Large System Performance of Downlink OFDMA with Limited Feedback

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Abstract— We consider allocation of sub-channels to users in a downlink OFDMA system. Each user feeds back one bit per sub-channel, which indicates whether or not the gain exceeds a threshold. Users are assigned priority weights, and the thresholds are selected to maximize the weighted sum capacity. We analyze the behavior of the optimal thresholds and growth in capacity, assuming *i.i.d.* Rayleigh fading sub-channels, in the large system limit in which users K and sub-channels tend to infinity with fixed ratio. If all users have the same priority weight, then the optimized threshold increases as $\log K$ minus a second-order term, which is asymptotically bounded between $\log \log K$ and $\log \log \log K$. Furthermore, the sum capacity per sub-channel increases as $\log \log K$ plus a second-order term, which decreases to a constant as $\log \log K / \log K$. We then consider two classes of users, each assigned a different weight, and show that the capacity of the low priority group tends to zero. Finally, we solve for the optimal thresholds given a fairness constraint on the ratio between the rates of different classes.

I. INTRODUCTION

Orthogonal Frequency Division Multiple Access (OFDMA) is currently being proposed for emerging wireless cellular and data systems. It has the advantage of eliminating intra-cell interference by assigning different sets of tones, or sub-channels to different users. Furthermore, when combined with opportunistic downlink scheduling [1], the achievable rate region obtained by allowing transmission to multiple users on different sets of sub-channels, which experience frequency-selective fading, is strictly larger than the rate region achieved with time-division multiplexing.

Here we consider downlink OFDMA in a single cell, and study the assignment of sub-channels to users to maximize a weighted sum capacity performance objective. We assume that each sub-channel can be assigned only to a single user. This assignment can be combined with a scheduler, which changes the weights in the sum capacity objective to satisfy fairness constraints.

Given perfect channel state information (CSI) at the transmitter, i.e., knowledge of all users' sub-channel gains, to maximize the sum capacity, the base station would assign a particular sub-channel to the user with the largest corresponding channel gain. Our model is motivated in part by prior work in [5], which considers scheduling for CDMA with a weighted capacity objective given perfect CSI at the transmitter. Other related work on power and rate allocation for downlink OFDMA given perfect CSI at the transmitter

is presented in [4] and [7], which maximize an unweighted sum rate performance objective, and in [3], which studies bit loading across sub-channels. Also, in [2] the downlink sum capacity for OFDMA is compared with water-filling and uniform power allocations.

In an OFDMA system with hundreds of sub-channels and many users, full downlink CSI at the transmitter may be difficult to obtain. Namely, hundreds of channel gains must be quantized and relayed to the transmitter during each coherence time, or scheduling interval. To limit the amount of feedback, we assume that the receivers can relay at most one bit per sub-channel back to the transmitter. Each bit informs the transmitter whether or not the channel gain falls above an announced threshold, which can differ across users. (In practice, each feedback bit would likely correspond to a coherence band containing many sub-channels.)

We assume that channel gains across users are *i.i.d.* Rayleigh distributed, and that the users are assigned weights in the sum capacity objective, which reflect priorities or fairness constraints. For each sub-channel there is a set of users with channel gains that exceed their assigned thresholds, and the sub-channel is assigned to the user in that set with the largest weight. (If no user's channel gain exceeds the threshold, then the sub-channel is not assigned, and ties can be settled randomly.) Furthermore, in our model the same power is transmitted on each active sub-channel. This is motivated in part by the results in [4] and [2], which show that optimizing the power spectrum across sub-channels offers only a marginal improvement relative to a uniform power assignment over an optimized subset of sub-channels. Our problem is to find the set of optimal thresholds across users, which maximizes the weighted sum capacity. The weights in the sum capacity objective and the thresholds determine the rates received by the users.

To obtain insight into the behavior of the weighted sum capacity objective and assigned rates as functions of system parameters, we analyze a large system in which the number of users and sub-channels both tend to infinity with fixed ratio. Furthermore, for purposes of analysis we partition the users into groups, corresponding to different priorities, or classes of service. Users within each group are assigned the same weight in the sum capacity objective, and the same threshold.

