MAC Trade-offs Between Age and Reachability of Information in Vehicular Safety Applications

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***Abstract*—Vehicular networking offers the promise of greatly improving transportation safety but has stringent requirements on information age as well as information reachability, where the later refers to the range over which information is propagated. We consider an idealized model of a one-dimensional vehicular networks and show that there is a basic trade-off between these two metrics: a smaller age can be obtained by reducing the reachability of information. We apply this to two current technologies: Cellular V2X (C-V2X) and Dedicated Short Range Communication (DSRC) and derive an equation that characterizes the trade-off between these two metrics for both technologies. In the case of exponential path loss and negligible noise, this relationship becomes a fixed invariant ratio. Given this relationship, under high congestion, these two protocols trade-off these metrics differently. C-V2X tends to achieve a smaller age while DSRC tends to maintain a larger reachability. The idealized model is also applied to analyze the steady state of rate control and power control mechanisms such as those in the SAE standard J2945/1. We show that the ratio of age and reachability is still governed by the same trade-off curve: rate control tries to maintain a large reachability, while power control helps improve the age.**

I. Introduction

Safety related applications of future connected vehicles are based on the periodic exchange of vehicular status. These messages, which are called Basic Safety Messages (BSMs) aim to assess potential road hazards by announcing the presence of a vehicle to other surrounding vehicles. It has been well recognized that the age of these status updates is an important metric (e.g. [1]–[4]). These updates are shared locally in a vehicular network via broadcast transmissions. Hence, another important metric is the *reachability* of these broadcasts, i.e., how many other vehicles receive updates from a given vehicle. Our goal in this paper is to understand the trade-offs between these two metrics for some common vehicular networking protocols.

As an initial standard for supporting such communication, the automotive industry standardized a WiFi extension called IEEE 802.11p (also known as DSRC) [5]. Recently, the adoption of cellular technologies (C-V2X) to support vehicular applications has emerged as an alternative to DSRC. In 3GPP Release 14, C-V2X is designed to operate in several modes: Vehicle-to-Vehicle (V2V), Vehicle-to-Infrastructure (V2I), Vehicle-to-Network (V2N) and Vehicle-to-Pedestrian (V2P) [6]. The V2V mode overcomes the limitation of the traditional LTE architecture by introducing a new sidelink alongside the downlink and uplink, which allows vehicles to transmit directly to each other. Here, we focus on V2V transmission mode 4 in 3GPP Release 14, under which vehicles can autonomously select their radio resources using a distributed scheduling scheme so that it can work without cellular coverage [7].

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In both DSRC and C-V2X, the medium access control (MAC) protocol plays a vital role in determining both the age and reachability of information. DSRC uses a variation of the 802.11 MAC protocol, which is based on carrier-sense multiple access (CSMA). When a node has a packet to send, it first listens to the channel. If the channel is idle, the node transmits the packet. If the channel is busy, the node waits for a random backoff time before transmitting the packet [8]. In C-V2X, a semi-persistent scheduling method is adopted, where vehicles transmit with a selected resource block and do not back-off even if it may cause a collision.

There have been a number of papers studying the performance of one or both of these MAC protocols. Different analytical models have been developed to study the performance of the CSMA-based MAC protocol of DSRC for broadcasting BSMs (e.g. [9]–[11]). Much of the prior work on the Age of information in vehicular networks has also focused on variations of this protocol (e.g. [1], [2], [4]). For C-V2X, the authors in [12]–[14] apply different models to evaluate the performance of semi-persistent scheduling in V2V communications. However, these works primarily focuses on selective metrics such as packet error rate and transmission delay.

While the previous work sheds some light into the performance of these two protocols, a basic understanding of the MAC layer trade-offs between them has not yet emerged and in particular how they trade-off age and reachability of information. In this paper, we propose an idealized framework to help provide such understanding. In this framework, we assume that at the MAC layer of both DSRC and C-V2X, the maximum number of concurrent transmissions are scheduled subject to basic constraints imposed by the protocols. We introduce two useful metrics, the *maximum hearing range* and *inter-packet gap* to characterize the reachability and age of information, respectively, of the two scheduling methods. We show that there is a trade-off between these two metrics: C-V2X tends to achieve a small inter-packet gap (age) while DSRC tends to maintain a large hearing range (reachability). We derive an equation governing this trade-off that applies to both our models of C-V2X and DSRC. We further show that in the special case of exponential path loss and negligible noise this reduces to specifying that the ratio of these two metrics is an invariant.

