Distributed Approaches For Exploiting Multiuser Diversity in Wireless Networks

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Abstract

In wireless fading channels, multiuser diversity can be exploited by scheduling users so that they transmit when their channel conditions are favorable. This leads to a sum throughput that increases with the number of users and, in certain cases, achieves capacity. However, such scheduling requires global knowledge of every user's channel gain, which may be difficult to obtain in some situations. This paper addresses contention-based protocols for exploiting multiuser diversity with only local channel knowledge. A variation of the classic ALOHA protocol is given in which users attempt to exploit multi-user diversity gains, but suffer contention losses due to the distributed channel knowledge. We characterize the growth rate of the sum throughput for this protocol in a backlogged system under both short-term and long-term average power constraints. A simple "fixed-rate" system is shown to be asymptotically optimal and to achieve the same growth rate as in a system with a centralized scheduler. Moreover, asymptotically, the fraction of throughput lost due to contention is shown to be $1/e$. Also, in a system with random arrivals and an infinite user population, a variation of this ALOHA protocol is shown to be stable for any total arrival rate, given that users can estimate the backlog.

I. Introduction

In a multi-user fading channel, different users experience peaks in their channel quality at different times; this effect is called *multi-user diversity* [19]. Multi-user diversity can be exploited by scheduling users to transmit during the times when they have favorable channel conditions. The more users present, the more likely it is that one user has a very good channel at any given time; hence, the total throughput of such a system tends to increase with the number of users. Multiuser diversity has its roots in the work of Knopp and Humblet [19], where they present a power control scheme for maximizing the capacity of the uplink in a wireless network, modeled as a Gaussian multiple-access channel with frequency flat fading. It is shown in [19] that the sum capacity of this channel is achieved by scheduling only the user with the best channel to transmit at any time (see also [33]). Similar results hold for a parallel Gaussian broadcast channel [34].

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Multiuser diversity underlies much of the recent work on “opportunistic” or “channel aware” wireless scheduling such as [3], [22], [23], [38], [37], as well as several recent systems such as Qualcomm’s High Data Rate (HDR) architecture (CDMA 1xEV-DO) [4], [43].

Our focus in this paper is on distributed approaches that exploit multiuser diversity. As in [19], we consider a multiple-access model where a group of users all communicate to a single receiver (e.g. a base station or access point). The approach in [19] requires a centralized scheduler with knowledge of each user’s channel state information (CSI). This could be gained by having the each user transmit a pilot signal to the base station; each user’s channel gain would then be estimated and a scheduler at the base station would tell the user with the best channel to transmit. More precisely, assume that each user sends an orthogonal pilot signal to the base-station for the purpose of channel estimation and that each pilot requires K_p degrees of freedom, where the length of a pilot signal depends on the amount of training needed for channel estimation and any additional overhead needed per transmission. For a given bandwidth of W Hz, this implies that each additional user requires approximately $T_p \equiv K_p/(2W)$ additional seconds of overhead, and so in a system with N users the total overhead will be approximately $NT_p + T_c$, where T_c is the delay required for the base station to signal which user should transmit.[1] This is illustrated in Fig. 1(a) for a case where the pilot signals are transmitted orthogonally in time. To effectively exploit multiuser diversity, the total overhead should be less than the channel’s coherence time or else the estimated channel gains would no longer be relevant. It is clear that with sufficiently many users this may not be satisfied. For a given number of users, whether or not this overhead is significant will depend on the coherence time, the bandwidth, and the signal-to-noise ratio (which will effect T_p).[2] In particular in systems with many users (for example, the “reachback” scenario in a dense sensor network [2]), the overhead required for such a centralized approach may be prohibitive.

Instead of a centralized approach, we consider a case where each user has knowledge of its own fading level, but no knowledge of the fading levels of the other users in the cell. As shown in Fig. 1(b), this *distributed CSI* may be acquired by having the base-station periodically broadcast a pilot signal, which each user uses to estimate its channel. This requires that reciprocity holds between the downlink and uplink channels, e.g., this can hold in a time-division duplex (TDD) system, assuming that the variation in the channel gains is due to multi-path fading and not to other-cell interference. The overhead required for this approach does not increase with the number of users; the price for this is that each user must now decide when to transmit without global channel knowledge. To address this, we propose a simple variation of the classic slotted ALOHA protocol [1], [7], which we call *channel-aware ALOHA* [28], [29]. In networking, a key reason

[1]Also note that T_c will increase logarithmically with the maximum number of users the system is designed to accommodate, due to the overhead needed to identify each user.

[2]For example, in multi-carrier systems the overhead required to estimate the channel condition in each narrow-band carrier quickly becomes excessive.

