Market Competition between LTE-U and WiFi

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Abstract—LTE-U is an extension of the Long Term Evolution (LTE) standard for operation in unlicensed spectrum. LTE-U differs from WiFi, the predominant technology used in unlicensed spectrum in that it utilizes a duty cycle mode for accessing the spectrum and allows for a more seamless integration with LTE deployments in licensed spectrum. There have been a number of technical studies on the co-existence of LTE-U and WiFi. In this paper, we instead investigate the impact of such a technology from an economic perspective. We consider a model where an incumbent service provider (SP) deploys a duty cycle-based technology like LTE-U in an unlicensed band along with operating in a licensed band and competes with one or more entrants that only operate in the unlicensed band using a different technology like WiFi. We characterize the impact of a technology like LTE-U on the market outcome and show that the welfare impacts are subtle, depending in part on the amount of unlicensed spectrum and number of entrants. The difference in spectral efficiency between LTE and WiFi also plays a role in the competition. Finally, we investigate the impact of the duty cycle and the portion of unlicensed spectrum used by the technology.

Index Terms—LTE-U, WiFi, Network Economics, Price Competition, Wardrop Equilibrium.

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1 INTRODUCTION

OFFLOADING traffic to unlicensed spectrum has been a vital approach for wireless service providers (SPs) to meet the ever rising demand for mobile data and retain control over profit margins [2]–[4]. Indeed, in 2016, there was more mobile data traffic offloaded to unlicensed bands than served in licensed spectrum worldwide [5]. These trends are expected to continue with 5G [6], [7] and have led to the development of technologies for unlicensed access that are based on the LTE technology that SPs utilize in licensed spectrum. The two main examples of this are LTE in unlicensed spectrum (LTE-U) and License Assisted Access (LAA). These differ in several ways from the WiFi technologies that are widely used in the same unlicensed spectrum. For example, both LTE-U and LAA utilize LTE's carrier aggregation capability to essentially combine a SPs licensed and unlicensed spectrum. Moreover, LTE-U differs in that it does not employ a listen-before-talk (LBT) protocol as used by WiFi, but instead is based on a duty-cycle based approach.[1] This led to much interest in studying the co-existence of WiFi and LTE-U from a technical point of view, e.g. [9]–[18]. Besides technical issues, economic issues are also important especially to spectrum regulators. There is considerable evidence that unlicensed spectrum has huge economic value. Past estimates, which already look too conservative, place the value created by current applications of unlicensed spectrum at 16-37 billion dollars a year in the United States alone. A goal of this work is try to understand

the potential economic impacts of technologies such as LTE-U on this key spectrum resource. Namely, we seek to understand the impact of a SP using a technology like LTE-U on the competition with other SPs that utilize a technology like WiFi.

We consider a scenario similar to that in [19], [20], where SPs compete for customers by announcing prices for their service (see also [21]–[27]). The customers select SPs based on the sum of the price they pay for service and a congestion cost that is incurred for using the given band of spectrum. In [19], [20], the SPs compete by announcing one price for service in an unlicensed band and a different price for service in any licensed band that the SP may own. When the LTE-U technology is adopted, we instead assume the SP can announce a single price because of the seamless integration of LTE technology on both the licensed and unlicensed bands. In our model, the duty cycle mechanism of LTE-U is considered. Under the duty cycle setting, the incumbent SP (with licensed spectrum) is able to use both its licensed spectrum and a portion of the unlicensed spectrum to serve customers when the duty cycle is 'ON' while it can only use its own licensed band when the duty cycle is 'OFF'. In such a scenario, we model customers as being sensitive to the average congestion they experience across the whole duty cycle. In addition, LTE-U can provide higher spectrum efficiency than WiFi systems [28]. This is because LTE is a synchronous system and adopts scheduling-based channel access instead of contention-based random access [29]. In our model, we use a spectrum efficiency factor to capture the difference while ignoring other complicated technical differences between LTE and WiFi.

We use α and β to denote the duty cycle and the portion of unlicensed spectrum that are used for LTE-U, respectively. We first consider α and β as fixed parameters, e.g. determined by a regulator. For example, currently LTE-U channel bandwidth is set to 20 MHz which corresponds to the smallest channel width in WiFi and Qualcomm recommends that LTE-U should use a period of 40, 80 or 160 ms, and limits the maximal duty cycle to 50% [30].

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1. LAA does employ LBT, which is required in some parts of the world. LTE-U was developed first and is being deployed in countries where LBT is not required for unlicensed channel access. According to [8], eleven operators have announced investments in LTE-U networks. Three LTE-U networks have been deployed/launched in three countries; eight other operators have been investing in the technology in the form of trials or pilots in seven countries.

We show that in the monopoly market, using LTE-U hurts the SP's revenue and social welfare. We then show that when there are multiple entrant SPs in the market, adopting LTE-U technology can help the incumbent SP to increase revenue and also benefit social welfare when the bandwidth of unlicensed spectrum is small. When there is only one entrant SP in the market, we show that it is possible for LTE-U technology to hurt the revenue of the incumbent. A simple intuition behind this result is that one entrant SP does not bring enough competition to the market, which makes it a intermediate case between the monopoly case and the fully competitive multiple entrants case. We also investigate the impact of differences in the spectral efficiency between LTE and WiFi. We show that if the efficiency advantage of LTE over WiFi is large, LTE-U may benefit the incumbent's revenue and customer welfare in different competition scenarios.

Next, we consider α as a controllable parameter with fixed β. We show that with multiple entrant competitors, the incumbent's revenue increases with α while with one entrant, the incumbent may prefer a small α. Finally, we consider varying α and β while keeping the utilization of unlicensed spectrum ($\alpha\beta$) constant. We show that when the unlicensed bandwidth is small, the incumbent may prefer lower α and higher β. But when the unlicensed bandwidth is large, the incumbent may prefer a higher α and a lower β.

In terms of other related work, [31] also considers an economic model of LTE-U and WiFi. In [31] the focus is not on competition between LTE-U and WiFi providers (there is only one licensed service provider) but rather on understanding how LTE-U impacts the service selection of a finite number of users, each with a "congestion tolerance" for the service they select. In their work, WiFi is a free option that is congestible, while the licensed service is not congestible but is available at a cost. In [32], the authors propose an auction based spectrum sharing framework to investigate the possibility of cooperation between LTE and Wi-Fi in an unlicensed band. The proposed mechanism makes SPs explore the potential benefits of cooperation before deciding whether to enter head-to-head competition. In [33], the authors mainly focus on competition instead of cooperation. They analyze the market impact when the incumbent SP offers a bundle price for service on licensed and unlicensed band to compete with entrant SPs. Different from bundling services, this paper focuses on the bundling of spectrum, because LTE-U aggregates the spectrum directly, which is able to provide more seamless service to customers.

The rest of the paper is organized as follows. Our model is described in Section 2. We first consider the monopoly case in Section 3. Then we treat α and β as fixed parameters in Section 4 and compare the results with the monopoly case and the model in [19], [20]. In Section 5, we look at how different spectral efficiencies impact the competition in the market. In Section 6, we view α and β as controllable variables and investigate their impact on the incumbent SP. Some numerical results are shown in Section 7. Finally, we conclude in Section 8. Some of proofs are omitted due to space limit. We provide complete proofs in our technical report [34].

2 SYSTEM MODEL

We consider a market with one incumbent SP and N entrant SPs, where the incumbent SP uses a duty-cycle based technology such as LTE-U. In the following we will simply refer to this as LTE-U, though as noted previously this is not intended to model every aspect of LTE-U. Also note that we only consider one incumbent SP in this paper, in part to simplify the model and in part to highlight a case where the incumbent has the greatest market power. Considering multiple incumbents adds several new dimension to the problem, which we leave for future work. The incumbent SP is assumed to possess its own licensed band of spectrum with bandwidth B, while entrants have no licensed spectrum. There is a single unlicensed band with bandwidth W that can be used by both the incumbent and entrant SPs. When the incumbent SP applies the LTE-U technology, it uses carrier aggregation on the unlicensed band and operates in a duty cycle mode. When LTE-U is in 'ON' mode, we assume that it is always using a portion of the unlicensed spectrum so that entrant SPs are not able to operate over this portion (e.g. due to LBT, the entrants would sense the incumbents presence and not transmit). We use α to denote the percentage of time that the SP aggregates the unlicensed spectrum. We use β to denote percentage of unlicensed spectrum that the incumbent uses when the duty cycle is in 'ON' state, i.e., when LTE-U is 'ON', the bandwidth that the incumbent can use becomes $B + \beta W$.

The SPs are assumed to compete for a common pool of infinitesimal customers by setting prices for their services. Without loss of generality, we assume that the incumbent is SP 1, and all the entrant SPs are indexed from 2 to $N + 1$. The price announced by SP i is denoted by p_i. The SPs serve all customers that accept their price. The revenue of SP i is then $x_i p_i$, where x_i is the customer mass that accept price p_i.