When traffic density becomes very high, congestion control mechanisms may be used by either of these protocols. For example the Society of Automotive Engineers (SAE) has developed the standard J2945/1 for congestion control in DSRC [15] and is developing a similar protocol for C-V2X. Other congestion control mechanisms are also proposed in [16], [17] to control the rate and power when sending BSMs. We use the proposed model to analyze the steady-state performance of these rate and power control mechanisms. We show that the same trade-off curve between inter-packet gap and maximum hearing range continues to hold when such congestion control algorithms are applied. In other words, congestion control simply changes the operating point on this curve. We show that rate control tends to maintain a large reachability while power control helps improve the age.

This paper is organized as follows. In Section II, we specify our idealized scheduling model for C-V2X. Section III defines the idealized scheduling model for DSRC. Section IV compares C-V2X and DSRC using these idealized models. In Section V, we derive the steady-state performance with congestion control for both the idealized and generalized models. Section VI concludes the paper.

II. Idealized scheduling of C-V2X

A. *Maximum hearing range*

In this section, we define our basic idealized model with the goal of calculating the maximum hearing range or reachability of a vehicle under C-V2X. The maximum hearing range of a vehicle is defined as the range within which the packet delivery ratio is greater than 0.

We consider a static scenario where an infinite number of vehicles are regularly spaced on a straight line road with infinite length. The density of vehicles per km is denoted as N_v. In C-V2X, each vehicle selects a virtual resource blocks that consists a group of OFDM symbols/time-slots that is large enough to send a BSM. This is selected from a set of N_r virtual resource blocks in a frame of length T_{tr} milliseconds.[1] Vehicles then repeatedly transmit using the same virtual resource block so that they send once every T_{tr} milliseconds.

We define the inter-packet gap as the expected time between two consecutive receptions of BSMs from a given transmitter at a given receiver. If another vehicle receives every transmission, then, under the above scheduling, the inter-packet gap is simply T_{tr}. This can be viewed as a measure of the peak age of information at the receiver [18] assuming that a new BSM is generated right before each transmission opportunity.[2]

Next we turn to the reachability of each transmission. This depends on how vehicles are assigned to virtual resource blocks. We consider an idealized model in which this assignment is done so as to maximize the minimum reachability across all vehicles.[3] It follows that the optimal schedule is simply a round robin assignment of vehicles to virtual resource blocks following the vehicles' order on the road. This maximizes the distance between two vehicles using the same virtual resource block. An illustration is shown in Fig. 1. Each colored block is a different virtual resource block used by a vehicle for transmission in the transmission period.

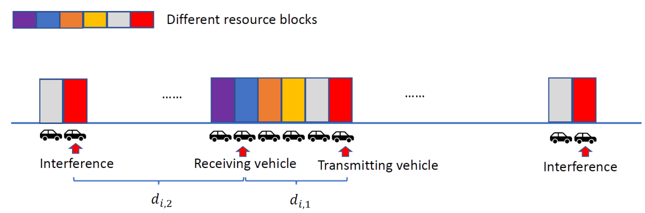


Fig. 1: Illustration of idealized scheduling of C-V2X.

We assume that the received power at a distance d, given a transmitted power P_t, is given by a path loss function $P_r(P_t, d)$, which is continuously increasing in P_t and continuously decreasing in d. We will refer to such a path loss function as *admissible*. For simplicity, in much of our discussion we will focus on the special admissible case where

$$P_r(P_t, d) = \frac{K_0 P_t}{d^k}, \tag{1}$$

where k is the path loss factor and K_0 is a constant. We assume all vehicles are transmitting with the same power and at times suppress the dependence of P_r on P_t by simply writing $P_r(d)$. We denote the SINR threshold in dB for successful reception as $SINR_{th}$.

To calculate the maximum hearing range, we view an arbitrary vehicle as a receiver. Suppose the minimum and second minimum distance between the receiver and a transmitter using virtual resource block i is $d_{i,1}$ and $d_{i,2}$, respectively. An illustration is shown in Fig. 1. In this derivation, we only consider the dominant interferer. We will later show that the impact of other interferers can be bounded. The distances $d_{i,1}$ and $d_{i,2}$ should satisfy the following inequality for successful reception of message transmitted on virtual resource block i:

$$10 \log_{10} \left(\frac{P_r(d_{i,1})}{P_r(d_{i,2}) + N_0} \right) \geq SINR_{th}, \tag{2}$$

where N_0 is the noise power.[4] Assuming $N_0 = 0$ and that $P_r(d)$ is given by (1), we can further reduce (2) to the following inequality[5]:

$$\frac{d_{i,2}}{d_{i,1}} \geq 10^{\frac{SINR_{th}}{10*k}}. \tag{3}$$

[1]For example nominally in C-V2X this frame consists of 100 time-slots of length 1 msec, with multiple OFDM symbols in each time-slot.