We start with a single class, and show that the opti-

mized threshold increases as $\log K$ minus a second-order term, which is asymptotically bounded between $\log \log K$ and $\log \log \log K$. Furthermore, the sum capacity per sub-channel increases as $\Theta(\log \log K)$[1] plus a second-order term, which decreases to a constant as $\log \log K / \log K$.[2] We then consider two classes of users, and show that with optimized thresholds, the capacity of the low priority group tends to zero. Finally, we solve for the optimal thresholds given a fairness constraint on the ratio between the rates of different groups. In that case, the capacity still grows as $\Theta(\log \log K)$, but with a smaller constant than in the unconstrained system.

II. System Model and Capacity

The base station transmits data to K users, and the total bandwidth is divided into N *i.i.d.* sub-channels. Let h_k^n denote the squared channel gain for the nth sub-channel of the kth user, where $1 \leq n \leq N$ and $1 \leq k \leq K$. We assume that the channel gains h_k^n, $n = 1, \cdots, N$, are *i.i.d.*, and are known at receiver k. Each user k is assigned a threshold u_k, and sends back one bit per sub-channel indicating whether or not h_k^n, $n = 1, \cdots, N$, exceeds the threshold.[3]

For the nth sub-channel let $\mathcal{U}_n = \{k : h_k^n \geq u_k\}$. If $|\mathcal{U}_n| > 1$, then the sub-channel is assigned to the user $k \in \mathcal{U}_n$ with the largest weight w_k. The weights therefore represent priorities, and are assumed to be adjusted by a scheduler to achieve a fairness objective. [4] The rate received by a particular user k then depends on w_k and u_k. Letting $\mathcal{N}_k$ denote the set of sub-channels assigned to user k, the achievable rate for user k, assuming that the transmitter is able to code over a large number of sub-channels, is $R_k = \sum_{n \in \mathcal{N}_k} E\left(\log(1 + Ph_k^n) | h_k^n \in \mathcal{N}_k\right)$, in nats per sub-channel,[5] where P is the power per sub-channel, the noise variance is normalized to unity, and the expectation is over h_k^n. We assume that the weights are fixed, and that the thresholds u_k are selected to maximize the performance objective $C = \sum_{k=1}^{K} w_k R_k$. We expect that the optimized threshold u_k will decrease as the weight w_k increases, so that R_k increases.

To obtain insight into the behavior of C, the optimized thresholds, and the rates $\{R_k\}$, we analyze a large system in which the number of users K and the number of sub-channels N both tend to infinity with fixed ratio $\beta = K/N$. Also, the users are partitioned into M groups corresponding to different classes of service. All users in the mth class, denoted $\mathcal{G}_m$, are assigned the same weight w_m and the same channel threshold u_m, and we consider the large system limit in which $K_m = |\mathcal{G}_m| \to \infty$ with fixed ratios $\beta_m = K_m/N$,

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[1] We use the notation: $x_K = O(y_K)$ if $\lim_{K \to \infty} \frac{|x_K|}{|y_K|} \leq M$; $x_K = \Omega(y_K)$ if $y_K = O(x_K)$; $x_K = \Theta(y_K)$ if $x_K = O(y_K)$ and $x_K = \Omega(y_K)$; $x_K \asymp y_K$ if $\lim_{K \to \infty} \frac{x_K}{y_K} = 1$.

[2] This is consistent with analogous results for a *single-user* OFDM channel with partial feedback presented in [6].

[3] The minimum feedback rate for this scheme is the entropy rate of the N-bit sequence.

[4] For example, following the *utility-based* approach to scheduling in [8], [9], we might take $w_k = U_k'(R)$, where $U_k(\cdot)$ is a concave increasing function of windowed throughput R.

[5] Throughout this paper we assume natural logarithms.

$m = 1, \cdots, M$. Without loss of generality, we assume that $w_1 > w_2 > \cdots > w_M$.