As in [21], [22], [33], a SP's service is characterized by a congestion cost. The congestion that the customers experience in a band is denoted by $g(X, Y)$, which is assumed to be increasing in the total customer mass X on the band and decreasing in the bandwidth used Y. Here, we assume a specific form $g(\frac{X}{Y})$, where $g(\cdot)$ is a convex increasing function with $g(0) = 0$ and $\frac{X}{Y}$ is the number of users per unit bandwidth. When the incumbent SP applies LTE-U technology, the congestion that the customers experience will vary across the duty cycle. We assume that customers are sensitive to the average congestion across the duty cycle.[2] The average congestion of customers served by the incumbent SP is then given by

$$\hat{g}_{in}(x_1) = \alpha g\left(\frac{x_1}{B + \beta W}\right) + (1 - \alpha)g\left(\frac{x_1}{B}\right).$$

The average congestion experienced by customers who choose an entrant SP is

$$\hat{g}_{en}(\mathbf{x}) = \alpha g\left(\frac{\sum_{j=2}^{N+1} x_j}{(1 - \beta)W}\right) + (1 - \alpha)g\left(\frac{\sum_{j=2}^{N+1} x_j}{W}\right).$$

2. This is reasonable as over the time-scale that customer select SPs they will receive service over many duty cycles.

Note that as in [19] [20], the congestion experienced in the unlicensed band by a customer of an entrant SP is the same for all entrants and depends on the total traffic across all entrants. This is modeling the fact that the entrants are all employing a technology like WiFi to share this band. Also note that we assume that when the LTE-U duty cycle is on, the entrant can only use the remaining $(1 - \beta)W$ of the spectrum.

As in [19], [20], we assume that customers seek to receive service from the SP with the lowest *delivered price*, which is given by the sum of the announced price and the average congestion cost of that SP's service. This captures the fact that customers are sensitive both to cost of service and the quality of service. Hence, for the incumbent SP, the delivered price $d_1(p_1, \mathbf{x})$ is denoted by $p_1 + \hat{g}_{in}(x_1)$. For an entrant SP i, $i \geq 2$, its delivered price $d_i(p_i, \mathbf{x})$ is given by $p_i + \hat{g}_{en}(\mathbf{x})$.

We assume that customers are characterized by an inverse demand function $P(q)$, which indicates the delivered price at which a mass of q customers are willing to pay for service. As in much of the prior literature, we assume $P(q)$ is concave and non-increasing. Each customer is infinitesimal so that a single customer has a negligible effect on the congestion in any band. Therefore, given the announced price by the SPs, the demand of service for each SP i is assumed to satisfy the Wardrop equilibrium conditions [35]. In our model, the conditions for the SPs are

$$d_i(p_i, \mathbf{x}) = P\left(\sum_{j=1}^{N+1} x_j\right), \text{ for } x_i > 0,$$

$$d_i(p_i, \mathbf{x}) \geq P\left(\sum_{j=1}^{N+1} x_j\right), \text{ for } \forall i.$$

The conditions imply that at the Wardrop equilibrium, all the SPs serving a positive amount of customers will end up with the same delivered price, which is given by the inverse demand function. A Nash equilibrium of the game is one in which the customers are in a Wardrop equilibrium and no SP can improve their revenue by changing their announced price (anticipating the impact this will have on the Wardrop equilibrium).

At an equilibrium, the customer surplus is defined as the difference between the delivered price each customer pays and the amount it is willing to pay, integrated over all the customers, i.e.,

$$CS = \int_0^Q P(q) - P(Q)dq, \tag{1}$$

where $Q = \sum_j x_j$. The social welfare of the market is the sum of consumer welfare and the SPs' revenue:

$$SW = CS + \sum_j p_j x_j. \tag{2}$$

3 MONOPOLY SCENARIO

We first examine a scenario in which there is only a single incumbent and no entrants. Hence, the incumbent is a monopolist and can use both the licensed and unlicensed band. Our goal in this section is to show that for our LTE-U model, such a monopolist would have no incentive to deploy the new technology. This shows that in later sections when the incumbent does deploy such a technology, it is due to competitive factors and not an inherent advantage of the technology.

In this section, we allow the incumbent to offer both service using LTE-U (with a given α and β) and additionally an "unlicensed service" that uses the remainder of the unlicensed band when the LTE-U duty cycle is off.[3] This ensures that the amount of spectrum the incumbent has access to stays the same with and without LTE-U. As a result, we can compare the incumbent's revenue in these two cases with the same total amount of spectrum. The incumbent's revenue optimization is then given by:

$$\max_{p_1^l, p_1^u} \quad p_1^l x_1^l + p_1^u x_1^u \tag{3}$$

$$\text{s.t.} \quad p_1^l + \alpha g\left(\frac{x_1^l}{B+\beta W}\right) + (1-\alpha)g\left(\frac{x_1^l}{B}\right) = P(x_1^l + x_1^u),$$

$$p_1^u + \alpha g\left(\frac{x_1^u}{(1-\beta)W}\right) + (1-\alpha)g\left(\frac{x_1^u}{W}\right) = P(x_1^l + x_1^u),$$

$$p_1^l, p_1^u \geq 0.$$

Here, p_1^l is the price the incumbent offers for serving x_1^l customers using LTE-U; p_1^u and x_1^u are the corresponding values for the unlicensed service. The first two constraints enforce the Wardrop equilibrium conditions for licensed and unlicensed services, respectively. Also note that if we set $\alpha = 0$ and $\beta = 0$, then this reduces to a model as in [19] in which the incumbent does not employ LTE-U and offers separate licensed and unlicensed service.

Consider the expected congestion for the LTE-U service in (3). Given the convexity of the congestion function $g(\cdot)$, we have the following inequality:

$$\hat{g}_{in}(x_1^l) \geq g\left(\frac{\alpha x_1^l}{B+\beta W} + (1-\alpha)\frac{x_1^l}{B}\right)$$

$$\triangleq g\left(\frac{x_1^l}{B_e}\right),$$

where B_e denotes the *equivalent licensed bandwidth* given by

$$B_e = B + \frac{\alpha\beta W}{1 + \beta(1-\alpha)W/B}. \tag{4}$$

Similarly, considering the congestion for the unlicensed service, we have the *equivalent unlicensed bandwidth* W_e given by

$$W_e = W - \frac{\alpha\beta W}{1 - \beta(1-\alpha)}. \tag{5}$$

Note that the congestion is no smaller than in a setting where the incumbent offers separate licensed and unlicensed services (without LTE-U) using the equivalent bandwidth and equality holds when the congestion function $g(\cdot)$ is linear. Also note that we call B_e and W_e equivalent bandwidth, because the congestion that customers experience when LTE-U is used is equivalent to the congestion on bands with the corresponding equivalent bandwidth without LTE-U. Based on this, we have the following result.

Theorem 3.1. In a monopoly scenario, the incumbent SP can gain no additional revenue by using the LTE-U technology.

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3. In subsequent sections, the incumbent will only offer service using LTE-U or unlicensed service, instead of this combination.

The detailed proof is in Appendix A. In the proof it is also shown that although the incumbent has the option to offer two different prices for licensed and unlicensed service, respectively, its optimal strategy is to offer the same price. Also notice that the equivalent licensed bandwidth increases and the equivalent unlicensed band decreases but the total amount of equivalent bandwidth in (4) and (5) decreases when adopting LTE-U. This is what prevents the incumbent from benefiting when adopting LTE-U.

Theorem 3.2. In a monopoly scenario, both customer surplus and social welfare are non-increasing when LTE-U is adopted.

The detailed proof is in Appendix B. As a result of the loss of equivalent bandwidth, customer surplus and social welfare are also non-increasing when LTE-U is used. Note that in this section, we assume the spectral efficiency of LTE and WiFi are the same. In Section 5, we will see that different results emerge when differences in spectral efficiency are accounted for.

4 COMPETITION WITH FIXED α AND β

We now study the case where there is competition between the incumbent and one or more entrants. We first consider the case where there are multiple entrants and then consider the special case of one entrant. In both cases we will see that unlike the previous section, the incumbent may now have an incentive to deploy LTE-U. Throughout this section we assume that α and β are fixed.

4.1 One incumbent & multiple entrants

Initially, we assume that there are $N \geq 2$ entrants. Hence, these entrants will compete with each other as well as the incumbent. The presence of this competition yields the following result on the entrants' equilibrium prices.

Lemma 4.1. If there are at least two entrant SPs in the market, in any Nash equilibrium every entrant SP i serving a positive mass of customers must have $p_i = 0$ and at least two SPs must announce this price.

Lemma 4.1 is similar to a result in [19] where firms compete in unlicensed spectrum without LTE-U, and so we omit the proof. Essentially, since the entrant SPs are all offering the same service due to sharing the same spectrum, they will be incentivized to compete the price for this service to zero. For example, every time a SP announces a positive price, its opponent can announce a price slightly lower to grab the whole market. Such competition will finally drive the service price to 0. Hence, all the entrant SPs obtain no revenue regardless of the incumbent's actions.[4] A corollary of this result is that if the incumbent SP also offers a separate service on the unlicensed band (as in the monopolist case), its price for this would also be zero so that the incumbent would have no incentive to offer such a service in this setting. Based on Lemma 4.1 we give the following result on the incumbent's revenue.

4. Note here we are ignoring any cost for offering service. If such a cost was included, the result would be that the price is competed down to cost, still yielding zero profit.

Theorem 4.1. Consider one incumbent and multiple entrants. Given a fixed $\alpha > 0$ and $\beta > 0$, the following hold:
1) The incumbent SP announces a higher price and attracts more customers when LTE-U is adopted. As a result the incumbent SP gets a higher revenue.
2) The customer mass served by the entrant SPs decreases when the incumbent SP uses LTE-U technology.
3) The total customer mass served by the incumbent and entrant SPs is lower when LTE-U is adopted.

As in the previous section, the use of LTE-U increases the equivalent licensed bandwidth. However, now that there are multiple entrants keeping the price on the unlicensed band zero, this benefit to the incumbent is not offset with a loss due to the reduced equivalent unlicensed bandwidth. Due to the improved service by adopting LTE-U, the incumbent is able to announce a higher price and at the same time attract more customers. This leads to an increase in the revenue. However the delivered price will increase, resulting in fewer customers served.