[2]In practice, this may not be true: e.g., if BSMs are generated every T_tr msecs at the application layer they may not coincide with the selected time-slots at the MAC layer and need to be queued. If the assignment is uniformly at random over the available time-slots, then this adds an additional term of $T_{tr}/2$ to the expected peak age.

[3]In other words, this ignores the randomness in the semi-persistent scheduling algorithm and assumes that a "genie" is able to assign vehicles to virtual resource blocks perfectly.

[4]Here, we are assuming that transmissions on separate resource blocks are orthogonal so that there is no cross-channel interference. We are also not considering the use of hybrid ARQ, which is an option in C-V2X.

[5]We will later numerically show that the assumption of $N_0 = 0$ is reasonable when the vehicle density is high.

With our idealized scheduling, the distance between two vehicles using the same virtual resource block is maximized, which means for any virtual resource block i, we have

$$d_{i,1} + d_{i,2} = \frac{N_r}{N_v}. \tag{4}$$

The maximum hearing range can be obtained by solving the following optimization problem.

$$\begin{aligned} R_{max} &= \max_i d_{i,1} \\ s.t. \quad & (3)(4). \end{aligned} \tag{5}$$

Assuming that the distances are continuous values, the solution to (5) satisfies the following equation:

$$P_r(R_{max}) = \left[P_r(\frac{N_r}{N_v} - R_{max}) + N_0\right] 10^{\frac{SINR_{th}}{10}}. \tag{6}$$

Under the same assumption as in (3), this simplifies to:

$$R_{max} = \frac{N_r}{\left(10^{\frac{SINR_{th}}{10*k}} + 1\right) N_v}. \tag{7}$$

Note that the definition for maximum hearing range is the range within which the packet delivery ratio is above 0 so that vehicles beyond this range will never receive any status updates from the transmitter. In fact in the idealized model, the tagged vehicle is able to receive all the messages transmitted by vehicles within the range R_{max}, which means the packet delivery ratio is 100% within the hearing range and thus the inter-packet gap is T_{tr} for these vehicles. From (7), the maximum hearing range increases with the resource/vehicle density ratio $\frac{N_r}{N_v}$, which represents the average amount of resources a vehicle can use. We can see that when the receiver requires a lower SINR threshold, i.e., $SINR_{th}$ gets smaller, the maximum hearing range gets larger. Additionally, we see that in (7) when the path loss factor k increases, R_{max} also increases. The intuition is that because there is no noise, a large path loss factor can reduce the interference benefiting the maximum hearing range.

B. Impact of multiple interferers

In the above setting, we only considered the dominant interferer. We investigate the impact of multiple interferers in this subsection for the path loss model in (1) and assuming noise is negligible. We first consider the second interferer and then extend to the case with multiple interferers. Assume the distance between the second interferer and the receiving vehicle i is $d_{i,3}$. Under idealized scheduling, we find that $d_{i,3} \geq \max\{d_{i,2}, 2d_{i,1}, \frac{N_r}{N_v}\}$. We can use the following SINR equation and $d_{i,3} \geq d_{i,2}$ to get a bound for the maximum hearing range:

$$SINR = \frac{1/(d_{i,1})^k}{1/(d_{i,2})^k + 1/(d_{i,3})^k} \geq 10^{SINR_{th}/10}.$$

The inequality can be simplified to

$$\frac{d_{i,2}}{d_{i,1}} \geq 2^{1/k} 10^{\frac{SINR_{th}}{10k}}.$$

The resulting maximum hearing range is

$$R_{max} = \frac{N_r}{\left(2^{1/k} 10^{\frac{SINR_{th}}{10k}} + 1\right) N_v} \geq 2^{-1/k} R^*_{max},$$

where R^*_{max} is the maximum hearing range when only the dominant interferer is considered. We can see that when the second interferer is considered, the resulting range is within a constant factor of the result with only the dominant interferer, where that factor is bounded by $\frac{1}{\sqrt{2}}$ for free space path loss ($k = 2$) and decreases with k.

