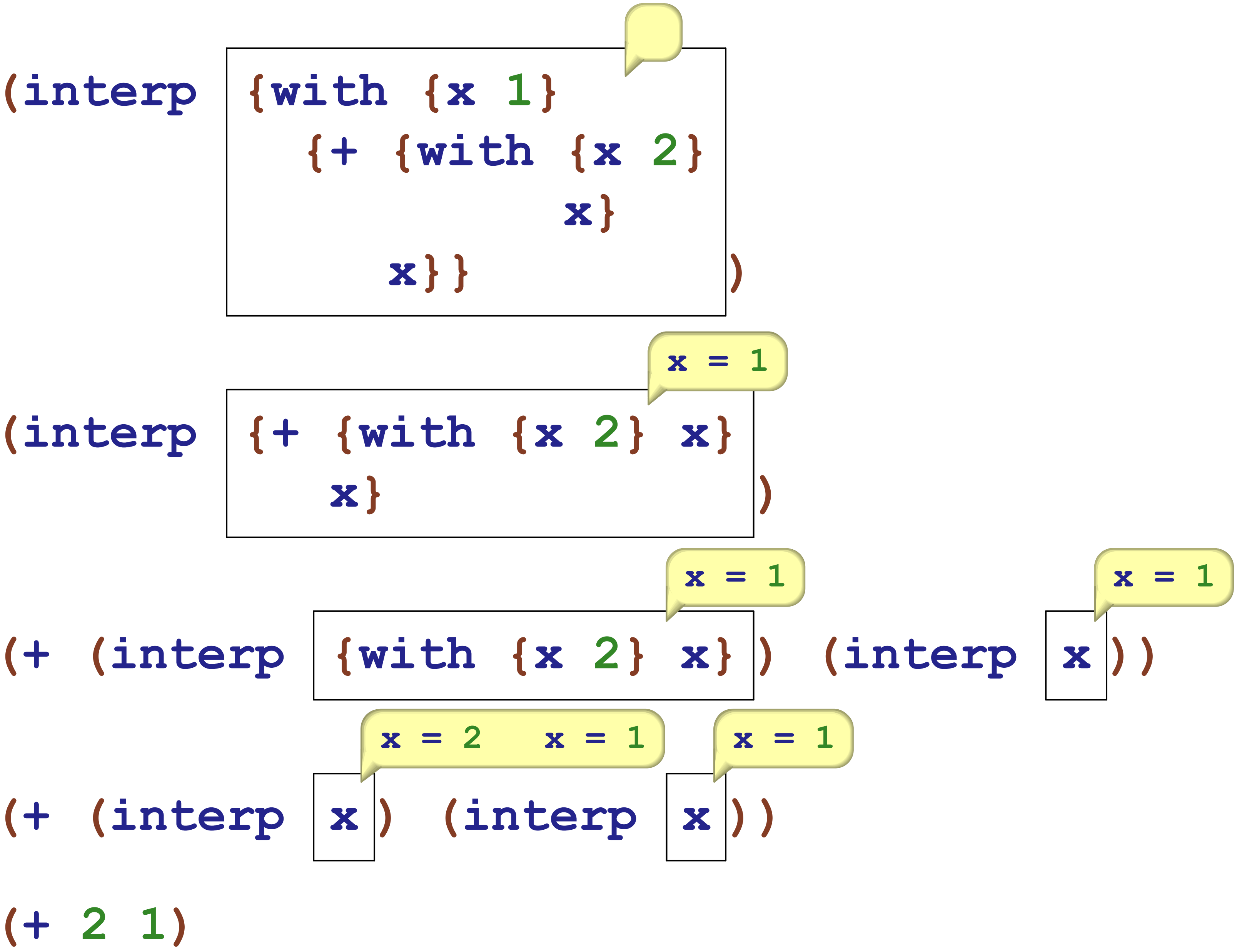
```
(interp {with {x 1}
           {+ {with {x 2}
                    x}
              x}})
```

```
                         x = 1
(interp {+ {with {x 2} x}
           x})
```

```
                          x = 1                   x = 1
(+ (interp {with {x 2} x}) (interp x))
```

```
           x = 2   x = 1   x = 1
(+ (interp x) (interp x))
```