Theorem 4.2. Customer surplus with LTE-U is non-decreasing as the amount of unlicensed bandwidth W increases but is always less than the consumer surplus achieved without LTE-U.

Theorem 4.2 shows that adding more unlicensed spectrum benefits customers while adopting LTE-U always hurts customers when there are multiple entrant SPs in the market. This is because increasing the bandwidth of unlicensed spectrum actually increases the amount of resources for both the incumbent and entrant SPs. However, when LTE-U is used, there is some loss in the total equivalent bandwidth and the incumbent is able to increase its price, leading to a loss in customer surplus. We can consider an extreme example when the congestion is linear. If we add a very large amount of unlicensed spectrum to the market so that $W \to \infty$, we lose $\frac{(1-\beta)W}{1-\beta+\alpha\beta} \to \infty$ on the unlicensed band while we only gain $\frac{\alpha B}{1-\alpha}$ on the licensed band. The overall resource loss would result in a decrease of customer surplus if LTE-U is used.

So far we have seen that with multiple entrants, LTE-U increases firm profits (namely those of the incumbent) but decreases consumer welfare. We next consider the impact of this technology on the overall welfare which includes both of these factors. We first use a simplified example to gain insight and then give a more general result.

Consider a flat inverse demand function and linear congestion cost, i.e., $P(q) = T$ if $q \leq A$, $P(q) = 0$, otherwise, and $g(x) = x$. Here, A can be viewed as the size of the market while T indicates the valuation of every consumer. We then have the following result.

Theorem 4.3. With a flat inverse demand function and linear congestion, if $W \leq \frac{\sqrt{A^2+B^2T^2}-BT+A}{2T}$, social welfare will always increase when LTE-U is adopted for any $\alpha, \beta > 0$. Otherwise, social welfare can either increase or decrease when LTE-U is adopted.

The proof is in Appendix C. Theorem 4.3 shows that under the assumption of a flat inverse demand function and linear congestion, when the unlicensed bandwidth W is small, adopting LTE-U is good for social welfare no matter what α and β are. Note that in these cases, customer

surplus is always zero and all the social welfare comes from the revenue of the incumbent SP. When W is beyond the threshold in the theorem, it becomes unclear how social welfare changes with LTE-U. It can depend on the choice of α and β. Also note that the threshold bandwidth is an increasing function of $\frac{A}{T}$, which is the ratio between market size and customer valuation. When there are more customers in the market or the customer's valuation goes down, the threshold goes up. This means that there is a larger range of W for which LTE-U increases welfare. This is because when there are more customers, or the customers have a lower valuation, the incumbent is incentivized to serve more customers to increase its revenue.

The following results extend Theorem 4.3 to a general inverse demand function and a general congestion function.

Theorem 4.4. Given a fixed $B > 0$, $\alpha > 0$ and $\beta > 0$, There exists some $W_{th} > 0$ such that when $W < W_{th}$ adopting LTE-U achieves a higher social welfare than that without LTE-U. But when W is large enough, LTE-U always hurts social welfare.

A linear approximation method as in [36] is used to prove the theorem. When LTE-U is adopted, Theorem 4.2 shows customer surplus decreases, which means the delivered price should increase. When the bandwidth of unlicensed spectrum is small, the increase in revenue of the incumbent is able to compensate for the customer surplus loss, so that the overall social welfare can increase. But when W is large, the advantage of LTE-U may not be large enough to raise the delivered price to make up for the loss of customer surplus, which will result in a loss of social welfare. The detailed proof is in Appendix D. Note that the threshold for W in Theorem 4.3 is an example of W_{th} in the special case of a flat inverse demand function and linear congestion function.

4.2 One incumbent & one entrant

We next consider the case with only one entrant. If only this entrant is offering unlicensed service, then Lemma 4.1 no longer applies and so this case requires a separate analysis. Before considering the impact of LTE-U, we first consider two possible ways the incumbent SP could act without this technology: it could compete with the entrant to serve customers on the unlicensed band or it could only serve customers on the licensed band. We call the first case *unlicensed sharing* and in this case, the results are the same as when an incumbent without LTE-U competes with multiple entrants. We call the second case *licensed sharing*; in this case, the entrant SP is able to use the unlicensed spectrum exclusively. The objective of each SP is still to maximize revenue while the Wardrop equilibrium conditions are satisfied. To be precise, in the licensed sharing case, the conditions for the entrant SP on the new band become

$$p_2 + g\left(\frac{x_2}{W}\right) = P\left(x_1 + x_2\right), \text{if } x_2 > 0$$
$$p_2 + g\left(\frac{x_2}{W}\right) \geq P\left(x_1 + x_2\right), \text{otherwise.}$$

We first give a brief result to compare the licensed sharing and unlicensed sharing cases without LTE-U.

Lemma 4.2. In the case with one incumbent and one entrant SP, both the incumbent and entrant SPs are able to gain higher revenue with licensed sharing than with unlicensed sharing.

Lemma 4.2 shows that rather than making the spectrum unlicensed, both the incumbent and entrant would prefer that it is exclusively licensed to the entrant SP. However, note that if the incumbent has the option of unlicensed sharing, then the licensed sharing case will not be an equilibrium as the incumbent would always want to enter the unlicensed market and capture some of the entrant's revenue (even though this would eventually lead it to earning lower revenue). In particular, when the incumbent and entrant are in the equilibrium of licensed sharing, both incumbent and entrant serve their customers with a positive price. Suppose at some point unlicensed sharing becomes allowed so that the incumbent is able to serve customers on the unlicensed band. In this case, if the incumbent offers a slightly lower price on the unlicensed band than the entrant, all users on the unlicensed band would switch to the incumbent because it is offering these users the same service quality at a lower price. This would improve the incumbent profits. However, the entrant can in turn announce an even lower price to retain the customers. Such competition will finally drive the service price on unlicensed band to 0, where both incumbent and entrant end up with lower revenue.

Next we study the impact of LTE-U and in particular compare this to the licensed sharing case (which as noted above gives an upper bound on the incumbent's revenue in the unlicensed sharing case). In this subsection, we assume a linear congestion function $g(x) = x$ and inverse demand function $P(x) = 1 - x$ to simplify the calculations and give some insights.

Theorem 4.5. With a linear congestion cost and inverse demand, we have the following comparisons with licensed sharing:

1) When $\frac{B}{1-\alpha} < \frac{4}{3}$, the incumbent SP can always gain higher revenue with LTE-U. Otherwise, the incumbent can be either better or worse off with LTE-U (depending on the parameter values).
2) For any $\alpha, \beta \in (0, 1)$, there always exists some W_{th}, such that when $W < W_{th}$, the incumbent SP can gain higher revenue with LTE-U.

Both statements in Theorem 4.5 give sufficient conditions to guarantee a larger revenue for the incumbent SP with LTE-U. Equations (4) and (5) show that LTE-U increases the amount of resources of the incumbent and at the same time reduces the amount of resources of the entrant. Intuitively, this should lead to higher revenue for the incumbent with LTE-U. The first statement in Theorem 4.5 shows that this intuition holds when the incumbent's licensed spectrum is sufficiently small. However, when there is a large enough amount of licensed spectrum, the incumbent SP may suffer a loss in revenue with LTE-U. This is because the incumbent can already serve a large amount of customers on the licensed band and reducing the entrant's resources causes it to reduce the delivered price, lowering the incumbent's revenue. The second statement of Theorem 4.5 claims that as long as there is not too much unlicensed spectrum, the incumbent is always willing to adopt LTE-U, which yields a higher revenue. That is because, when W is relatively

small, using LTE-U can decrease the equivalent bandwidth of the entrant competitor, which increases the congestion on the unlicensed band significantly, giving an advantage to the incumbent SP. But when W is large, the decrease of the entrants' spectrum resource does not have a significant impact on congestion. As a result LTE-U can not increase the customer mass served by the incumbent enough to compensate for the lowered price due to competition. So the incumbent may not want to use LTE-U.
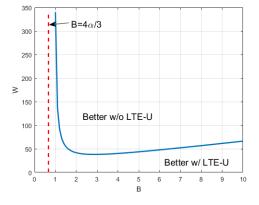

Fig. 1: The bandwidth regions where LTE-U is better and worse for the incumbent's revenue with $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$.

We use Fig. 1 to illustrate the region where the incumbent SP can get more revenue with LTE-U. We choose $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{2}$ in the figure. When B and W lie below the blue curve, the incumbent is better off with LTE-U. The red dashed line represents $B = \frac{4\alpha}{3}$. The blue curve approaches to the red line asymptotically when $W \to \infty$. We can also see that the unlicensed bandwidth threshold W_{th} is relatively large compared to the threshold for licensed bandwidth (the red dashed line). That means in most practical cases, the incumbent SP would be willing to use the LTE-U technology.

As we have seen, when there is one entrant SP, using LTE-U can yield lower revenue for the incumbent in the licensed sharing case. However, in the unlicensed sharing case, we have the following result.

Theorem 4.6. When there is only one entrant SP and unlicensed sharing is used, the incumbent always achieves higher revenue with LTE-U.

This result follows Theorem 4.1 and Lemma 4.2. The reason is that when unlicensed sharing is used without LTE-U, the price on the unlicensed band is zero due to the competition between the incumbent and entrant SPs, which hurts the incumbent's revenue. If LTE-U is used, the incumbent does not compete with the entrant on the unlicensed band directly. Instead, the incumbent accesses the unlicensed band through LTE-U. In this case, the service price on the unlicensed band is not zero and the incumbent gains more revenue consequently.