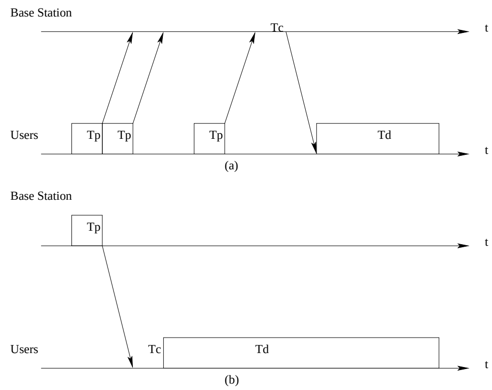

Fig. 1. An example of the overhead required for centralized (a.) and distributed (b.) channel measurement.

for using contention-based protocols, such as ALOHA, is because with bursty sources, the overhead in determining which source has traffic to send becomes prohibitive. Here, instead we use this protocol to compensate for the overhead in learning the channel information. Indeed, our results suggest that even without bursty arrivals, a contention-based protocol may be useful in this setting.

Multiaccess channels have a long history of research from both the information theory and networking communities; however, as pointed out in [14], these two communities often use very different models and approaches. In this paper, we borrow elements from both; we consider a "collision model" for the underlying channel as is often used in networking; given a successful transmission, we assume that the user's rate can approach the capacity of the underlying channel within a time-slot. This is reasonable when there are enough degrees of freedom available to use sophisticated codes. In this way information theoretic (capacity) results provide a useful abstraction of the underlying channel within a time-slot. This type of approach has been increasingly used to study scheduling and various queue control problems, often motivated by wireless applications (e.g. [5], [6], [8], [39]).

From an information theoretic perspective, related models with this type of distributed CSI have been studied in [17], [32], [36] for "distributed power control" in multiple-access channels.[3] These papers consider the expected sum mutual information at the receiver; transmitting at this rate with distributed CSI requires coding over many time-slots to average over the joint fading process. Here, we focus on a model where there is no coding over multiple time-slots. Also within each time-slot we assume that each user transmits a single codeword and single user decoding is used. This precludes approaches as in [10], [25], [26], which employ ideas from multiuser information theory (i.e., rate-

[3]In CDMA systems distributed power control has also been considered (e.g. [12]); in these systems, the distributed knowledge is typically the received signal-to-interference plus noise ratio (SINR) instead of the channel gain.

splitting and superposition coding) to recover information when collisions occur. These constraints could arise, for example, due to delay or complexity concerns. We emphasize that the basic ideas considered here can clearly be extended to these more sophisticated systems. From a random access perspective, distributed CSI has been considered in [2] for a ALOHA model with multi-packet reception [15], [41]. In [2], the CSI affects the reception probability of the transmitted packets, while the transmission rate per packet is fixed. Here, we use a simple collision model without multiuser reception or any power capture effects [24], [21]. Again, our basic ideas could be extended to such settings. Without capture, ALOHA with distributed CSI has been addressed in [40], which builds on the conference version of this paper [28] and considers stability issues. A carrier sensing random access protocol based on distributed CSI is given in [42] for optimizing the energy efficiency in a sensor network.

Our focus is on characterizing how the throughput of the channel aware ALOHA protocol scales as the size of the network increases. We primarily consider a backlogged or saturated system with n users, where each user always has data to send. Our basic model is described in the next section. In this setting, we show that this contention-based system can still exploit multiuser diversity and has a sum throughput that increases with n. In Section III, the rate at which this throughput is increasing as well as the first order constants are given under both long-term and short-term power constraints, for a broad class of fading distributions. It is also demonstrated that a simple “fixed-rate” policy can achieve the optimal growth rate. In Section IV, this distributed approach is compared to an optimal centralized system. We prove that the throughput of both systems increases at the same rate. Asymptotically, the ratio of the throughput of the channel-aware ALOHA to the throughput with a centralized scheduler is shown to be $1/e$, the same as the well-known ratio achieved by a standard slotted ALOHA system in an unfaded channel. This can be interpreted as saying that the only loss due to distributed channel knowledge is the loss due to random access for the channel. For a finite number of users, it is shown that the loss in throughput due to contention when fading is present is less than the loss in a channel without fading. In other words, lack of centralized control is less harmful in a fading environment. Finally, in Section V, we consider a variation of the ALOHA protocol for random arrivals. For an infinite user Rayleigh fading model, it is shown that the channel-aware ALOHA is stable for any total arrival rate. This stability is achieved by leveraging the increasing multiuser diversity as the number of backlogged users increases.