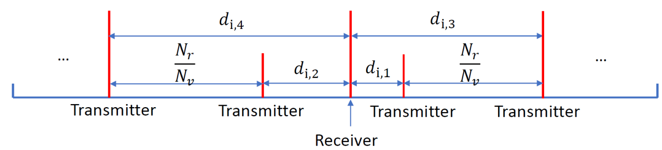


Fig. 2: Illustration of distances with multiple interferers.

We next extend the result to multiple interferers. In the idealized scheduling model, the distance between two vehicles using the same virtual resource block is $\frac{N_r}{N_v}$. We use $d_{i,j}$ to denote the distance between the receiver and jth closest transmitter using virtual resource block i. An illustration is shown in Fig. 2. Observe that $d_{i,j+2} = d_{i,j} + \frac{N_r}{N_v}$, $\forall i = 1, 2, ..., N_r, j = 1, 2, ...$ It follows that

$$d_{i,j} \geq t \times d_{i,2}, \text{for} \quad j = 2t, 2t + 1. \tag{8}$$

Combining (8) and the SINR equation, we have

$$SINR = \frac{1/(d_{i,1})^k}{\sum_{j=2}^{\infty} 1/(d_{i,j})^k} \geq \frac{1/(d_{i,1})^k}{2/(d_{i,2})^k \sum_{j=1}^{\infty} 1/j^k}. \tag{9}$$

Note that in (9), $\sum_{j=1}^{\infty} \frac{1}{j^k}$ is the well known Riemann Zeta function $\zeta(k)$. Therefore, we can express the bound for maximum hearing range when considering multiple interferers as

$$R_{max} \geq \frac{R^*_{max}}{\sqrt[k]{2\zeta(k)}}, \tag{10}$$

where R^*_{max} is the maximum hearing range when only the dominant interferer is considered. This shows that again the maximum hearing range with just the dominant interferer is within a constant factor of the range when all interferers are accounted for. Note that the constant factor $\frac{1}{\sqrt[k]{2\zeta(k)}}$ is an increasing function of k. When $k = 2$, which represents the free space path loss model, the value is around 0.55. When $k = 4$, the value increases to around 0.82. Hence, when k is large, the maximum hearing range with multiple interferers can be very close to that calculated with only the dominant interferer.

C. Impact of noise and path loss factor

So far we have ignored noise. When vehicle density is low, the closest interferer is far away making the interference low and noise the major limit of the hearing range. When vehicle density is high, interference dominates noise, in which case the maximum hearing range should be close to the value obtained without noise. We use an example with thermal noise to illustrate this. In C-V2X, the thermal noise is around -100 dBm. The result is shown in Fig. 3. When a free space path loss model is used ($k = 2$), there is little difference between the case with noise and without for densities above 100 vehicles/km. When we change the path loss factor to $k = 3$, the difference is larger with densities below 300 vehicles/km because the high path loss factor limits the interference. For larger densities there is again little difference between these two curves. Our main focus in this paper is on the performance of these protocols at high densities, and so we will continue to assume no noise in much of the following when the idealized model is considered.

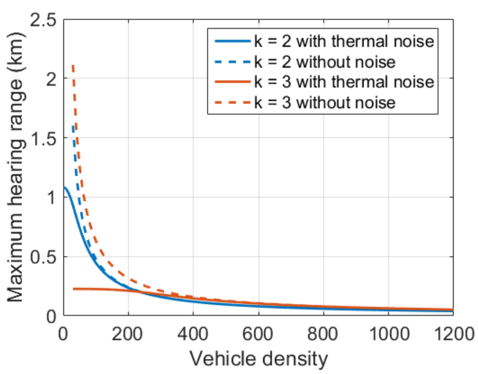


Fig. 3: Impact of noise and path loss factor on maximum hearing range.

From the expression of R_{max} in (7), we can see that R_{max} increases with the path loss factor when noise is ignored. But when noise is included, as can be seen from Fig. 3, a larger path loss factor can degrade the hearing range when vehicle density is low enough but still leads to a larger hearing range with a high enough density. Essentially, when the density is low, the hearing range is *noise limited* in which case path loss reduces signal strength relative to noise leading to a loss of hearing range. When the density is high, the system becomes *interference limited*, in which case the dominant effect of path loss is on reducing interference, which expands the hearing range.

III. Idealized scheduling model for DSRC

Next we consider a similar idealized scheduling model for DSRC. A major difference between the scheduling of C-V2X and DSRC is that C-V2X does not back-off, while the CSMA mechanism in DSRC enforces vehicles to listen before talk and back-off if the channel is sensed busy.