Next we characterize the customer surplus in the case of one incumbent and one entrant SP.

Theorem 4.7. When there is one incumbent and one entrant SP, for any values of B, α and β, customer surplus is non-decreasing with unlicensed bandwidth W. Also,

there exists some $W_{th} \geq B$, such that when $W \leq W_{th}$, customer surplus decreases when LTE-U is adopted compared to licensed sharing and when $W > W_{th}$, customer surplus increases.

The first result in Theorem 4.7 is consistent with that in the multiple entrants case, which shows customer surplus increases with the bandwidth of unlicensed spectrum. The second result is slightly different; it shows that LTE-U can hurt customer surplus when W is relatively small but it is able to improve customer surplus when W is large, while in the multiple entrants case, customer surplus always becomes worse with LTE-U. The reason is that when there is only one entrant SP, the price on the unlicensed band is not zero, which means a certain amount of consumers in the market are not served. When W is large and LTE-U is used, the incumbent can use the additional unlicensed spectrum to alleviate congestion without hurting the entrant SP too much. As a result, more consumers can be served and customer surplus increases. Also note that when W is relatively small, the loss in consumer surplus is balanced out by the increase in the SP's revenue.

However, when comparing to unlicensed sharing, the competition between the incumbent and entrant drives the price on the unlicensed band to 0. As a result, the results in Theorem 4.2 still hold if the incumbent and entrant SP use unlicensed sharing.

We next examine how social welfare changes when LTE-U is adopted in the asymptotic case when $W \to \infty$.

Theorem 4.8. In the case with one incumbent and one entrant SP with licensed sharing, if $W \to \infty$, social welfare always increases when LTE-U is adopted.

Theorem 4.8 shows that in the single entrant case, social welfare is higher with LTE-U when there is a large amount of unlicensed spectrum. Recall that in Theorem 4.4, we show in the case with multiple entrant SPs, social welfare is larger with LTE-U when the bandwidth of unlicensed spectrum is small. The intuition is that with multiple entrants, the price is competed to 0 on the unlicensed band. As a result, the increment in revenue can only cover the loss of customer surplus when a small mass of customers are served in the market, which implies a small W. However, in the single entrant case, the price is not zero and there can be a larger amount of customers unserved in the market, which leaves enough room for customer surplus to improve. When W is large, the improvement of customer surplus can make up for the loss of revenue by the SPs. Note that, $W \to \infty$ is a sufficient condition for social welfare to increase when LTE-U is adopted. It holds for any value of B, α, β. For some choices of B, α, β, W does not need to be extremely large to increase social welfare with LTE-U.

Theorem 4.8 also suggests that if a regulator wants to increase social welfare with LTE-U, it would be better to limit the number of entrant SPs to one in the market and to encourage licensed sharing if the bandwidth of the unlicensed band is large. However, when the amount of unlicensed spectrum is limited, it is better to encourage more entrant SPs to enter the market, which is able to benefit social welfare according to Theorem 4.4.

5 IMPACT OF DIFFERENT SPECTRAL EFFICIENCIES

Compared to 802.11ac, LTE-U offers a higher spectral efficiency due to features such as Hybrid ARQ and CSI feedback [37], [38]. In this section, we examine the market impact of these gains in different scenarios.

First, we look at the model used in the analysis. Without loss of generality, we assume the spectral efficiency of WiFi is 1. We assume that the spectral efficiency of LTE is $\gamma \in [1, \infty)$.[5] A higher spectral efficiency leads to lower congestion on the same band, which we model by scaling the bandwidth on that band. As a result, the congestion experienced by customers when using LTE-U is given by

$$\hat{g}_{in}(x_1) = \alpha g \left(\frac{x_1}{\gamma (B + \beta W)} \right) + (1 - \alpha) g \left(\frac{x_1}{\gamma B} \right). \quad (6)$$

The resulting equivalent bandwidth of the incumbent SP is

$$B_e = \gamma \left[B + \frac{\alpha \beta W}{1 + \beta (1 - \alpha) W / B} \right]. \quad (7)$$

The equivalent bandwidth of the entrant SP is still the same as (5), because only LTE traffic is affected by γ.

5.1 Monopoly market

We again begin by looking at a monopoly market. In Section 3, we showed that the incumbent SP cannot increase its revenue with LTE-U. But when spectral efficiency is different between LTE and WiFi, this is no longer true as shown next:

Theorem 5.1. In a monopoly scenario, the incumbent SP can gain more revenue by using LTE-U if and only if

$$\gamma > \frac{1 + \beta (1 - \alpha) W / B}{1 - \beta (1 - \alpha)}. \quad (8)$$

This theorem shows that the gain in spectral efficiency of LTE-U over WiFi, γ, is large enough, then it is possible for a monopolist to gain more revenue with LTE-U. The reason is that when γ is large, serving customers with LTE-U leads to less congestion, which allows the incumbent to serve more customers. We can also look at the equivalent bandwidth in (7). When LTE-U is used, the increase of licensed equivalent bandwidth is multiplied by the spectral efficiency γ. When γ is large enough, it is possible for the total equivalent bandwidth to increase, which helps the incumbent SP make more revenue. In this case, the incumbent SP is wiling to use LTE-U instead of separately serving customers on the licensed and unlicensed bands.

Note that when α and β are fixed, the threshold of γ in (8) increases with the ratio $\frac{W}{B}$. This means that when W is relatively small comparing to B, the incumbent is willing to use LTE-U even if its advantage over WiFi is not as large. However when W is relatively large, the incumbent prefers serving customers on the two bands separately, because the gain from serving all customers with LTE cannot make up for the loss on the total equivalent bandwidth. When the bandwidth of licensed and unlicensed spectrum is fixed, the threshold of γ in (8) is decreasing with α. This shows that in a monopoly market, the incumbent is more likely to use LTE-U if the duty cycle is large. Moreover, the threshold of

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5. We also include any MAC layer efficiency gains in this "spectral efficiency" term.

γ is increasing with β. This means that the larger the portion of unlicensed spectrum the incumbent is allowed to use, the less profitable it will be for the SP to use LTE-U. This is because if β is large, the congestion on the unlicensed band can be very high when the LTE-U mode is on, which causes a significant loss on both the amount of customers served and the revenue on the unlicensed band.

The next result characterized LTE-U's impact on customer surplus and total welfare.

Theorem 5.2. In a monopoly scenario, both customer surplus and social welfare increase when LTE-U is used if and only if (8) holds.

We see that the condition for customer surplus and social welfare to increase is the same as the condition for the incumbent's revenue to increase. This is because all of these quantities depend on the total equivalent bandwidth, which only increases when (8) holds.

Recall that in Section 3, customer surplus and social welfare both decrease if LTE-U is used when there is no difference in spectral efficiency between LTE and WiFi. However, if we assume LTE-U has a better spectral efficiency than WiFi, operating LTE-U is able to increase the overall efficiency of spectrum usage and consequently can benefit customer surplus and social welfare.

5.2 Competition with multiple entrant SPs

Now we turn to the case where multiple entrant SPs compete. We consider a linear model where the inverse demand function is $P(x) = 1 - x$ and the congestion function is $g(x) = x$.

First we look at the incumbent SP's revenue with multiple entrant SPs.

Theorem 5.3. Consider one incumbent and multiple entrants. Given a fixed $\alpha > 0$ and $\beta > 0$, when LTE-U is adopted, for any γ, the incumbent SP always gets a higher revenue and the customer mass served by entrant SPs decreases. Also the revenue of the incumbent SP increases with γ.

As in Theorem 4.1 (when $\gamma = 1$), LTE-U still results in an increase in the equivalent bandwidth of the incumbent, which leads to an increase in its revenue. For $\gamma > 1$, this advantage only increases and grows with γ.

While the incumbent's revenue behaves similarly for $\gamma > 1$, customer surplus behaves differently when considering $\gamma > 1$. LTE-U basically operates as LTE on a certain portion of the unlicensed band. Thus, if LTE has a better spectral efficiency, it is possible to serve more customers in the market with the same amount of spectrum resources. As a result, customer surplus can increase in this case, which is different from the result in Theorem 4.2. We characterize the customer surplus in the following theorem.

Theorem 5.4. In the case with multiple entrants with a linear congestion cost and inverse demand function, for any α and β, if B and γ satisfy the following conditions:

$$B \leq \frac{\left(\sqrt{2} - 1 \right) (1 - \beta + \alpha \beta) \alpha \beta}{2 (1 - \beta)}, \quad (9)$$

$$\frac{\alpha \beta - 2(1 - K)B - \sqrt{\Delta}}{4(1 - K)} \leq \gamma \leq \frac{\alpha \beta - 2(1 - K)B + \sqrt{\Delta}}{4(1 - K)}, \quad (10)$$

where $K = \frac{\alpha\beta}{1-\beta(1-\alpha)}$, and $\Delta = 2\alpha^2\beta^2 - [2(1-T)B + \alpha\beta]^2$, there always exists some W_{th}, such that when $W < W_{th}$, customer surplus increases when LTE-U is used. Otherwise, customer surplus always decreases if LTE-U is used.

This theorem shows that when licensed bandwidth is small and LTE has a relatively large efficiency advantage over WiFi, using LTE-U can lead to serving more customers. This also requires that the bandwidth of unlicensed spectrum be small. Because if W is large, the loss on the unlicensed band can no longer be compensated by the increase in the efficiency. Note that the threshold for γ in (10) shows that the efficiency gain cannot be too large, because when γ is too large, the incumbent serves a large amount of customers even without LTE-U. There is no room for customer surplus to increase when LTE-U is used. Also, γ cannot be too small, because otherwise using LTE-U would cause some loss of equivalent bandwidth, which leads to a loss of customer surplus. An extreme case is that $\gamma = 1$, which basically falls into the case in Section 4.1 and is consistent with the results in Theorem 4.2.