II. Model Description

We consider a multiple access model with n users all communicating to a single receiver over a common bandwidth of WHz. The channel between each user and the receiver is modeled as a frequency-flat fading channel with additive white Gaussian noise. Specifically, at each time t, the received signal $y(t)$ is given by

$$y(t) = \sum_{i=1}^{n} \sqrt{H_i(t)}x_i(t) + z(t), \tag{1}$$

where $x_i(t)$ and $H_i(t)$ are the transmitted signal and channel gain for the ith user, and $z(t)$ is additive white Gaussian noise with power spectral density $N_0/2$. The sequence of channel gains, $\mathbf{H}(t) = (H_1(t), \ldots, H_n(t))$ is modeled as a block-fading process [27], so that for $k = 1, 2, \ldots$, $\mathbf{H}(t) = \mathbf{H}_k \equiv (H_{1,k}, \ldots, H_{n,k})$ for all $t \in [kT, (k+1)T)$, where T is the length of each time-slot. Between time-slots, $\mathbf{H}_k$ changes randomly. Each component of $\mathbf{H}_k$ is independent, i.e. each user has independent fading. For simplicity, we focus on the case where $\{\mathbf{H}_k\}_{k=1}^{\infty}$ is an i.i.d. sequence of random vectors and each component $H_{i,k}$ has a probability density denoted by $f_{H_i}(h)$. For example, in the case of Rayleigh fading, $f_{H_i}(h) = \frac{1}{h_0} e^{-\frac{h}{h_0}}$, where $h_0 = \mathbb{E}(H_{i,k})$. Much of the following analysis also applies when for each i, $\{H_{i,k}\}_{k=1}^{\infty}$ is an arbitrary stationary ergodic process, in which case $f_{H_i}(h)$ can be interpreted as the steady-state distribution. However, when the channel has memory, this memory can be exploited to improve the performance over the approaches considered here, e.g., see [30]. We assume that $\mathbb{E}(H_{i,k}) < \infty$ and that $f_{H_i}(h) > 0$ for all $h > 0$ and is differentiable, so that the corresponding distribution function $F_{H_i}(h)$ is strictly increasing and twice differentiable. We mainly address the case where the fading statistics are the same for each user, i.e. for each slot k, $\{H_{i,k}\}_{i=1}^{n}$ are i.i.d.[4] In this case, we denote $f_{H_i}(h)$ by $f_H(h)$ for all i. Asymmetric models, where the fading statistics vary across users, are discussed in Section III-D.

We assume that each user has perfect distributed CSI, i.e., at the start of the kth time-slot, each user i knows $H_{i,k}$ but not $H_{j,k}$ for all $j \neq i$. We also assume that each user knows the distribution of its own channel gain; this is a more questionable assumption. In practice adaptive schemes which attempt to estimate the channel distributions from past observations would be needed. We briefly discuss one such approach that is suggested by our work in Section III. Given this distributed channel knowledge, let $P_{i,k}(H_{i,k})$ denote the transmission power of user i during time-slot k as a function of the user's channel gain. We assume that each transmitter i is subject to one of the following power constraints (see e.g. [9]):

- *Long-term average power constraint:* $\mathbb{E}_{H_i} P_i(H_i) \leq \bar{P}$.
- *Short-term maximum power constraint:* $P_i(H_i) \leq \check{P}$ for all H_i.

Here, we have dropped the time-index k to simplify notation. A long-term power constraint limits the total power used over many time-slots, while a short-term constraint limits the power used in each time-slot. The former may reflect constraints due to limited available energy, while the later may reflect regulatory constraints.

In [19], it is shown that given $\mathbf{H}_k$, the sum capacity of (1) in the symmetric case under a long-term average power constraint is achieved by setting $P_i(H_i) > 0$ only for some user i such that $H_i \geq H_j$ for all j; the exact value of $P_i(H_i)$ is determined by using a water-filling power allocation. In this case, during each time-slot only one user

[4] We note that as the the number of users in the network increases, this i.i.d. assumption becomes more questionable. If the spatial area of the network increases with the users, then differences in the channel statistics due to path loss will become more pronounced. On the other hand, if the users are confined to a given area, then as the number of users increases, the correlation between neighboring users channels will increase.

is transmitting.[5] We note that under a short-term power constraint the sum capacity is achieved if every user transmits in each time-slot with full power. However, in this case, a centralized controller needs to know $\mathbf{H}_k$ to determine the resulting rates that can be achieved during each time-slot (i.e. the resulting capacity region). Also, achieving the sum capacity will require successive interference cancellation or some other type of multi-user decoding. If one restricts themselves to approaches where at most one user transmits in a slot, then under a short-term power constraint, the sum throughput is again clearly maximized by having the user with the best channel transmit in each slot. Under both a short-term or long-term average power constraint the sum capacity will increase with the number of users, due to the increased multi-user diversity gain.