With DSRC, we again begin by modeling the maximum hearing range. This will still depend in part on the received power, which we still assume is given by a decreasing function $P_r(d)$. Again, for concreteness, we will initially focus on a model assuming the same path loss model as in (1), ignoring noise and considering only the dominant interferer. We continue to assume that BSMs are generated once every T_{tr} msec by each vehicle. Assume time is divided into small time slots with length T_0, which is the transmission duration of each BSM. By idealized scheduling, we again assume that each slot is utilized (i.e., we ignore collisions) and that the slots are allocated in a round robin manner to each vehicle scheduled so that interference is minimized.

To model CSMA, we assume each vehicle has a sensing range R. Each vehicle can perfectly sense the transmission status and avoid collisions with other vehicles within the range R. Assume all vehicles transmit with power P_t and let P_{sen} to denote the minimum power threshold to be sensed. Under the path loss model in (1), the sensing range can then be written as the following function of P_t and P_{sen}:

$$R = \sqrt[k]{\frac{K_0 P_t}{P_{sen}}}. \tag{11}$$

As is illustrated in Fig. 4, within the sensing range R of any one vehicle, maintaining the collision-free property of CSMA may results in some vehicles not being allowed to transmit in a given transmission period. When a vehicle is not scheduled in a transmission period, that vehicle's BSM in that period is considered a loss, reducing the packet delivery ratio.[6]

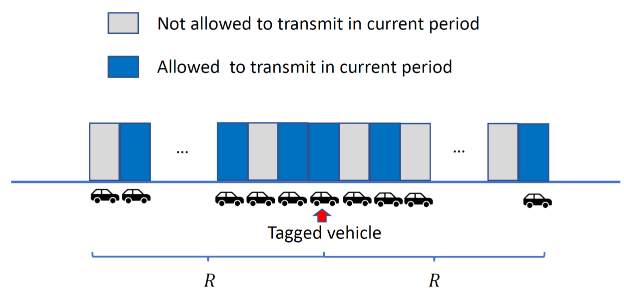


Fig. 4: Illustration of idealized scheduling in DSRC.

A. Transmission probability and inter-packet gap

Under idealized scheduling of DSRC, at most $\frac{T_{tr}}{T_0}$ vehicles can access the channel in one transmission period. When the number of vehicles within the sensing range is no greater than $\frac{T_{tr}}{T_0}$, all the vehicles within that range can transmit successfully without interfering with each other. When the number of vehicles within the sensing range is greater than $\frac{T_{tr}}{T_0}$, then some vehicles cannot access the channel. In this case, we assume that each vehicle has an equally likely chance to access the channel, which models the random access nature of CSMA. Thus, the transmission probability P_{tr} can be expressed as

$$p_{tr} = \min\left(\frac{T_{tr}}{2T_0 N_v R}, 1\right), \tag{12}$$

where the first term represents the probability that a given vehicle is assigned one of the $\frac{T_{tr}}{T_0}$ slots out of the $2RN_v$ vehicles in the sensing range. Under our idealized scheduling assumption, the packet delivery ratio for a vehicle within the hearing range will then be equal to p_{tr}.

[6]If a BSM is not transmitted before the next BSM is generated, it is dropped and replaced by the next BSM.

Another way to look at the impact of the transmission probability is via the inter-packet gap. In C-V2X, the inter-packet gap is always T_{tr} within the hearing range. However, in DSRC, the inter-packet gap can be expressed as

$$T_{gap} = \frac{T_{tr}}{p_{tr}}, \tag{13}$$

which now varies with N_v via expression for p_{tr} in (12).

B. Maximum hearing range

Although our ideal CSMA model removes any interference within the sensing range, there is still interference from vehicles outside of the sensing range. As we are still assuming perfect round robin scheduling (as in Section II), the maximum hearing range has a similar form as (7). When the number of vehicles within the sensing range is below the maximum supported number of vehicles, i.e., $2N_vR \leq \frac{T_{tr}}{T_0}$, the expression should be the same as (7). But when the vehicle density is above the threshold, the effective vehicle density cannot keep increasing, because CSMA restricts the access to the channel to avoid collisions (see Fig. 5). As a result, the maximum hearing range can be written as

$$R_{max} = \frac{\frac{T_{tr}}{T_0}}{\left(10^{\frac{SINR_{th}}{10*k}}+1\right)\min\left(N_v, \frac{T_{tr}}{2RT_0}\right)}. \tag{14}$$

Likewise, under a general admissible path loss model and including noise, R_{max} must satisfy an equation similar to that in (6).