For social welfare, the result in Theorem 4.4 still holds. A very simple way to verify this is that when W is small, it is possible to increase customer surplus by using LTE-U. Since the incumbent's revenue increases with LTE-U and entrants always end up with no revenue, social welfare also increases when LTE-U is used. In the case with different spectral efficiencies, it is possible to have customer surplus and social welfare increase at the same time, which is not possible when the spectral efficiency is the same.

5.3 Competition with one entrant SP

We next consider the case with only one entrant. We focus on the licensed sharing case, where the entrant SP is able to use the unlicensed spectrum exclusively unless the incumbent SP uses LTE-U. We first look at how the revenue of the incumbent changes when LTE-U is used.

Theorem 5.5. With a linear congestion cost and inverse demand, when $\frac{\gamma B}{1-\alpha} < \frac{4}{3}$, the incumbent's revenue is higher when LTE-U is used and is increasing with γ.

Similar to Theorem 4.7, when B is small, the incumbent is able to gain more revenue with LTE-U. But the threshold of B decreases when we consider a spectral efficiency advantage of LTE. The intuition is again that when $\gamma > 1$, the equivalent bandwidth of the incumbent SP increases. When $\frac{4(1-\alpha)}{3\gamma} < B < \frac{4(1-\alpha)}{3}$, the increased equivalent bandwidth helps the incumbent to serve more customers to gain more revenue. In such a case, it is possible that the increased equivalent bandwidth brought by LTE-U reduces the revenue of incumbent, because it may lower the delivered price due to a more intense competition between the incumbent and entrant SPs. The theorem also states that when the given condition holds, the incumbent's revenue increases with the spectral efficiency. However, when the given condition does not hold, a higher spectral efficiency may result in a loss of revenue. This is again caused by increased competition with the entrant SP.

Next we characterize the impact on customer surplus and social welfare.

Theorem 5.6. In the case with one incumbent and one entrant SP, both customer surplus and social welfare always increase with γ.

Although it is possible for the incumbent to lose some revenue when γ increases, customer surplus and social welfare always benefit from an increase in spectral efficiency. This means that when there is a loss in revenue for the incumbent with increasing spectral efficiency, the gain in customer surplus is able to make up for the loss.

6 IMPACT OF α AND β

The duty cycle, α, and the percentage of the band for LTE-U use, β, are two important parameters to maintain fair and efficient coexistence of LTE-U and other unlicensed spectrum users. These parameters may be managed by a regulator or an industry standard, our goal here is to understand how such decisions impact both the incumbent's revenue and social welfare. We investigate the impact of α and β both with multiple entrants and with one entrant. To simplify our analysis we again assume no spectral efficiency gains (i.e., $\gamma = 1$) and again consider a linear model where the inverse demand function is $P(x) = 1 - x$ and the congestion function is $g(x) = x$.

6.1 Impact of duty cycle

First we consider when β is fixed and only vary the duty cycle α to see its impact. We begin with the case of one incumbent and multiple entrant SPs in the market. Before proceeding with our analysis of varying the duty cycle, we give the following proposition which characterizes the market equilibrium in the assumed scenario.

Proposition 6.1. Assuming a linear inverse demand function and congestion function and multiple entrants, the equilibrium announced price of the incumbent SP and the customer mass served is

$$p_1 = \frac{1}{2(1+W_e)}, \quad x_1 = \frac{B_e}{2(1+B_e+W_e)},$$

where B_e and W_e are defined in (4) and (5) respectively. The announced prices of entrant SPs are all zero and the total customer mass served by the entrants is

$$w_t = \frac{W_e(2+2W_e+B_e)}{2(1+W_e)(1+B_e+W_e)}.$$

This proposition shows that the equilibrium price can be expressed with the equivalent bandwidth in (4) and (5). We next investigate how the equivalent bandwidth B_e, W_e and their sum change with the duty cycle α.

Lemma 6.1. The equivalent bandwidth B_e increases with α and W_e decreases with α. If $W > \frac{B}{1-\beta}$, for $\alpha \in (0, \frac{1}{2})$, the total amount of equivalent bandwidth $B_e + W_e$ always decreases with α.

This lemma shows that when B is relatively smaller than W, the total equivalent bandwidth decreases with α in the range $(0, \frac{1}{2})$. As mentioned previously, the duty cycle is usually limited below 50%. That implies that in practice, the total equivalent bandwidth usually decreases with α.

Theorem 6.1. When there is one incumbent SP and multiple entrant SPs, the revenue of the incumbent always increases with the duty cycle α.

Theorem 6.1 is a natural result of Lemma 6.1. Because the incumbent SP gets more equivalent bandwidth with LTE-U while the entrants lose more resources with increasing α, the incumbent's revenue increases with α. Consequently, if there was no regulation for choosing α, the incumbent SP may want to raise α to a value close to 1.

Things become different when we consider the case with only one entrant SP in the market. In this case, Lemma 6.1 still holds, but the incumbent SP may not want to choose a large α all the time. The following theorem describes such a case.

Theorem 6.2. When there is only one incumbent and one entrant in the market and $W \to \infty$, the optimal α for the incumbent SP to maximize its revenue is $\alpha^* = \max\{1 - \frac{3B}{4}, 0\}$.

Theorem 6.2 shows that the revenue of incumbent is no longer increasing with α when there is only one entrant SP in the market. Fig. 2 shows how revenue changes with α when $B = 1, W \to \infty$ and $\beta = 0.2$. We can see the revenue of the incumbent reaches a maximum when $\alpha = \frac{1}{4}$ and is higher than that without LTE-U. In the case with one entrant SP, the incumbent SP may want to choose a small α or even does not want to use LTE-U technology ($\alpha^* = 0$) when there is plenty of licensed resource. Another thing to notice is that when $W \to \infty$, the optimal α is non-increasing with licensed bandwidth B. This implies the more licensed spectrum the SP possesses, the smaller duty cycle it may prefer.
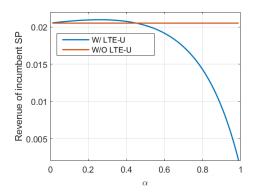

Fig. 2: Revenue of the incumbent in the case with one entrant SP when $B = 1, W \to \infty$ and $\beta = 0.2$.

Next we look at the social welfare. Theorem 4.8 states with one incumbent, one entrant and $W \to \infty$, LTE-U yields higher social welfare. We characterize the gap between the two cases in the following theorem.

Theorem 6.3. When there is one incumbent and one entrant SP, if $W \to \infty$, the social welfare gap between the cases with and without LTE-U is non-decreasing in α.

Theorem 6.3 shows that when there is a sufficient amount of unlicensed spectrum, a regulator may prefer a larger duty cycle α, because it increases the total amount of effective resources in the market, which results in a larger increase in social welfare.

6.2 Fixed utilization ratio

The total time-frequency resources allocated to LTE-U is given by the product $\alpha\beta$. In this section, we fix this product and seek to understand the impact of trading off a regulation for a small bandwidth with a long duty cycle (a large α) or a large bandwidth with a short duty cycle (a large β). We focus on the case of one incumbent and multiple entrants. We set $\alpha\beta = k$, where k is a constant. We then view α as a variable in the analysis. In this case, α can vary in the range $(k, 1)$. The equivalent bandwidths can be rewritten as

$$B_e = B + \frac{kW}{1 + (k/\alpha - k)W/B}, \quad W_e = W - \frac{kW}{1 - (k/\alpha - k)}. \tag{11}$$

Note that both B_e and W_e are increasing in α. As a result, the total amount of equivalent bandwidth increases with α. But it remains unclear what impact this has on the incumbent's revenue. The following theorem addresses this.

Theorem 6.4. In the case with one incumbent SP and multiple entrant SPs under the linear setting, if $\alpha\beta = k$ and k is a constant in $(0, 1)$, then:

1) If $B > \frac{\sqrt{2}}{2}$ and $W \leq B$, the incumbent's revenue always decreases with α in the range $(k, 1)$;
2) For any choice B, there always exists some $W_{th} > 0$ and $k_{th} \in (k, 1)$, such that when $W > W_{th}$, the incumbent's revenue decreases with β in the range (k, k_{th}).

Theorem 6.4 shows that in different situations, the incumbent SP has different preferences on higher α or β when the product $\alpha\beta$ is fixed. When the bandwidth of the unlicensed spectrum is relatively small, the amount of equivalent bandwidth increases with α, but the revenue decreases with α. In this case, using a larger portion of unlicensed spectrum is more profitable than using a small portion for a longer time. However when W is relatively large, the incumbent's revenue decreases with β in some range, which implies that the incumbent may prefer a larger α. In this case, a small portion of the spectrum may be enough for the incumbent to serve its customers. As a result a larger duty cycle α might be more profitable for the incumbent SP.

Figure 3 is an example of these two cases. We fix $k = 0.2$ and $B = 1$. We can see in Fig. 3(a), when W is relatively small, the incumbent may prefer a lower α. But when W is relatively large, the incumbent may prefer a higher α as is shown in Fig. 3(b).