In the distributed case, we assume that at most one user can successfully transmit in each time-slot. Given that only user i transmits, let $R(\gamma_i)$ be a function that indicates the maximum rate at which the user can reliably transmit as a function of the received signal-to-noise ratio (SNR), $\gamma_i \equiv \frac{H_i P_i(H_i)}{N_0 W}$. To simplify notation, we normalize $N_0 W = 1$, so that $\gamma_i = H_i P_i(H_i)$. We assume that $R(\gamma)$ is an increasing, twice differentiable and strictly concave function of γ with $R(0) = 0$, $R(\infty) = \infty$, $R'(\infty) = 0$, and $R'(\gamma) > 0$ for all $\gamma \in [0, \infty)$.[6] We also assume that $R(\gamma)$ has *zero asymptotic elasticity*[7] meaning that

$$\limsup_{\gamma \to \infty} \frac{R'(\gamma)\gamma}{R(\gamma)} = 0.$$

This condition requires that the marginal change in rate per marginal change in SNR is asymptotically going to zero. The main example we consider is

$$R(\gamma_i) = \log(1 + \gamma_i), \tag{2}$$

which models the case where a user can transmit at rates approaching the Shannon capacity of (1) in each time-slot. This satisfies the preceding assumptions. Other functions, $R(\gamma)$, could also be used, for example, to model the achievable rate under a specific adaptive modulation and coding scheme; for most common schemes, the resulting $R(\gamma)$ will also satisfy these assumptions.[8] When user i transmits, it sends a single packet of $R(\gamma_i)T$ bits. This packet may be encoded, but no coding is done between consecutive packets. If multiple users transmit during a time-slot, a collision occurs and no data is received. As in the standard ALOHA model, after each time-slot the users receive instantaneous $(0, 1, e)$ feedback [7] indicating whether a slot was idle, contained a successful transmission or contained a collision.[9] In most practical systems, additional feedback will be available to

[5]In the asymmetric case, the optimal power allocation is also to allow only one user to transmit; however, in this case it will be the user with the largest weighted channel gain, where the weights depend on the channel distribution.

[6]We use the standard notation $f'(x)$ to denote the derivative of $f(x)$ with respect to its argument.

[7]We borrow this terminology from economics, see e.g. [20]

[8]In particular, for any such function, as long as $R(\gamma)$ asymptotically grows no faster than logarithmically with γ, then it will satisfy the zero asymptotic elasticity condition.

[9]In terms of the throughput of the backlogged system studied in the next section, it does not matter if the feedback is instantaneous or delayed.

the users, allowing for more elaborate protocols to be employed. Again, here we chose to illustrate the basic ideas in the simplest setting.

Given only H_i, each user i must decide in which slots to transmit and how much power to use when it transmits. For this purpose, we consider the following class of *channel aware ALOHA* protocols. In standard ALOHA, each backlogged user independently sends a packet in every slot with probability p. With channel aware ALOHA, each user i bases its transmission probability on its available CSI, H_i. Specifically, for a given threshold h_i', each user i transmits with probability one when $H_i > h_i'$, and otherwise sends nothing. Thus, user i will transmit with probability $p_i = \bar{F}_{H_i}(h_i')$, where $\bar{F}_{H_i}(h) \equiv 1 - F_{H_i}(h)$ is the complimentary distribution function of H_i, which by assumption is strictly decreasing in h. The average throughput of this protocol when all n users are always backlogged is given by

$$s(\boldsymbol{p}, n) = \sum_i \left(p_i \prod_{j \neq i} (1 - p_j) \mathbb{E}_{H_i} \left\{ R(P(H_i) \big| H_i > \bar{F}_{H_i}^{-1}(p_i) \right\} \right), \tag{3}$$

where $\boldsymbol{p} = (p_1, \ldots, p_n)$ denotes the vector of transmission probabilities and $\bar{F}_{H_i}^{-1}(\cdot)$ denotes the inverse function of $\bar{F}_{H_i}(\cdot)$. Each user's power allocation must also satisfy the given power constraint.

III. Throughput Scaling for Backlogged Systems

In this section, we analyze the throughput scaling of channel aware ALOHA protocols in a backlogged system, where all n users always have data to send. We emphasize that the number of backlogged users, n, is known by all users in the system. This is a reasonable assumption when the backlog is constant over a long time-scale, as assumed here. Given the backlog, first, we consider a heuristic "fixed-rate" protocol for a symmetric system, where each user transmits at a fixed rate (for a given number of users) with probability $p = \frac{1}{n}$. We characterize the order at which the throughput of this system increases with n. We then show that asymptotically this choice of probability is optimal for any fixed-rate system and, furthermore, such a fixed-rate system is asymptotically optimal within the larger class of variable-rate systems. Both long-term and short-term power constraints are considered. Systems with heterogeneous users will also be discussed, and finally, the performance of the channel aware ALOHA protocol will be compared with several other approaches.