The ergodic capacity summed over users in group m is

$$C_m^{K_m} = \frac{1}{N} \sum_{k \in \mathcal{G}_m} R_k = \Pr(\text{sub-channel is assigned to } \mathcal{G}_m)$$
$$\times E[\log(1 + Ph_k^n) \mid h_k^n \geq u_m \text{for some } k \in \mathcal{G}_m]$$
(1)

Let $F_m(u) = \Pr\{h_k^n < u | k \in \mathcal{G}_m\}$ denote the cumulative distribution function (cdf) of the channel gains for group m, and $f_m(x) = F_m'(x)$ denote the associated probability density function (pdf). (We allow different distributions for the different groups.) The probability that a sub-channel is assigned to $\mathcal{G}_m$ is therefore $(1 - g_m) \prod_{i=1}^{m-1} g_i$ where $g_m = F_m^{K_m}(u_m) = \Pr\{h_k^n < u_m \forall k \in \mathcal{G}_m\}$, so that

$$C_m^{K_m} = (1 - g_m) \left(\prod_{i=1}^{m-1} g_i \right) \int_{u_m}^{\infty} \log(1 + Px) \frac{f_m(x)}{1 - F_m(u_m)} dx.$$
(2)

Since the base station allocates constant power P across all active sub-channels, the total transmitted power scales linearly with the number of users. That is, the total power is

$$K\mathcal{P} = PN[1 - \prod_{i=1}^{M}(1 - g_i)].$$
(3)

where $\mathcal{P}$ is the power per user. Given $\mathcal{P}$, we wish to maximize $\sum_m w_m C_m^{K_m}$ over the thresholds $u_1, \cdots, u_M$.

To achieve the rate in (2), each user must compute its own achievable rate, given the assigned set of sub-channels, and relay this rate to the transmitter. That enables the transmitter to select the appropriate codebook for each user. (Another possibility, which requires much more feedback, is for each user to feed back a modulation and coding format for each sub-channel, as in [3].) To reduce feedback requirements further, the transmitter could simply compute the rate assuming that $h_k^n = u_m$ for each sub-channel assigned to $\mathcal{G}_m$, i.e., $\log(1 + Pu_m)$. This requires no additional feedback, but gives a lower ergodic capacity for group m, namely,

$$\underline{C}_m^{K_m} = (1 - g_m) \left(\prod_{i=1}^{m-1} g_i \right) \log(1 + Pu_m)$$
(4)

Proposition 1: Given a set of channel gain cdf's, which satisfy $0 < \frac{1 - F_m(x)}{f_m(x)} < \infty$ for all $x > 0$ and $1 \leq m \leq M$, as $u_m \to \infty$, $C_m^{K_m} = \underline{C}_m^{K_m} + O(\frac{1}{Pu_m})$.

The proof is based on integrating the expression for $C_m^{K_m}$ by parts. In the large system limit $(K, N) \to \infty$, we will see that the optimized threshold $u_m \to \infty$. If $\underline{C}_m^{K_m} \to \infty$, then $\underline{C}_m^{K_m} \asymp C_m^{K_m}$. In what follows, we will focus on the large system behavior of $\underline{C}_m^{K_m}$.

III. Single Class

We start with a single group of users, i.e., $M = 1$, $K_1 = K$. We assume that the distribution $F(x)$ is continuous, twice

differentiable, and that the density $f(x)$ is non-zero for $x > 0$. Furthermore, we assume that

$$\lim_{x \to \infty} \frac{d}{dx}\left(\frac{1 - F(x)}{f(x)}\right) = 0.$$

From Theorem 2.7.2 in [10], as $K \to \infty$,

$$\lim_{K \to \infty} F^K(a_K + b_K x) \to \exp(-e^{-x}) \qquad (5)$$

uniformly in x, where a_K and b_K satisfy

$$F(a_K) = 1 - \frac{1}{K} \qquad b_K = \frac{1}{Kf(a_K)} < \infty. \qquad (6)$$

In other words, the given conditions ensure that with suitable normalization the cdf of the maximum converges to a Gumble cdf. These conditions are satisfied by common fading distributions, such as Rayleigh or Ricean. In what follows, we will assume Rayleigh fading with variance σ^2 for which $a_K = \sigma^2 \log K$ and $b_K = \sigma^2$.