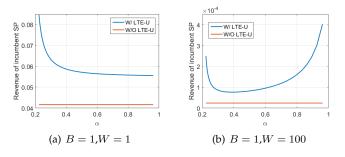(a) $B = 1, W = 1$ (b) $B = 1, W = 100$

Fig. 3: Revenue of incumbent SP versus α with multiple entrant SPs and $k = 0.2$.

Theorem 6.5. In the case with one incumbent SP and multiple entrant SPs, if $\alpha\beta = k$ and k is a constant in $(0, 1)$, then customer surplus always increases with α in $(k, 1)$.

The intuition behind Theorem 6.5 is that consistent bandwidth between LTE-U on and off can help reduce the congestion experienced by customers. For example, LTE-U using 25% of the bandwidth, 100% of the time would be better than using 50% of the bandwidth, 50% of the time in terms of congestion, even though the total amount of unlicensed resources used by LTE-U is the same. If we only look at the expected congestion experienced by the users on unlicensed band, it is higher when $\alpha = 50\%, \beta = 50\%$ than $\alpha = 100\%, \beta = 25\%$, because congestion is a function of $\frac{x}{W}$ and a 50% loss on bandwidth will at least double the congestion. Therefore, a higher α actually reduces the expected congestion, which helps increase the customer surplus. We can also verify the results from the expression of equivalent bandwidth in (11). We know both B_e and W_e are increasing with α, which implies the total amount of equivalent spectrum resources increases with α. So in this case, a higher α can help increase the amount of virtual resources and serve more customers. This suggests that a regulator should set a large α to increase customer surplus given a fixed utilization ratio.
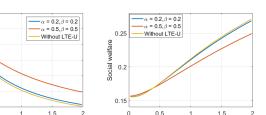
7 NUMERICAL RESULTS

In this section we give some additional numerical examples illustrating our results. We again consider a model with a linear inverse demand function and congestion function where $P(x) = 1 - x$, $g(x) = x$. Both cases with fixed α, β and varying α, β are considered.

7.1 Fixed α and β

First, we examine how the incumbent's revenue and social welfare change with the amount of unlicensed spectrum when there is one incumbent SP and multiple entrant SPs. We fix the licensed bandwidth as $B = 1$ and set α, β to different values. The results are shown in Fig. 4. As is described in Theorem 4.1, the incumbent is always gaining more revenue with LTE-U; this is illustrated in Fig. 4(a). Also we can see that when more spectrum can be used by LTE-U, and a higher duty cycle is allowed, the revenue is higher. The resulting social welfare is show in Fig. 4(b). We can see that when the bandwidth of the unlicensed spectrum is small, social welfare increases slightly with the adoption of LTE-U technology. But when more unlicensed spectrum is available, social welfare decreases with LTE-U. Another thing to notice is that social welfare decreases with W when W is small. This effect is also noted for the case without LTE-U in [19]. The use of LTE-U makes the social welfare loss smaller.

Next we look at the impact of different spectral efficiencies when there are multiple entrant SPs. We showed that when spectral efficiency is not considered, customer surplus is always worse with LTE-U, but it is possible to end up with higher customer surplus when the efficiency difference is considered. In Fig. 5, we show how customer surplus changes with W while fixing B and γ. We can see that when W is below a certain value, customer surplus increases with

(a) Revenue of incumbent   (b) Social welfare

Fig. 4: Comparison of revenue and welfare versus W in the case with multiple entrant SPs.

LTE-U. However, for larger values of W, customer surplus is worse with LTE-U.
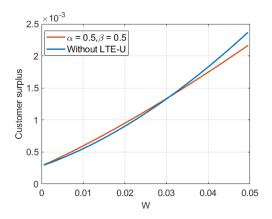

Fig. 5: Customer surplus versus W with $B = 0.01$ and $\gamma = 5$.

Next we take a look at the case with one incumbent and only one entrant SP in the market. We fix $B = 5$, $\alpha = 0.5, \beta = 0.5$ and assume that $\gamma = 1$ (i.e., there is no difference in spectral efficiency). Results are shown in Fig. 6. We also include the entrant's revenue in Fig. 6(a). We can see that when W is relatively small, the incumbent is able to gain more revenue with LTE-U while the entrant SP suffers a loss in revenue. However, when W is large, LTE-U hurts the revenue of both SPs. The results for social welfare are shown in Fig. 6(b). We can see that when W is large, social welfare increases with LTE-U and there is a social welfare gap between the cases with and without LTE-U. Next, we let $W \to \infty$ and see how this gap changes with B under different α (β makes no difference when $W \to \infty$). Results are shown in Fig. 7. We can see that the social welfare gap first increases then decreases with B and always increases with α. When B is small, LTE-U is able to increase the amount of spectrum resources of the incumbent SP to serve more customers, which benefits the incumbent's revenue and customer surplus and as a result leads to a gain in the social welfare. However, when B is large, the gap is smaller because both cases approach maximum possible social welfare in the market so that the increase in resources does not have as large an impact as when B is smaller.
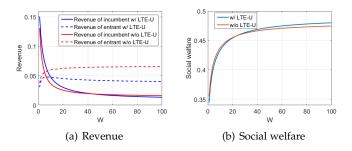
(a) Revenue

(b) Social welfare

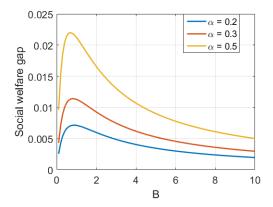Fig. 6: Comparison of revenue and welfare versus W in the case with one entrant SP.

Fig. 7: Social welfare gap between the cases with and without LTE-U as a function of α and B for $W \to \infty$.

7.2 Varying α and β

Next, we consider the impact of the duty cycle α with β fixed when there are multiple entrants in the market. We have already shown that the incumbent's revenue increases with α in Theorem 6.1. In Fig. 8, we show how social welfare changes with α for different values of licensed bandwidth, unlicensed bandwidth, and β. We can see that when W is small (Fig. 8(a)), social welfare increases with the duty cycle α. In this case, a higher α is desirable by both the incumbent SP and a social planner. Also we can see that a larger β helps increase the social welfare. But when W is slightly larger (Fig. 8(b)), social welfare first decreases then increases with α. Additionally, we can see that when β increases, social welfare decreases.

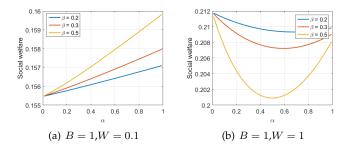(a) $B = 1, W = 0.1$

(b) $B = 1, W = 1$

Fig. 8: Social welfare in the case with multiple entrants with fixed β.

Next we show how social welfare changes with α when $\alpha\beta$ is fixed in the case with multiple entrant SPs. Results

are shown in Fig. 9. We can see in Fig. 9(a) that when W is relatively small, social welfare first decreases then increases with α. That is because there is some welfare loss when adding a small amount of unlicensed spectrum to the market as is described in [19]. Recall that the equivalent bandwidth of unlicensed spectrum increases with α when $\alpha\beta$ is fixed. As a result the social welfare may suffer when α increases in the case of small W. But when W is large as in Fig. 9(b), social welfare always increases with α.

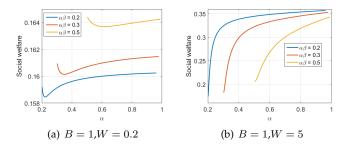(a) $B = 1, W = 0.2$

(b) $B = 1, W = 5$

Fig. 9: Social welfare versus α in the case of multiple entrants and fixed $\alpha\beta$.

8 CONCLUSION

In this paper, we analyzed the market impact of LTE-U technology on the competition among incumbent and entrant SPs with licensed and unlicensed spectrum. We first analyzed the case where the duty cycle, α, and the portion of unlicensed spectrum that can be used by LTE-U, β, are fixed. Our results show that when there are multiple entrant SPs competing on the unlicensed band, the incumbent SP can get more revenue by using LTE-U. However when there is only one entrant SP in the market, the incumbent's revenue may decrease when LTE-U is adopted. We also show that the welfare impact of LTE-U depends on the market parameters - in some cases it can lead to a gain and in others a loss. We also investigated the case where LTE-U has a better spectral efficiency than WiFi. In general, the incumbent benefits from the efficiency gain. When there are multiple entrants in the market both revenue and customer surplus can increase when LTE-U is used. We also investigated the impact of α and β on the market. Our results show that when there are multiple entrants and if β is fixed, the incumbent's revenue increases with α. However, when there is only one entrant SP using unlicensed spectrum, the optimal α is not necessarily 1 and can even be 0. We also fixed the product $\alpha\beta$ to see whether the incumbent prefers a high α or a high β. Results show that when the unlicensed bandwidth is relatively small, the incumbent prefers high β and when the unlicensed bandwidth is relatively large, the incumbent may prefer high α.

There are many ways this work could be extended. Extensions include considering the investment costs for an SP to upgrade to LTE-U, competition among multiple incumbents and different types of customers.

REFERENCES

[1] X. Wang and R. A. Berry, "Price competition with LTE-U and WiFi," in *IEEE Conference on Computer Communications (INFO-COM)*. IEEE, 2019, pp. 901–909.

[2] J. Jia and Q. Zhang, "Bandwidth and price competitions of wireless service providers in two-stage spectrum market," in *IEEE International Conference on Communications (ICC)*, 2008, pp. 4953–4957.

[3] E. Kavurmacioglu, M. Alanyali, and D. Starobinski, "Competition in secondary spectrum markets: Price war or market sharing?" in *IEEE DySPAN*, 2012, pp. 440–451.

[4] A. Ghosh and S. Sarkar, "Quality sensitive price competition in spectrum oligopoly," in *Information Theory Proceedings (ISIT), 2013 IEEE International Symposium on*. IEEE, 2013, pp. 2770–2774.