A. *Fixed-rate algorithm,* $p = \frac{1}{n}$.

To begin, we focus on a symmetric, backlogged system with a long-term average power constraint. Consider a channel aware ALOHA protocol, where, for a given number of users n, every user has the same transmission probability p, and whenever a user transmits it does so at a fixed rate, R_f, which requires a fixed received power of $P_f = R^{-1}(R_f)$.[10] Each user will then simply invert the channel when they transmit and use power $P(h) = P_f/h$. To satisfy the long-term power constraint, p must satisfy

$$\int_{\bar{F}_H^{-1}(p)}^{\infty} f_H(h) \tfrac{P_f}{h} \, dh \le \overline{P}, \tag{4}$$

where $\bar{F}_H^{-1}(p)$ is the transmission threshold used by each user. It follows that given p, the maximum fixed rate a user can transmit at is

$$R_f(p) = R\left(\left(\int_{\bar{F}_H^{-1}(p)}^{\infty} f_H(h) \frac{1}{h} \, dh \right)^{-1} \bar{P} \right). \tag{5}$$

Assuming that this rate is used, the average sum throughput of the fixed-rate system under an average power constraint is

$$\bar{s}_f(p,n) = \left(np(1-p)^{n-1}\right) R_f(p). \tag{6}$$

The transmission probability p can be chosen to maximize this expression. Initially, we consider the sub-optimal choice of $p = \frac{1}{n}$ which results in a throughput of $\bar{s}_f(n) \equiv \bar{s}_f(\frac{1}{n}, n)$; this choice maximizes the first term in (6) and simplifies the following analysis.

We consider how $\bar{s}_f(n)$ scales as n increases. Notice that the first term in (6) is decreasing with n and approaches the well-known asymptote of $\frac{1}{e}$. However, $h' = \bar{F}_H^{-1}(\frac{1}{n})$ increases as n increases, and thus so will $R_f(1/n)$. The total throughput is increasing with n; the rate of increase is given in Proposition 1 below. To describe this rate, we use the following notation: Two sequences $f(n)$ and $g(n)$ are defined to be *asymptotically equivalent*, denoted by $f(n) \asymp g(n)$, as $n \to \infty$, if $\frac{f(n)}{g(n)} \to c > 0$. This implies that both f and g asymptotically grow at the same rate. In the special case where $c = 1$, we write $f(n) \doteq g(n)$ and say that $f(n)$ and $g(n)$ are *strongly asymptotically equivalent*; in this case, we indicate both the growth rate and the first order constant. Note that both $\doteq$ and $\asymp$ are equivalence relations.

For Proposition 1, we also require that the fading distribution satisfies the following definition:

Definition 1: A fading density, $f_H(h)$ on $[0, \infty)$ has a *well-behaved tail* if,

$$\lim_{h \to \infty} \frac{\bar{F}_H(h)}{h f_H(h)} = 0.$$

In most common fading models, such as Rayleigh or Ricean fading, $f_H(h)$ has an exponential tail, i.e., as $h \to \infty$, $f_H(h) \asymp e^{-\alpha h}$ for some $\alpha > 0$. It can be shown that such densities always satisfy the above definition. More generally, it can be shown that if

[10] Note, here the rate is fixed for a given number of users n, but it can vary with n, i.e. as the system scales the rate used may change.

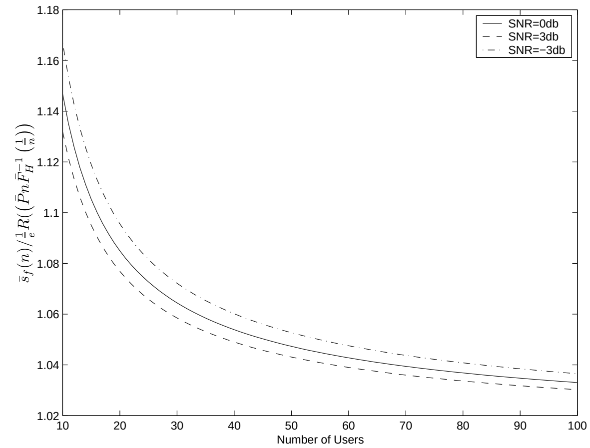

Fig. 2. Ratio of the total throughput $\bar{s}_f(n)$ of the fixed rate algorithm to $\frac{1}{e}R((\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right))$ as a function of the number of users, n for a Rayleigh fading channel with the indicated average received SNRs.

this limit exists and $\mathbb{E}(H) < \infty$, then it must lie in $[0, 1]$; however, examples of densities for which this limit does not exist can be found.[11]

Proposition 1: If $f_H(h)$ has a well-behaved tail, then $\bar{s}_f(n) \doteq \frac{1}{e}R\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)$.