Deferring Substitution with the Same Identifier



```
(interp {with {x 1}
          {+ {with {x 2}
                   x}
             x}})

                                 x = 1
(interp {+ {with {x 2} x}
           x})

                                  x = 1                 x = 1
(+ (interp {with {x 2} x}) (interp x))

              x = 2    x = 1        x = 1
(+ (interp x) (interp x))

(+ 2 1)
```

Representing Deferred Substitution

Change

```
; interp : WAE -> num
```

to

```
; interp : WAE DefrdSub -> num
```

Representing Deferred Substitution

Change

```
; interp : WAE -> num
```

to

```
; interp : WAE DefrdSub -> num
```

```
(define-type DefrdSub
  [mtSub]
  [aSub (name symbol?)
        (value number?)
        (rest DefrdSub?)])
```

Interp with DefrdSub

```
(interp  {with {x 1}
           {with {y 2}
             {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}}
         (mtSub))
```

Interp with DefrdSub

```
(interp {with {x 1}
          {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}}
        (mtSub))

⇒ (interp {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}
          (aSub 'x 1 (mtSub)))
```

Interp with DefrdSub

```
(interp {with {x 1}
          {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}}
        (mtSub))

⇒ (interp {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}
          (aSub 'x 1 (mtSub)))

⇒ (interp {+ 100 {+ 99 {+ 98 ... {+ y x}}}}
          (aSub 'y 2 (aSub 'x 1 (mtSub))))
```

Interp with DefrdSub

```
(interp {with {x 1}
          {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}}}
        (mtSub))

⇒ (interp {with {y 2}
            {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}}
          (aSub 'x 1 (mtSub)))

⇒ (interp {+ 100 {+ 99 {+ 98 ... {+ y x}}}}}
          (aSub 'y 2 (aSub 'x 1 (mtSub))))

⇒ ...

⇒ (interp y (aSub 'y 2 (aSub 'x 1 (mtSub))))
```

WAE Interpreter with Deferred Substitutions

```
; interp : WAE DefrdSub -> num
(define (interp a-wae ds)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l ds) (interp r ds))]
    [sub (l r) (- (interp l ds) (interp r ds))]
    [with (bound-id named-expr body-expr)
      ...]
    [id (name) ...]))
```

WAE Interpreter with Deferred Substitutions

```
; interp : WAE DefrdSub -> num
(define (interp a-wae ds)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l ds) (interp r ds))]
    [sub (l r) (- (interp l ds) (interp r ds))]
    [with (bound-id named-expr body-expr)
      ...]
    [id (name) (lookup name ds)]))
```

WAE Interpreter with Deferred Substitutions

```
; lookup : symbol DefrdSub -> num
(define (lookup name ds)
  (type-case DefrdSub ds
    [mtSub () (error 'lookup "free variable")]
    [aSub (sub-name num rest-ds)
          (if (symbol=? sub-name name)
              num
              (lookup name rest-ds))]))
```

WAE Interpreter with Deferred Substitutions

```
; interp : WAE DefrdSub -> num
(define (interp a-wae ds)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l ds) (interp r ds))]
    [sub (l r) (- (interp l ds) (interp r ds))]
    [with (bound-id named-expr body-expr)
      ...]
    [id (name) (lookup name ds)]))
```

WAE Interpreter with Deferred Substitutions

```
; interp : WAE DefrdSub -> num
(define (interp a-wae ds)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l ds) (interp r ds))]
    [sub (l r) (- (interp l ds) (interp r ds))]
    [with (bound-id named-expr body-expr)
      ... (interp named-expr ds) ...]
    [id (name) (lookup name ds)]))
```

WAE Interpreter with Deferred Substitutions

```
; interp : WAE DefrdSub -> num
(define (interp a-wae ds)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l ds) (interp r ds))]
    [sub (l r) (- (interp l ds) (interp r ds))]
    [with (bound-id named-expr body-expr)
      ...
      (aSub bound-id (interp named-expr ds) ds)
      ...]
    [id (name) (lookup name ds)]))
```

WAE Interpreter with Deferred Substitutions

```
; interp : WAE DefrdSub -> num
(define (interp a-wae ds)
  (type-case WAE a-wae
    [num (n) n]
    [add (l r) (+ (interp l ds) (interp r ds))]
    [sub (l r) (- (interp l ds) (interp r ds))]
    [with (bound-id named-expr body-expr)
      (interp
       body-expr
       (aSub bound-id (interp named-expr ds) ds))]
    [id (name) (lookup name ds)]))
```