With some abuse of notation, in this section u_K denotes the threshold for a system with K users. Without loss of generality, we can write

$$u_K = a_K + b_K x_K = \sigma^2(\log K + x_K),$$

We consider a sequence of thresholds for which the corresponding sequence x_K either converges, or tends to $+/-\infty$. For such sequences, the uniform convergence in (5) implies that

$$\lim_{K \to \infty} F^K(u_K) = \begin{cases} 0, & \text{if } x_K \to -\infty, \\ \exp(-e^{-x_0}), & \text{if } x_K \to x_0, \\ 1, & \text{if } x_K \to \infty. \end{cases}$$

For a sequence of thresholds, we study the asymptotic behavior of the corresponding system capacity,

$$\underline{C}_1^K(u_K) = w_1[1 - F^K(u_K)] \log\left(1 + \frac{\beta \mathcal{P} u_K}{1 - F^K(u_K)}\right).$$

Proposition 2: As $K \to \infty$, if $x_K \to -\infty$ and $x_K = o(\log K)$, then $\underline{C}_1^K(u_K) \asymp w_1 \log(\log K)$. Furthermore, if the sequence x_K does not satisfy these assumptions, then $C_1^K(u_K) = O(\log(\log K))$ with constant strictly less than w_1.

Proof: Since $x_K = o(\log K)$, the threshold $u_K \to \infty$. We therefore have

$$\begin{aligned} \underline{C}_1^K &= w_1[1 - F^K(u_K)] \log\left(1 + \frac{\beta \mathcal{P}}{1 - F^K(u_K)} u_K\right) \\ &\asymp w_1[1 - F^K(u_K)] \log(u_K) \end{aligned} \qquad (7)$$

The second asymptotic equivalence follows since $w_1[1 - F^K(u_K)] \log[\beta \mathcal{P}/(1 - F^K(u_K)]$ is bounded when $0 \le F^K(u_K) \le 1$. Since $x_K \to -\infty$, we have $F^K(u_K) \to 0$ and $\underline{C}_1^K \asymp w_1 \log(u_K) \asymp w_1 \log \log K$.

To show that the capacity cannot grow faster with K, we consider the cases $x_K \to x_0 < \infty$ and $x_K \to \infty$. If $x_K \to x_0$, then $F^K(u_K) \to \exp(-e^{-x_0})$ and

$$\begin{aligned} \underline{C}_1^K &\asymp w_1[1 - \exp(-e^{-x_0})] \log(\sigma^2 \log K + \sigma^2 x_0) \\ &\asymp w_1[1 - \exp(-e^{-x_0})] \log \log K. \end{aligned} \qquad (8)$$

Finally, if $x \to \infty$, so that $F^K(u_K) \to 1$, following the same approach, we can show that

$$\underline{C}_1^K \asymp w_1 o(\log \log K) \qquad (9)$$

Hence the capacity with a single group and optimized threshold goes to infinity as $\Theta(\log \log K)$, the optimized threshold u_K grows as $\Theta(\log K)$, and $F^K(u_K) \to 0$. ∎

The fraction of active channels in a system with K users is denoted as $\eta_K(u_K) = 1 - F^K(u_K)$. For a sequence of thresholds as in Proposition 2, $\eta_K(u_K) \to 1$ as $K \to \infty$.

Lemma 1: Given a sequence of thresholds as in Prop. 2,

$$1 - \eta_K(u_k) \asymp \exp(-e^{-x_K}).$$

Proof: We have

$$\begin{aligned} 1 - \eta_K(u_K) &= F^K(\sigma^2 \log(K) + \sigma^2 x_K) \\ &= \left(1 + \frac{-e^{-x_K}}{K}\right)^K. \end{aligned}$$

Without loss of generality, assume that $x_K = -\log(w_K)$, where $w_K \to \infty$. Then

$$\log\left(\frac{1 - \eta_K(u_K)}{\exp(-e^{-x_K})}\right) = K \log\left(1 + \frac{-w_K}{K}\right) + w_K \le 0, \quad (10)$$

where the last step follows because $K \log(1 + \frac{x}{K}) \le x$. Hence,

$$\limsup_{K \to \infty} \frac{1 - \eta_K(u_K)}{\exp(-e^{-x_K})} \le 1.$$

Next note that

$$\frac{\log(1 - \eta_K(u_K))}{\log(\exp(-e^{-x_K}))} = \frac{K}{-w_K} \log\left(1 + \frac{-w_K}{K}\right) \to 1, \quad (11)$$

since if u_K satisfies Proposition 2, then $w_K = o(K)$, and so $\frac{K}{w_K} \to \infty$. Thus, given any ϵ, for K large enough,