The proof is given in Appendix A. As an example, consider a Rayleigh fading channel ($f_H(h) = \frac{1}{h_0}e^{-\frac{h}{h_0}}$), and assume that $R(\gamma)$ is given in (2). In this case, $\bar{F}_H^{-1}\left(\frac{1}{n}\right) = h_0\log(n)$, and, from Proposition 1,

$$\begin{aligned}\bar{s}_f(n) &\doteq \frac{1}{e}\log\left(1 + \bar{P}h_0 n\log(n)\right)\\ &\asymp \log(n) + \log(\log(n)).\end{aligned}$$

Figure 2 shows the ratio of $\bar{s}_f(n)$ to $\frac{1}{e}\log(1 + \bar{P}h_0 n\log(n))$ as a function of n for this example with three different values of the average received SNR ($h_0\bar{P}$). As expected, this ratio converges to 1. Even for small values of n, the ratio is only slightly larger than 1, suggesting that the asymptotic analysis is relevant for moderate values of n.

For a fixed-rate system with n users, let $\tilde{P}_f(n)$ be the maximum power used in any time-slot. Under a long-term average power constraint $\bar{P}$, from (4),

$$\tilde{P}_f(n) = \frac{P_f}{\bar{F}_H^{-1}(\frac{1}{n})} = \frac{\overline{P}}{\bar{F}_H^{-1}(\frac{1}{n})\int_{\bar{F}_H^{-1}(\frac{1}{n})}^{\infty} f_H(h)\frac{1}{h}\,dh}.$$

From the proof of Proposition 1, it follows that if $f_H(n)$ has a well-behaved tail, then

$$\int_{\bar{F}_H^{-1}(\frac{1}{n})}^{\infty} f_H(h)\frac{1}{h}\,dh \doteq \frac{1}{\bar{F}_H^{-1}(\frac{1}{n})n}.$$

[11]Prop. 1 can be generalized for the weaker assumption that this limit exists, but is not necessarily 0; in this case, we have only asymptotic equivalence instead of strong asymptotic equivalence.

Therefore we have,

Corollary 1: If $f_H(h)$ has a well-behaved tail, then as $n \to \infty$, $\tilde{P}_f(n) \doteq n\bar{P}$.

In other words, under an average power constraint, the maximum short-term power per time-slot is increasing linearly with the number of users. This is because each user is transmitting $1/n$th of the time and so it can use on average n times $\bar{P}$ when it does transmit.[12] It follows that if there is both a short-term and long-term power constraint, the short term power constraint eventually limits the throughput growth in Proposition 1.

Next, consider a version of the fixed-rate algorithm under a short-term power constraint. Each user still has the same transmission probability p and uses a fixed transmission rate whenever it transmits, and so requires a fixed received power of $P_f = hP(h)$. To satisfy the short-term maximum power constraint, $P(h)$ must be no greater than $\check{P}$ for all $h \geq \bar{F}_H^{-1}(p)$. Hence, for a given p the maximum fixed-rate is given by $R(\bar{F}_H^{-1}(p)\check{P})$. In a symmetric system, the resulting sum-throughput is given by

$$\check{s}_f(p,n) \equiv \left(np(1-p)^{n-1}\right) R\left(\bar{F}_H^{-1}(p)\check{P}\right). \tag{7}$$

Again, choosing $p = 1/n$, it is straightforward to see that as $n \to \infty$

$$\check{s}_f(n) \equiv \check{s}_f(\tfrac{1}{n}, n) \doteq \frac{1}{e} R\left(\check{P} F_H^{-1}\left(\frac{1}{n}\right)\right). \tag{8}$$

Note that compared to the average power constraint, the argument of $R(\cdot)$ is now $\frac{1}{n}$ times smaller. This is because the short-term power is now constant, instead of increasing with n as in Corollary 1. Indeed, if the average power per user is normalized by the number of users, the growth rate under the average power constraint will be the same as with a short-term power constraint. For the example of a Rayleigh fading channel with $R(\gamma)$ given by (2),

$$\begin{aligned}\check{s}_f(n) &\doteq \frac{1}{e}\log(1 + \check{P}h_0 \log(n)) \\ &\asymp \log(\log(n)).\end{aligned}$$

B. Fixed-rate algorithm, optimal p

So far, we set $p = \frac{1}{n}$ under both the long-term and short-term power constraint. In general, this is a suboptimal choice of p in the sense of maximizing the total throughput of the fixed-rate algorithm. However, we will show that asymptotically there is no loss in this choice of p.