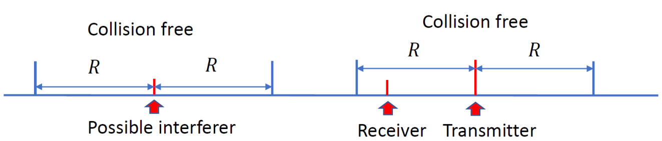


Fig. 5: Illustration of interference in DSRC.

IV. Comparison between C-V2X and DSRC

In this section, we compare the performance of C-V2X and DRSC under our idealized scheduling model. Our focus is on the differences between these two protocols at the MAC layer under high loads. To accomplish this, we assume that the maximum number of BSMs that can be transmitted in one transmission period is the same for C-V2X and DSRC, i.e., $N_r = \frac{T_{tr}}{T_0}$ and additionally assume that both schemes have the same SINR requirement (i.e., $SINR_{th}$ is the same for both schemes).[7] We again focus on the case where the path loss is given by (1) and noise is neglected.

We can see from (7) that the maximum hearing range of C-V2X is always decreasing with the vehicle density. Moreover, under C-V2X, the inter-packet gap within the hearing range is always equal to 1 under idealized scheduling since vehicles always transmit.

For DSRC, when the vehicle density satisfies $N_v \leq \frac{T_{tr}}{2RT_0}$, the maximum hearing range in (14) has the same form as (7) for C-V2X. However, when vehicle density $N_v > \frac{T_{tr}}{2RT_0}$, the hearing range becomes a constant. But at the same time, the inter-packet gap increases, because the transmission probability in (12) decreases.

[7] In practice, these protocols may differ in both of these dimensions. Such differences can easily be accommodate in our model. Here we assume them to be the same to isolate the MAC layer differences.

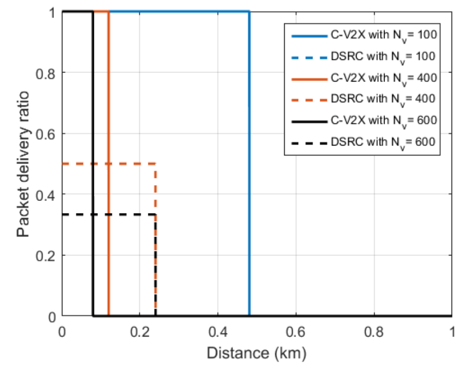
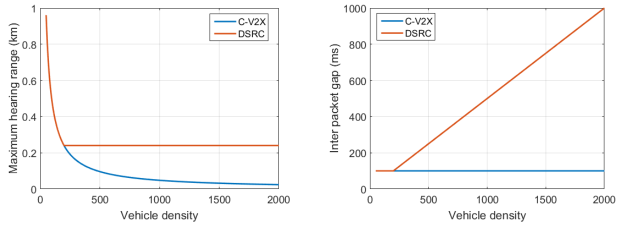


Fig. 6: Packet delivery ratio versus distance between receiving and transmitting vehicles under idealized scheduling for both C-V2X and DSRC.

To illustrate the differences between these two protocols, we define *packet delivery ratio* at certain distance d as the probability that one BSM can be successfully delivered to the tagged vehicle from a transmitter at distance d. We plot in Fig. 6 the packet delivery ratio with idealized scheduling versus distance for each protocol under several different vehicle densities. These curves have a "box" shape, where the width of the box corresponds to the maximum hearing range and the height is the packet delivery ratio within the hearing range. In these figures, for each protocol we set $T_{tr} = 100$ms, $N_r = \frac{T_{tr}}{T_0} = 200$, $SINR_{th} = 10$dB and $k = 2$. The sensing range in DSRC is set to $R = 0.5$km. In this case, when $N_v \leq 200$, the curve for DSRC coincides with the curve of C-V2X as neither system is congested. When N_v is greater than 200, for C-V2X only the width of the box area decreases. But for DSRC, instead of the width, the height of the box decreases. This shows a key difference between how C-V2X and DSRC respond to congestion. C-V2X tends to maintain a high packet delivery ratio (and thus a small peak age) but only within a shrinking hearing range. However, DSRC tends to maintain a large hearing range by limiting access to the channel with CSMA. The price is that packet delivery ratio within the hearing range can be very low (and the peak age can increase). As a result the inter-packet gap of DSRC within the hearing range can be much larger than T_{tr}. Fig. 7 illustrates this trade-off in another way. In Fig. 7(a), the hearing range for DSRC stops shrinking when vehicle density is above around 200 vehicles/km. Fig. 7(b) shows the corresponding inter-packet gaps versus vehicle density. At the point the hearing range stops shrinking for DSRC, the inter-packet gap for DSRC begins linearly increasing, while it remains constant for C-V2X.