$$\log(1 - \eta_K(u_K)) \ge (1 - \epsilon) \log(\exp(-e^{-x_K})),$$

and so,

$$1 - \eta_K(u_K) \ge (\exp(-e^{-x_K}))^{1-\epsilon}.$$

Dividing both sides by $(\exp(-e^{-x_K}))^{1-\epsilon}$ we have

$$1 \le \frac{1 - \eta_K(u_K)}{(\exp(-e^{-x_K}))^{1-\epsilon}} \le \left(\frac{1 - \eta_K(u_K)}{\exp(-e^{-x_K})}\right)^{1-\epsilon},$$

where the last inequality follows since $1 - \eta_K(u_K) \le 1$. Therefore,

$$\liminf_{K \to \infty} \frac{1 - \eta_K(u_K)}{\exp(-e^{-x_K})} \ge 1,$$

and so $1 - \eta_K(u_K) \asymp \exp(-e^{-x_K})$, as desired. ∎

Proposition 2 states that the optimal x_K's decrease at a slower rate than $-\log(K)$. From Lemma 1, if x_K increases at a faster rate, then the number of active channels approaches one faster. However, the rate per channel, $\log(\log K + x_K)$ grows at a slower rate. Next we develop tighter bounds on the optimal growth rate of the x_K-sequence. For this we consider the "remainder" term

$$\Delta^K(u_K) = \underline{C}_1^K(u_K) - w_1 \log(a_K).$$

This captures the higher order behavior of the total capacity.

Proposition 3: The optimal sequence of thresholds satisfies $x_K = O(\log(\log(K)))$ and $x_K = \Omega(\log(\log(\log(K))))$ as $K \to \infty$. With such a sequence,

$$\Delta^K(u_K) - w_1 \log(\beta\mathcal{P}) \asymp -w_1 \frac{\log(\log(K))}{\log K}.$$

In other words, an optimal sequence x_K decreases to $-\infty$ no faster than $\log(\log K)$ and no slower than $\log(\log(\log K))$. With such a sequence the remainder term approaches $w_1 \log(\beta\mathcal{P})$ at the indicated rate. The proof is omitted to save space.

IV. Multiple Classes

First we consider two groups of users $\mathcal{G}_1$ and $\mathcal{G}_2$. Continuing to abuse notation, let u_{K_m} denote the threshold for $\mathcal{G}_m$, and $\mathbf{u}_K = (u_{K_1}, u_{K_2})$. The system capacity with K users is given by

$$\underline{C}_{tot}^K(\mathbf{u}_K) = w_1 \underline{C}_1^{K_1}(\mathbf{u}_K) + w_2 \underline{C}_2^{K_2}(\mathbf{u}_K),$$

where $\underline{C}_m^{K_m}$ is given by (4). Here we assume that users in group $\mathcal{G}_m$ experience Rayleigh fading with variance σ_m^2, $m = 1, 2$.

Proposition 4: As $K \to \infty$, if $x_{K_1} \to -\infty$ and $x_{K_1} = o(\log K_1)$, then $\underline{C}_{tot}^K(\mathbf{u}_K) \asymp w_1 \underline{C}_1^{K_1}(\mathbf{u}_K) \asymp w_1 \log(\log K_1)$. Furthermore, if x_k does not satisfy these assumptions, then $\underline{C}_{tot}^K(\mathbf{u}_K) = O(\log(\log K))$ with constant strictly less than w_1.

In other words, with two groups, the higher priority group determines the asymptotic growth rate. The growth rate of the higher priority group is unaffected by the other group, since it receives all channels requested by both groups. From the single-class analysis, the fraction of channels allocated to $\mathcal{G}_1$ therefore approaches one.

From the proof of Proposition 4, it follows that with an optimal sequence of thresholds

$$\lim_{K \to \infty} \frac{\underline{C}_2^{K_2}(\mathbf{u}_K)}{\underline{C}_1^{K_1}(\mathbf{u}_K)} = 0,$$

i.e., the throughput of class 2 users is asymptotically negligible compared to the throughput of class 1 users. The next proposition states that in fact the class 2 throughput asymptotically tends to zero. To see this we consider the remainder term,

$$\Delta_{tot}^K(\mathbf{u}_K) = \underline{C}_{tot}^K(\mathbf{u}_K) - w_1 \log(a_{K_1})$$

Proposition 5: For the two-class system considered, $\Delta_{tot}^K(\mathbf{u}_K) \to w_1 \log(\beta\mathcal{P})$ and $\underline{C}_2^{K_2}(\mathbf{u}_K) \to 0$ as $K \to \infty$.