Again, we first examine the system with a long-term average power constraint. We are still considering the fixed-rate algorithm with the total throughput $\bar{s}_f(p,n)$ given by (6). Let $p^*(n)$ be the optimal transmission probability for a given number of users n, i.e.,

$$p^*(n) = \arg\max_{0 \leq p \leq 1} \bar{s}_f(p,n).$$

[12] Asymptotically, this can be viewed as a type of "flash" or "peaky" signaling scheme, which are known to be capacity achieving for wide-band multipath fading channels, see e.g. [18], [35].

The resulting throughput is $\bar{s}_f(p^*(n), n)$. From (6) it can be seen that $p^*(n) \to 0$ as $n \to \infty$; otherwise, the total throughput would go to zero. For any such sequence, $p(n)$, the next lemma gives a direct generalization of Proposition 1.

Lemma 1: Let $p(n)$ be any sequence of transmission probabilities with $\lim_{n\to\infty} p(n) = 0$. If $f_H(h)$ has a well-behaved tail, then as $n \to \infty$,

$$\bar{s}_f(p(n), n) \asymp np(n)\,(1 - p(n))^{n-1}\, R\left(\bar{P}\frac{\bar{F}_H^{-1}(p(n))}{p(n)}\right).$$

The proof follows essentially the same steps as that for Proposition 1 and so is omitted. This lemma implies that $\bar{s}_f(p^*(n), n) \asymp \bar{s}_f(\tilde{p}(n), n)$, where for each n,

$$\tilde{p}(n) = \arg\max_{0\le p\le 1} np\,(1 - p)^{n-1}\, R\left(\bar{P}\frac{\bar{F}_H^{-1}(p)}{p}\right). \tag{9}$$

In other words, to characterize the asymptotic behavior of $\bar{s}_f(p^*(n), n)$, it is sufficient to study the behavior of $\bar{s}_f(\tilde{p}(n), n)$. The next lemma shows that asymptotically $\tilde{p}(n)$ cannot go to zero much faster than $\frac{1}{n}$; e.g., $p(n) = \frac{1}{n^2}$ does not satisfy this lemma.

Lemma 2: If $f_H(h)$ has a well-behaved tail, then there exists a constant $\check{\alpha} > 0$ and an integer $N > 0$ such that for all $n \ge N$, $\tilde{p}(n)$ in (9) satisfies

$$\frac{\check{\alpha}}{n} \le \tilde{p}(n) \le \frac{1}{n}.$$

The proof is given in Appendix B. Before stating the main result for this section, we state one other useful lemma regarding the tail of the fading distribution.

Lemma 3: Given any constant, $\alpha \in (0, 1)$,

$$\limsup_{y\to 0^+} \frac{\bar{F}_H^{-1}(\alpha y)}{\bar{F}_H^{-1}(y)} < \frac{1}{\alpha}.$$

The proof is given in Appendix C. Note that as $y \to 0^+$, both $\bar{F}_H^{-1}(y)$ and $\bar{F}_H^{-1}(\alpha y)$ are approaching infinity; this lemma implies that these quantities increase at essentially the same rate.[13]

Using these two lemmas, we have the following proposition which states that as $n \to \infty$ asymptotically there is no loss in throughput by choosing $p = \frac{1}{n}$.

Proposition 2: If $f_H(h)$ has a well-behaved tail, then as $n \to \infty$

$$\bar{s}_f(p^*(n), n) \asymp \bar{s}_f(n).$$

The proof is given in Appendix D. For example this implies that with Rayleigh fading and $R(\gamma)$ given in (2), then under the optimal transmission probability the total throughput also increases like $\log(n)$.

[13] We use the notation $y \to 0+$ to indicate that y approaches zero from the right.

Next we consider the optimal transmission probability for a fixed-rate algorithm under a short-term power constraint. In this case, let

$$\begin{aligned} p^*(n) &= \arg\max_{0\leq p\leq 1} \check{s}_f(p,n) \\ &= \arg\max_{0\leq p\leq 1} np(1-p)^{n-1}R(\check{P}\bar{F}_H^{-1}(p)). \end{aligned} \tag{10}$$

Notice that the only difference between this and (9) is the argument of $R(\cdot)$. In this case, it can again be shown that $p^*(n)$ decays like $\frac{1}{n}$.

Lemma 4: If $f_H(h)$ has a well-behaved tail, then there exists a constant $\check{\alpha} > 0$ and an integer $N > 0$ such that for all $n \geq N$, $p^*(n)$ in (10) satisfies

$$\frac{\check{\alpha}}{n} \leq p^*(n) \leq \frac{1}{n}.$$

The proof is given in Appendix E. Using this property, we have that under a short-term power constraint, there is again no loss asymptotically in choosing $p = \frac{1}{n}$.