[5] "Cisco visual networking index: Global mobile data traffic forecast update, 2016–2021," 2017.

[6] F. Mekuria and L. Mfupe, "Spectrum sharing for unlicensed 5G networks," in *2019 IEEE Wireless Communications and Networking Conference (WCNC)*. IEEE, 2019, pp. 1–5.

[7] M. Ali, S. Qaisar, M. Naeem, J. J. Rodrigues, and F. Qamar, "Resource allocation for licensed and unlicensed spectrum in 5G heterogeneous networks," *Transactions on Emerging Telecommunications Technologies*, vol. 29, no. 10, p. e3299, 2018.

[8] Global mobile Suppliers Association (GSA). LTE Unlicensed - unlicensed and shared spectrum report – July 2019. Https://gsacom.com/paper/unlicensed-shared-spectrum-report-july-2019/. [Online]. Available: https://gsacom.com/paper/unlicensed-shared-spectrum-report-july-2019/

[9] V. Maglogiannis, A. Shahid, D. Naudts, E. De Poorter, and I. Moerman, "Enhancing the coexistence of LTE and Wi-Fi in unlicensed spectrum through convolutional neural networks," *IEEE Access*, vol. 7, pp. 28 464–28 477, 2019.

[10] P. Gawlowicz, A. Zubow, and A. Wolisz, "Enabling cross-technology communication between LTE unlicensed and WiFi," in *IEEE INFOCOM 2018-IEEE Conference on Computer Communications*. IEEE, 2018, pp. 144–152.

[11] T. LeAnh, N. H. Tran, D. T. Ngo, Z. Han, and C. S. Hong, "Orchestrating resource management in LTE-Unlicensed systems with backhaul link constraints," *IEEE Transactions on Wireless Communications*, vol. 18, no. 2, pp. 1360–1375, 2019.

[12] A. Manzoor, N. H. Tran, W. Saad, S. A. Kazmi, S. R. Pandey, and C. S. Hong, "Ruin theory for dynamic spectrum allocation in LTE-U networks," *IEEE Communications Letters*, vol. 23, no. 2, pp. 366–369, 2018.

[13] A. K. Bairagi, N. H. Tran, W. Saad, and C. S. Hong, "Bargaining game for effective coexistence between LTE-U and WiFi systems," in *NOMS 2018-2018 IEEE/IFIP Network Operations and Management Symposium*. IEEE, 2018, pp. 1–8.

[14] I. Elgendi, K. S. Munasinghe, and A. Jamalipour, "Break-even point-based radio resource management for fair coexistence between LTE-U and Wi-Fi," in *2018 IEEE International Conference on Communications (ICC)*. IEEE, 2018, pp. 1–6.

[15] C. Cano and D. J. Leith, "Coexistence of WiFi and LTE in unlicensed bands: A proportional fair allocation scheme," in *Communication Workshop (ICCW), 2015 IEEE International Conference on*. IEEE, 2015, pp. 2288–2293.

[16] M. Sriyananda, I. Parvez, I. Güvene, M. Bennis, and A. I. Sarwat, "Multi-armed bandit for LTE-U and WiFi coexistence in unlicensed bands," in *WCNC 2016*, pp. 1–6.

[17] E. Almeida, A. M. Cavalcante, and et al., "Enabling LTE/WiFi coexistence by LTE blank subframe allocation," in *ICC*, 2013.

[18] K. Hamidouche, W. Saad, and M. Debbah, "A multi-game framework for harmonized LTE-U and WiFi coexistence over unlicensed bands," *IEEE Wireless Communications*, vol. 23, no. 6, pp. 62–69, 2016.

[19] T. Nguyen, H. Zhou, R. A. Berry, M. L. Honig, and R. Vohra, "The impact of additional unlicensed spectrum on wireless services competition," in *IEEE DySPAN*, 2011, pp. 146–155.

[20] T. Nguyen, H. Zhou, R. A. Berry, M. L. Honig, and R. V. Vohra, "The cost of free spectrum," *Operations Research*, vol. 64, no. 6, 2016. [Online]. Available: http://arxiv.org/abs/1507.07888

[21] P. Maillé and B. Tuffin, "Price war with partial spectrum sharing for competitive wireless service providers," in *IEEE Global Telecommunications Conference (GLOBECOM)*, 2009, pp. 1–6.

[22] C. Liu and R. A. Berry, "Competition with open spectrum access: Sharing vs. unlicensed access," in *9th International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CROWNCOM)*. IEEE, 2014, pp. 486–491.

[23] A. K. Bairagi, N. H. Tran, W. Saad, Z. Han, and C. S. Hong, "A game-theoretic approach for fair coexistence between LTE-U and WiFi systems," *IEEE Transactions on Vehicular Technology*, vol. 68, no. 1, pp. 442–455, 2018.

[24] Y. Zhu and R. A. Berry, "Contracts as investment barriers in unlicensed spectrum," in *IEEE INFOCOM 2018-IEEE Conference on Computer Communications*. IEEE, 2018, pp. 1925–1933.

[25] A. Ghosh, V. Aggarwal, and P. Chakraborty, "Tiered spectrum measurement markets for joint licensed and unlicensed secondary access," *IEEE Transactions on Network Science and Engineering*, 2019.

[26] K. Hamidouche, W. Saad, M. Debbah, M. T. Thai, and Z. Han, "Contract-based incentive mechanism for LTE over unlicensed channels," *IEEE Transactions on Communications*, 2019.

[27] A. Ghosh and R. Berry, "Spot markets for spectrum measurements," in *2019 IEEE International Symposium on Dynamic Spectrum Access Networks (DySPAN)*, 2019, pp. 1–10.

[28] A. Al-Dulaimi, S. Al-Rubaye, Q. Ni, and E. Sousa, "5G communications race: Pursuit of more capacity triggers LTE in unlicensed band," *IEEE vehicular technology magazine*, vol. 10, no. 1, pp. 43–51, 2015.

[29] R. Zhang, M. Wang, L. X. Cai, Z. Zheng, X. Shen, and L.-L. Xie, "LTE-unlicensed: the future of spectrum aggregation for cellular networks," *IEEE Wireless Communications*, vol. 22, no. 3, pp. 150–159, 2015.

[30] P. Gawłowicz, A. Zubow, and A. Wolisz, "LtFi: Cross-technology communication for RRM between LTE-U and IEEE 802.11," *arXiv preprint arXiv:1707.06912*, 2017.

[31] M. Bykowsky and W. W. Sharkey, "An economic welfare analysis of shared spectrum use: A case study of LTE-U/LAA," *Available at SSRN: https://ssrn.com/abstract=2857632 or http://dx.doi.org/10.2139/ssrn.2857632*, 2016.

[32] H. Yu, G. Iosifidis, J. Huang, and L. Tassiulas, "Auction-based coopetition between LTE unlicensed and Wi-Fi," *IEEE Journal on Selected Areas in Communications*, vol. 35, no. 1, pp. 79–90, 2017.

[33] X. Wang and R. A. Berry, "The impact of bundling licensed and unlicensed wireless service," in *INFOCOM 2018*. IEEE, pp. 1–9.

[34] ——. Price competition with LTE-U and WiFi. Https://arxiv.org/abs/2001.01388. [Online]. Available: https://arxiv.org/abs/2001.01388

[35] J. G. Wardrop, "Some theoretical aspects of road traffic research," *Proceedings of the institution of civil engineers, Part II*, vol. 1, pp. 325–378, 1952.

[36] X. Wang and R. A. Berry, "The impact of short-term permits on competition in unlicensed spectrum," in *IEEE DySPAN*, March 2017, pp. 1–10.

[37] Qualcomm Inc. LTE in unlicensed spectrum research. Https://www.qualcomm.com/invention/research/projects/lte-unlicensed. [Online]. Available: https://www.qualcomm.com/invention/research/projects/lte-unlicensed

[38] M. Paolini and S. Fili, "LTE unlicensed and WiFi: Moving beyond coexistence," *Senza Fili Report*, 2015.

APPENDIX A
PROOF OF THEOREM 3.1

Proof: First, we claim that when congestion on both bands decreases, the optimal revenue of the SP increases; this is because the SP can just announce the same price and attract more customers while keeping the potential to increase revenue by adjusting its price. As a result we only need to show that when the congestion level meets the lower bound, i.e., with linear congestion cost, the incumbent SP can gain no higher revenue than that without LTE-U.

We use the equivalent licensed and unlicensed bandwidth to rewrite the optimization in (3) as follows:

$$\max_{p_1^l, p_1^u} \quad p_1^l x_1^l + p_1^u x_1^u$$

$$\text{s.t.} \quad p_1^l + g\left(\frac{x_1^l}{B_e}\right) = P(x_1^l + x_1^u),$$

$$p_1^u + g\left(\frac{x_1^u}{W_e}\right) = P(x_1^l + x_1^u),$$

$$p_1^l, p_1^u \geq 0.$$

For the optimization problem above, we can equivalently use x_1^l and x_1^u instead of p_1^l and p_1^u as optimization variables.