From the proof of Proposition 5, it follows that an optimal sequence $\{x_{K_2}\}$ satisfies $x_{K_2} \to \infty$ at a rate that satisfies the conditions in Proposition 3; i.e. the thresholds for both groups increase at approximately the same rate. The fraction of channels requested by $\mathcal{G}_2$ also converges to one, except that due to the tie-breaking rule, this group gets an arbitrarily small portion of those. Also, we note that to first order, the thresholds for $\mathcal{G}_2$ do not effect the growth rate of the total capacity. One conclusion from these results is that in such a

system, it may be worth reducing the feedback by having only the users in $\mathcal{G}_1$ report their CSI.

This analysis extends to $M > 2$ groups. In that case, again asymptotically only the highest priority group has a capacity that is increasing with K at a rate of $w_1 \log \log(K)$. The capacity of every other group tends to zero. Moreover, it can be shown that for all $m \leq M$, under the optimal thresholds,

$$\lim_{K \to \infty} \frac{\underline{C}_{m+1}^{K_{m+1}}}{\underline{C}_m^{K_m}} = 0,$$

i.e., the capacity of class $m + 1$ goes to zero faster than the capacity of class m.

V. Multiple Classes with Fixed Rate Ratios

If instead of using the optimal thresholds derived in the previous section, users in $\mathcal{G}_1$ chose x_{K_1} to decrease slowly enough so that $\exp(-e^{-x_{K_1}}) \log(\log(K))$ did not converge to zero, then the class 2 users would asymptotically receive a non-zero rate. Moreover, as long as x_{K_1} satisfied the conditions in Proposition 2, the total capacity would still be asymptotically equivalent to $w_1 \log(\log K)$. Alternatively, if $x_{K_1} \to x_0$, then both classes would achieve a throughput that increased at rate $\Theta(\log(\log K))$; however, the first-order constant of the total throughput would be strictly less than w_1. In this section, we study such a variation, where the rates allocated to classes increase asymptotically with a fixed ratio. One reason to consider this modification is to introduce "fairness" among classes. However, we note that if the weights w_i dynamically change over time, such modifications may not be needed, e.g., if class weights are inversely proportional to the windowed throughput, as in a proportional fair scheduler [8].

More precisely, we again consider the asymptotic growth rate of $\underline{C}_{tot}^K(\mathbf{u}_K)$ in a system with M groups of users with the additional constraint that as $K \to \infty$,

$$\underline{C}_m^K(\mathbf{u}_K) \asymp \alpha_m \underline{C}_M^K(\mathbf{u}_K), \tag{12}$$

for $m = 1, \ldots, M - 1$. The parameters α_m are positive constants that specify the asymptotic ratio of each group's (unweighted) capacity to that of $\mathcal{G}_M$.

Proposition 6: Given the constraints (12), as $K \to \infty$ the optimal thresholds for $\mathcal{G}_M$ satisfy $x_{K_M} \to \infty$ and $x_{K_M} = o(\log K)$. For $m = 1, \ldots, M-1$, the optimal thresholds satisfy $x_{K_m} \to x_m^*$, where

$$\exp(-e^{-x_m^*}) = \frac{1 + \sum_{l=m+1}^{M-1} \alpha_l}{1 + \sum_{l=m}^{M-1} \alpha_l}.$$

With such a sequence of thresholds

$$\underline{C}_{tot}^K \asymp \left(\sum_{m=1}^{M-1} w_m \alpha_m + w_M \right) \theta^* \log(\log K),$$

where $\theta^* = \frac{1}{1 + \sum_{m=1}^{M-1} \alpha_m}$.

Note that

$$\left(\sum_{m=1}^{M-1} w_m \alpha_m + w_M \right) \theta^* < w_1 \left(\sum_{m=1}^{M-1} \alpha_m + 1 \right) \theta^* = w_1.$$

Hence the asymptotic growth of C_{tot}^K with the fairness constraints is strictly less than the unconstrained growth. Also, it follows directly from the proof that each class m has a weighted throughput that increases as $w_m \alpha_m \theta^* \log(\log K)$. As an example, suppose that $\alpha_m = 1$ for each class $m < M$. Then it can be shown that $\theta^* = 1/M$, i.e., adding classes asymptotically reduces the capacity of each class as $1/M$.