Fig. 7 illustrates the trade-off between the inter-packet gap and hearing range . If the inter-packet gap is smaller, the hearing range is also smaller. As shown in the next theorem, there is a basic equation that characterizes this trade-off for our idealized models of C-V2X and DSRC.



(a) Maximum hearing range (b) Inter-packet gap

Fig. 7: Comparison between C-V2X and DSRC.

Theorem 1: In the idealized scheduling model, with a general admissible path loss function $P_r(P_t, d)$ and noise power N_0, for both C-V2X and DSRC, the maximum hearing range satisfies $R_{max} = f\left(P_t, \frac{T_{gap}}{T_0 N_v}\right)$, for a given function $f(x, y)$ which is nondecreasing in y given fixed x. Here, T_0 is average time duration for each BSM transmission and N_v is the vehicle density (in C-V2X, we view $T_0 = \frac{T_{tr}}{N_r}$).

The proof is omitted due to space consideration. If transmission power is the same for C-V2X and DSRC, we can see that R_{max} is always nondecreasing with T_{gap}. If we make some further assumptions, a more concise invariant between inter-packet gap and maximum hearing range can be found in the following corollary.

Corollary 1: In the idealized scheduling model for both the DSRC and CV2X, if the received power has the form of (1) and the noise power is 0, the following equation always holds:

$$\frac{T_{gap}}{R_{max}} = T_0 N_v (1 + 10^{\frac{SINR_{th}}{10k}}). \tag{15}$$

The invariant on the right-hand side of (15) is a function of physical layer performance (which determines both T_0 and the SINR threshold), vehicle density and the path loss model. A smaller value for this invariant means a better trade-off curve between T_{gap} and R_{max}. For example, a better physical layer performance or a better receiver reduces this value. The MAC layers in C-V2X and DSRC, each are constrained by (15), but manage the trade-off between T_{gap} and R_{max} differently at high loads.

V. Congestion control

So far we have assumed that neither C-V2X or DSRC used congestion control when the vehicle density increased. In this section we extend our models to include congestion control at high loads. Two natural ways of doing congestion control are either rate control or power control. In rate control, the algorithm adjusts the rate of generating BSMs. In power control, transmission power is adjusted. We begin next by considering the impact of these on our idealized scheduling model.

In Theorem 1 we showed that there is a basic trade-off between hearing range and inter-packet gap that holds for both our idealized models of C-V2X and DSRC. The next result shows that this is still true when rate and/or power control are applied to these models.[8]

[8]Note this result applies to the steady-state values obtained by rate/power control and is not considering the underlying dynamics.

Proposition 1: Under the idealized scheduling model for both the DSRC and CV2X, no matter how rate and power are adjusted, Theorem 1 and Corollary 1 still hold.

The proposition shows that when congestion control is applied, there is still a trade-off between T_{gap} and R_{max}. Rate control and power control each manage this trade-off in different ways. To illustrate the difference between rate and power control, we assume a path loss function as in (1) and ignore noise. We use a congestion control mechanism motivated by the SAE J2945/1 standard [15].[9]

The following four major parameters are used in the congestion control algorithm:

(a) N_{est}: this denotes the estimated number of vehicles within the range of 100m around a vehicle. This estimation is based on the messages received by the vehicle within a time-window and so is closely related to the packet error ratio.
(b) T_{tr}: this denotes the transmission period of a BSM. The range of T_{tr} is [100ms, 600ms], and it is determined as a function of N_{est}.
(c) CBP: this is the channel busy percentage. In DSRC, it denotes the percentage of time the channel was sensed busy. In C-V2X, CBP can be viewed as the percentage of resource blocks that are above an energy threshold.
(d) P_t: this denotes the transmission power. The range of transmission power is $[10\text{dBm}, 20\text{dBm}]$ and is determined by the sensed CBP before each BSM transmission.