Function Calls

```
{deffun {f x} {+ 1 x}}

(interp  {with {y 2}
           {f 10}}   )
```

Function Calls

```
{deffun {f x} {+ 1 x}}

(interp {with {y 2}
          {f 10}} )

⇒
```

y = 2

```
(interp {f 10} )
```

Function Calls

```
{deffun {f x} {+ 1 x}}

(interp {with {y 2}
          {f 10}})
```

⇒

```
                y = 2
(interp {f 10})
```

⇒

```
                 ...
(interp {+ 1 x})
```

Function Calls

```
{deffun {f x} {+ 1 x}}

(interp {with {y 2}
          {f 10}} )

⇒
                  y = 2
(interp {f 10} )

⇒
                   x = 10
(interp {+ 1 x} )
```

Interpreting function body starts with only one substitution

Function Calls

What goes wrong if you extend the old substitution?

```
{deffun {f x} {+ y x}}
```

```
(interp {with {y 2}
           {f 10}} )
```

Function Calls

What goes wrong if you extend the old substitution?

```
{deffun {f x} {+ y x}}

(interp {with {y 2}
           {f 10}})
```

⇒

```
                y = 2
(interp {f 10})
```

Function Calls

What goes wrong if you extend the old substitution?

```
{deffun {f x} {+ y x}}

(interp {with {y 2}
           {f 10}})
```

⇒

```
                y = 2
(interp {f 10})
```

⇒

```
                 x = 10 y = 2
(interp {+ y x})
```

⇒ 12 wrong!

Function Calls

What goes wrong if you extend the old substitution?

```
{deffun {f x} {+ y x}}
```

```
(interp {with {y 2}
           {f 10}})
```

⇒

y = 2

```
(interp {f 10})
```

⇒

x = 10

```
(interp {+ y x})
```

⇒ "free var: y"

Interpreting function body starts with only one

F1WAE Interpreter with Deferred Substitutions

```
; interp : F1WAE list-of-FunDef DefrdSub -> num
(define (interp a-f1wae fundefs ds)
  (type-case F1WAE a-f1wae
    ...
    [app (name arg-expr)
         ...]))
```

F1WAE Interpreter with Deferred Substitutions

```
; interp : F1WAE list-of-FunDef DefrdSub -> num
(define (interp a-f1wae fundefs ds)
  (type-case F1WAE a-f1wae
    ...
    [app (name arg-expr)
         (local [(define a-fundef
                   (lookup-fundef name fundefs))]
           (interp (fundef-body a-fundef)
                   fundefs
                   ...
                   (interp arg-expr fundefs ds)
                   ...))]))
```

F1WAE Interpreter with Deferred Substitutions

```
; interp : F1WAE list-of-FunDef DefrdSub -> num
(define (interp a-f1wae fundefs ds)
  (type-case F1WAE a-f1wae
    ...
    [app (name arg-expr)
         (local [(define a-fundef
                   (lookup-fundef name fundefs))]
           (interp (fundef-body a-fundef)
                   fundefs
                   (aSub (fundef-arg-name a-fundef)
                         (interp arg-expr fundefs ds)
                         (mtSub))))]))
```

Timing tests

```
(define (mk-sums n)
  (cond
    [(zero? n) 1]
    [else
     (let ([varn (string->symbol (format "x~a" n))])
       `{+ ,varn ,(mk-sums (- n 1))})]))

(define (mk-withs n body)
  (cond
    [(zero? n) body]
    [else
     (let ([varn (string->symbol (format "x~a" n))])
       `{with {,varn 1}
              ,(mk-withs (- n 1) body)})]))
```

Timing tests, 2

```
(define (mk-exp n) (mk-withs n (mk-sums n)))

(test (mk-exp 2)
      `{with {x2 1}
             {with {x1 1}
                   {+ x2 {+ x1 1}}}})

(define (run n)
  (let ([expr (parse (mk-exp n))])
    (time (interp-expr expr '()))))
```

Timing tests, 3

With the substitution-based interpreter, expect the difference between adjacent timings to be growing linearly. With the environment-based one, you will also see linear growth, but if you make the environment use a more efficient data structure, that'll go away

(you may need to make the numbers bigger or smaller to see what is going on here)

```
(collect-garbage) (collect-garbage)
(collect-garbage) (collect-garbage)
(run 100) (run 110) (run 120)
(run 130) (run 140) (run 160)
```