VI. NUMERICAL RESULTS

In this section we provide numerical examples, which illustrate our asymptotic results. We assume two classes with weights $w_1 = 1.1$ and $w_2 = 1$. For both groups the channel variance $\sigma_m^2 = 1$, and $K_1 = K_2$. Also, we set the ratio $\beta = 0.5$, and the power per user $\mathcal{P} = 1$ (0 dB).

Figure 1 shows the weighted capacity per sub-channel for each group, $w_1 \underline{C}_1^{K_1}$ and $w_2 \underline{C}_2^{K_2}$, and the weighted sum capacity $\underline{C}_{tot}^K$ as a function of the number of users in the system K. As expected, $\underline{C}_{tot}^K$ and $w_1 \underline{C}_1^{K_1}$ both increase as $\log(\log K)$, and $\underline{C}_{tot}^K$ converges to $w_1 \log(\log K) + w_1 \log(\beta \mathcal{P})$, which is also shown in the figure. The capacity for group 2, $w_2 \underline{C}_2^{K_2}$, decreases very slowly to zero.

Figure 2 shows the optimal thresholds for the two groups, u_{K_1} and u_{K_2}, as a function of K. Both tend to infinity no faster than $\log(K)$, and no slower than $\log(K) - \log(\log(K))$, which are also shown.
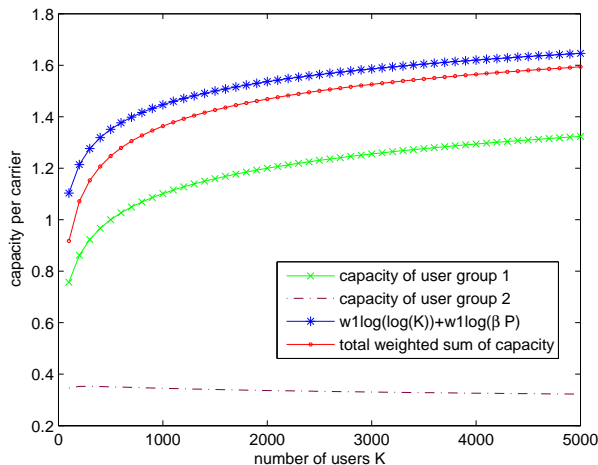

Fig. 1. Rates for two classes vs. total number of users K.

VII. CONCLUSION

We have characterized the asymptotic behavior of a sub-channel assignment scheme for downlink OFDMA system. We have assumed that each user feeds back one bit per sub-channel, although in practice, the feedback can be reduced further by feeding back one bit for a block of contiguous sub-channels (i.e., within a coherence band). Different quality of service requests are taken into account by assigning the users different priority weights, and by adjusting the feedback thresholds. We have shown that for multiple classes, optimizing the thresholds to maximize the weighted sum capacity
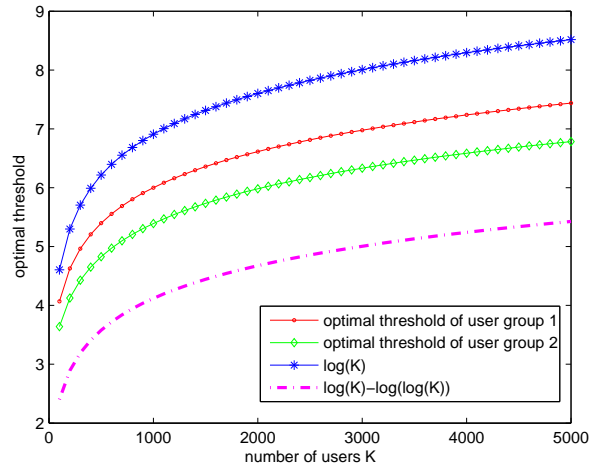

Fig. 2. Optimal thresholds for two classes vs. K.

results in the group with highest priority getting all of the sub-channels asymptotically. Hence to enforce fairness constraints, either the sum capacity objective must be compromised, as in the fixed rate ratio scenario, or the priorities can be adjusted by a scheduler (or both). Adjusting priorities may enable a reduction in feedback with little loss in spectral efficiency, since only the high priority group would need to report CSI. Although our results were derived for Rayleigh fading channels, we expect that analogous results can be derived for a more general class of fading distributions.

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