From the first order conditions of the objective function over x_1^l and x_1^u, we can show $\frac{x_1^l}{B_e} = \frac{x_1^u}{W_e} = \frac{x_1^l + x_1^u}{B_e + W_e}$, which means the congestion levels on the licensed and unlicensed bands are the same. Since we can verify $B_e + W_e \leq B + W$, the total customer mass served becomes less when LTE-U is applied. As a result, the revenue of the incumbent SP decreases. □

APPENDIX B
PROOF OF THEOREM 3.2

Proof: In Appendix A, we show that in the monopoly case, the congestion on the licensed and unlicensed bands are the same. As a result the price for service on these bands must also be the same. Thus, we can rewrite the problem in the following form:

$$\max_{p} \quad p_1 x_1$$
$$\text{s.t.} \quad p_1 + g\left(\frac{x_1}{B_e + W_e}\right) = P(x_1),$$
$$p_1, x_1 \geq 0.$$

Again using x_1 as the optimization variable, the first order condition for optimality is

$$\frac{\partial p_1 x_1}{\partial x_1} = x_1 P'(x_1) + P(x_1) - g\left(\frac{x_1}{B_e + W_e}\right)$$
$$-\frac{x_1}{B_e + W_e} g'\left(\frac{x_1}{B_e + W_e}\right) = 0. \quad (12)$$

Since $P(x)$ is concave and non-increasing and $g(x)$ in convex and increasing, the solution to (12) decreases when $B_e + W_e$ decreases. Again using that $B_e + W_e < B + W$, we conclude that the total customer mass served decreases when LTE-U is used.

Next we show customer surplus is an increasing function of the total customer mass served. Differentiating the customer surplus in (1) with respect to the total customer mass served, x, yields

$$\frac{\partial CS}{\partial x} = -x P'(x).$$

Again, because $P(x)$ is a concave, non-increasing function, customer surplus, CS, is always a non-decreasing function in the total customer mass served. Therefore, customer surplus is always non-increasing if LTE-U is used.

Because both the incumbent's revenue and customer surplus are non-increasing, the overall social welfare is non-increasing as well. □

APPENDIX C
PROOF OF THEOREM 4.3

Proof: First we consider the case without LTE-U. This case is analyzed in [22]; we summarize the key results next:

1) When $W \leq \max\{\frac{A}{T} - \frac{B}{2}, 0\}$, customer surplus is zero and

$$SW = \frac{BT^2}{4}; \quad (13)$$

2) When $\max\{\frac{A}{T} - \frac{B}{2}, 0\} < W \leq \frac{\sqrt{A^2 + B^2 T^2} - BT + A}{2T}$, customer surplus is zero and

$$SW = (A - WT)\left[T - \frac{(A - WT)}{B}\right]; \quad (14)$$

3) When $W > \frac{\sqrt{A^2 + B^2 T^2} - BT + A}{2T}$, we have positive customer surplus and

$$SW = AT - \frac{A^2(B + 4W)}{4W(B + W)}. \quad (15)$$

We next show that in the first two cases, the adoption of LTE-U increases the social welfare. Recall that with a linear congestion function, LTE-U increases B to B_e and decreases W to W_e, where B_e and W_e are defined in (4) and (5), respectively.

In the first case, when we change B to B_e and W to W_e, since $W - W_e > B_e - B > 0$, we always have $W_e \leq \max\{\frac{A}{T} - \frac{B_e}{2}, 0\}$. This implies that (B_e, W_e) still falls in the region of case 1), so that we can still use equation (13) to calculate the social welfare. Obviously, when B increases to B_e, social welfare also increases.

In the second case, we claim that when LTE-U is adopted, the equivalent unlicensed bandwidth W_e can never go beyond the boundary $\frac{\sqrt{A^2 + B_e^2 T^2} - B_e T + A}{2T}$. Consider the following function :

$$f(b) = \frac{\sqrt{A^2 + b^2 T^2} - bT + A}{2T}.$$

Given that $f(B) > W$ and $W - W_e > B_e - B$, we have

$$f(B_e) = f(B) + \int_B^{B_e} f'(b) db$$
$$> f(B) + \int_B^{B_e} -1 db = f(B) - (B_e - B)$$
$$\geq W - (W - W_e) = W_e.$$

So in this case, when LTE-U is adopted, B_e and W_e can only fall into case 1) and case 2). Since all of the social welfare functions increase with B, it suffices to show that when reducing W to W_e, social welfare is nondecreasing. When fixing B, it can be shown that social welfare in (14) is decreasing in W when $W \geq \frac{A}{T} - \frac{B_e}{2}$ and achieves maximum $\frac{BT^2}{4}$ when $W = \frac{A}{T} - \frac{B_e}{2}$. Consequently, when decreasing W to W_e, if it still falls in the range of case 2), social welfare increases. If it falls into the range of case 1), it then becomes a constant with respect to unlicensed bandwidth W. As a result social welfare is non-decreasing when decreasing W to W_e.

□

APPENDIX D
PROOF OF THEOREM 4.4

Proof: When $W \to 0$, social welfare is the same with and without LTE-U. In this limit, we have $w_t \to 0$ and $x_1 \to x^*$, where x^* represents the optimal customer mass in the monopoly case. Note that here x^* is a constant if B is fixed and can be characterized by the following equation

$$x^* P'(x^*) + P(x^*) = g\left(\frac{x^*}{B}\right) + \frac{x^*}{B} g'\left(\frac{x^*}{B}\right). \quad (16)$$

Consequently, when $W \to 0$, we have $SW^{LTE-U} = SW^{un}$, where SW^{LTE-U} and SW^{un} denote the social welfare with and without LTE-U, respectively. Hence, it is sufficient to show that $\lim_{W \to 0} \frac{\partial SW^{LTE-U}}{\partial W} > \lim_{W \to 0} \frac{\partial SW^{un}}{\partial W}$. Here, we use a linear approximation method as in [36] to characterize the social welfare for both cases when $W \to 0$.

In both cases, differentiating the social welfare with respect to W and letting $W \to 0$ gives

$$\lim_{W \to 0} \frac{\partial SW}{\partial W} = [-x^* P'(x^*)] \lim_{W \to 0} \frac{\partial x_1}{\partial W}. \tag{17}$$

Next we show how to calculate the value of $\lim_{W \to 0} \frac{\partial x_1}{\partial W}$. First, consider the case without LTE-U. In this case, the incumbent's revenue maximization problem can be transformed to the following optimization formulation:

$$\max_{x_1} \quad \left[P(x_1 + \Delta w_t^{un}) - g\left(\frac{x_1}{B}\right) \right] x_1, \tag{18}$$
$$\text{s.t.} \quad x_1 \geq 0,$$

where $\Delta w_t^{un} = g^{-1}\left[P(x^*)\right] W$ is the customer mass increment on the unlicensed band as $W \to 0$. Here $g^{-1}(\cdot)$ is the inverse function of $g(\cdot)$ and is well defined, because $g(\cdot)$ is an increasing function.

The first order optimality condition for the optimization problem (18) is

$$(x^* + \Delta x_1^P) P'(x^* + \Delta x_1^P + \Delta w_t^P) + P(x^* + \Delta x_1^P + \Delta w_t^P)$$
$$= g\left(\frac{x^* + \Delta x_1^P}{B}\right) + \frac{x^* + \Delta x_1^P}{B} g'\left(\frac{x^* + \Delta x_1^P}{B}\right). \tag{19}$$

Linearly approximating each term in (19) at point x^* and applying equation (16), we find:

$$\Delta x_1^{un} = \frac{\left[P'(x^*) + x^* P''(x^*)\right] g^{-1}\left[P(x^*)\right] W}{\left[\frac{2g'\left(\frac{x^*}{B}\right)}{B} + \frac{x^* g''\left(\frac{x^*}{B}\right)}{B^2} - 2P'(x^*) - x^* P''(x^*) \right]}.$$

We can use similar method in the case with LTE-U technology. The resulting Δx_1^{LTE-U} is

$$\Delta x_1^{LTE-U} = \frac{\left[P'(x^*) + x^* P''(x^*)\right] h^{-1}\left[P(x^*)\right] W}{\left[\frac{2g'\left(\frac{x^*}{B}\right)}{B} + \frac{x^* g''\left(\frac{x^*}{B}\right)}{B^2} - 2P'(x^*) - x^* P''(x^*) \right]},$$

where $h(t)$ is defined as

$$h(t) = (1 - \alpha)g(t) + \alpha g\left(\frac{t}{1 - \beta}\right).$$

It can be seen that $h(t)$ is also a convex increasing function and has a well defined inverse function $h^{-1}(t)$. Since we always have $h(t) > g(t)$ for $\alpha, \beta > 0$, we have $g^{-1}\left[P(x^*)\right] > h^{-1}\left[P(x^*)\right] > 0$. By substituting these values back into equation (17), we have

$$\lim_{W \to 0} \frac{\partial SW^{LTE-U}}{\partial W} > \lim_{W \to 0} \frac{\partial SW^{un}}{\partial W}.$$

Therefore, we can reach the conclusion that for some small W, social welfare increases when LTE-U is used.
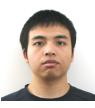
Next we show that when W is large enough, social welfare decreases when LTE-U is used. From the definition of the congestion function, we have $\lim_{W \to \infty} g\left(\frac{w}{W}\right) = 0$, because the customer mass is always bounded. When LTE-U is used,

because both α and β are fixed, the average congestion on the unlicensed band is

$$\lim_{W \to \infty} \left[\alpha g\left(\frac{w}{(1 - \beta)W}\right) + (1 - \alpha)g\left(\frac{w}{W}\right) \right] = 0.$$

The Wardrop equilibrium conditions suggests the delivered price is 0 in such a condition, which leads to 0 profit for the incumbent SP. Then, from the definition of social welfare, we have

$$\lim_{W \to \infty} SW = \lim_{W \to \infty} \sum_j p_j x_j = CS.$$

Theorem 4.2 states that customer surplus always decreases when LTE-U is used. Hence, social welfare also decreases in such a case. □

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