The rate or the transmission period is controlled by N_{est}. The rate is adjusted according to the following function:

$$T_{tr}(\text{in ms}) = \begin{cases} 100, & N_{est} \le 25 \\ 4N_{est}, & 25 < N_{est} \le 150 \\ 600, & N_{est} > 150. \end{cases} \tag{16}$$

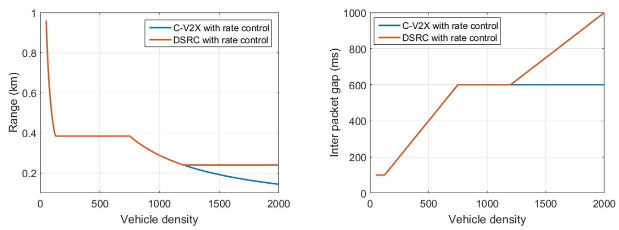
Using the idealized scheduling model and the rate control function in (16), we can calculate the maximum hearing range and inter-packet gap in steady-state. When $N_{est} \le 25$ or $N_{est} > 150$, T_{tr} is constant, so the calculation is the same as in Section II and III. When $25 < N_{est} \le 150$, T_{tr} grows linearly with N_{est}. From (7) and (14), we can see that the hearing range under either protocol is a constant in this range.

Figure 8 shows the same two plots as in Fig. 7, but with rate control used. In Fig. 8(a), we can see that for both C-V2X and DSRC, the hearing range stops shrinking when rate control is in the linear range. But at the same time, the inter-packet gap actually increases, because BSMs are sent with a slower rate. Rate control helps maintain a large hearing range but the price is to hear from the surrounding vehicles less often. Also, when rate control is applied, our idealized C-V2X and DSRC only differ at very high vehicle density, in which case we see the same behavior as in Section IV.

Transmission power is adjusted based on CBP according to following function:

$$P_t = \begin{cases} 20, & CBP \le 50\% \\ 20 - \frac{10(CBP-50\%)}{30\%}, & 50\% < CBP \le 80\% \\ 10, & CBP > 80\%. \end{cases} \tag{17}$$

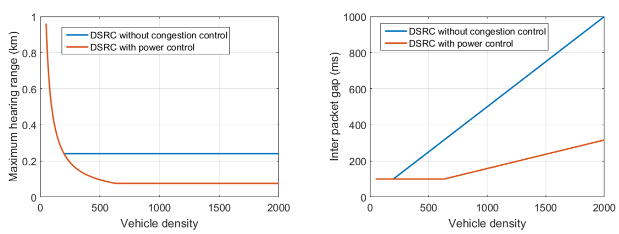
[9]This standard was developed for DSRC. A similar standard is being developed for C-V2X; here, we simply assume that the same mechanism is used in the C-V2X case.



(a) Maximum hearing range (b) Inter-packet gap

Fig. 8: Comparison between C-V2X and DSRC with rate control.

For C-V2X, we can see from (7), the transmission power has no impact on hearing range and inter-packet gap when noise is ignored, as the system is interference limited. However transmission power matters for DSRC even when noise is ignored, because the sensing range is dependent on the transmission power as in (11). When the transmission power is lower, the sensing range of each vehicle is reduced so that the transmission probability in (12) is non-decreasing. As a result, the inter-packet gap should be non-increasing. However, the price for sending BSMs more frequently is that the hearing range is reduced. From (14) we can see that the maximum hearing range in non-increasing when transmission power is lower. An example is shown in Fig. 9, where we can see the trade-off from using power control clearly.



(a) Maximum hearing range (b) Inter-packet gap

Fig. 9: Comparison for DSRC with and without power control.

We can see that power control tends to improve the inter-packet gap by sacrificing some hearing range, while rate control does the opposite. Therefore, both rate and power control can be viewed as different ways of managing the trade-off between inter packet gap and hearing range given the fixed invariant in (15).

VI. Conclusion

In this paper, we considered the trade-off between the peak age of information (measured by the inter-packet gap) and the reachability of information (measured by the maximum hearing range) in a vehicular safety application. We proposed an idealized model of both C-V2X and DSRC, where resource are maximally allocated to vehicles to keep interferers as far as possible. We showed that these two protocols trade-off peak age and reachability in different ways at high loads and characterized an underlying relation governing this trade-off. Furthermore, we used the proposed model to analyze the performance of rate and power control mechanisms.

We considered the age-reachability trade-off for two specific protocols. One possible future direction would be to understand if this relationship holds for a larger class of MAC protocols. We also focused on an idealized model, studying a more realistic MAC protocol is another possible extension as would be looking at more complicated models of vehicle deployments.

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