Proposition 3: If $f_H(h)$ has a well-behaved tail, then as $n \to \infty$

$$\check{s}_f(p^*(n), n) \eqsim \check{s}_f(n).$$

The proof is in Appendix F.

These results suggest that with either a short-term or long-term power constraint, there is little loss in simply setting the transmission probability equal to $1/n$. This choice of transmission probability also facilitates adaptive algorithms when the users do not know their channel distributions. Assuming that all users have the same, but unknown channel distribution, each user would want to set a channel threshold so that it transmits $1/n$th of the time, where n is the current backlog. Each user could then simply track the fraction of time it transmits within a given window and adjust its channel threshold depending on whether this is less than or greater than $1/n$.

C. Variable-rate algorithms

We now turn to variable-rate algorithms, where for a given n, each user may transmit at a variable-rate $R(HP(H))$, which will depend on the user's channel gain H and power allocation $P(H)$. We first consider the system under a short-term power constraint. In this case, given that a user transmits, it should use the maximum power $\check{P}$, resulting in a rate of $R(H\check{P})$. In a symmetric network if each user transmits with probability p, then the sum throughput for a variable-rate system under a short-term power constraint will now be

$$\check{s}_v(p,n) = n(1-p)^{n-1}\int_{\bar{F}_H^{-1}(p)}^{\infty} f_H(h)R(\check{P}h)\,dh. \tag{11}$$

Clearly, for any p, $\check{s}_v(p,n) > \check{s}_f(p,n)$, i.e. the variable-rate system will have a larger throughput than a fixed-rate system with the same transmission probability. However, the next proposition shows that asymptotically these two systems are equivalent.

Proposition 4: Let $\{p_n\}$ be a non-negative, decreasing sequence of probabilities such that $p_n \to 0$ as $n \to \infty$. If $f_H(h)$ has a well-behaved tail, then $\check{s}_v(p_n, n) \asymp \check{s}_f(p_n, n)$.

The proof is given in Appendix G. Let $p_v^*(n)$ be the probability which maximizes $\check{s}_v(p, n)$ for each n. It follows from this proposition that $\check{s}_v(p_v^*(n), n) \asymp \check{s}_f(p_v^*(n), n)$. Also, letting $p^*(n)$ be the probability that maximizes $\check{s}_f(p, n)$, we have $\check{s}_v(p^*(n), n) \asymp \check{s}_f(p^*(n), n)$. Therefore, it must be that

$$\check{s}_v(p_v^*(n), n) \asymp \check{s}_f(p^*(n), n) \asymp \check{s}_f(n),$$

where the last relation follows from Proposition 3. In other words, the optimal variable-rate throughput is also strongly asymptotically equivalent to $\check{s}_f(n)$.

Next, we examine a variable-rate system under a long-term power constraint. We restrict ourselves to the case where $R(\gamma)$ is given by (2). In this case, if a user transmits with probability p, to maximize the sum throughput each user should choose a power allocation which maximizes the average throughput given a success subject to the average power constraint. This power allocation will be the solution to the following optimization problem:

$$\begin{aligned} \underset{P(h)}{\text{maximize}} \quad & \int_{\bar{F}_H^{-1}(p)}^{\infty} f_H(h) \log(1 + hP(h))\, dh \\ \text{subject to} \quad & \int_{\bar{F}_H^{-1}(p)}^{\infty} f_H(h) P(h)\, dh = \overline{P}. \end{aligned} \tag{12}$$

The solution to this will be a “water-filling” power allocation [13] over those channel states, $h > \bar{F}_H^{-1}(p)$. This is given by

$$P(h) = \left(\frac{1}{\lambda_p} - \frac{1}{h}\right)^+$$

for all $h \geq \bar{F}_H^{-1}(p)$, where λ_p is chosen to satisfy the average power constraint. Note that when p is large, the solution to (12) may result in $P(h) = 0$ for some $h > \bar{F}_H^{-1}(p)$. Specifically, this occurs when $\lambda_p > \bar{F}_H^{-1}(p)$. In this case, each user is actually transmitting with a probability smaller than p. However, it can be seen that as p decreases, $\bar{F}_H^{-1}(p)$ will increases and the corresponding parameter λ_p will be non-increasing. Hence, for small enough p, $\lambda_p < \bar{F}_H^{-1}(p)$; in this case, we have

$$\lambda_p = \frac{p}{\overline{P} + \int_{\bar{F}_H^{-1}(p)}^{\infty} \frac{1}{h} f_H(h)\, dh}. \tag{13}$$

Let $s_v(p, n)$ denote the optimal sum throughput given by (12) for a given n and p. Assuming that p is small enough so that (13) holds, then

$$\bar{s}_v(p, n) = n(1-p)^{n-1} \int_{\bar{F}_H^{-1}(p)}^{\infty} f_H(h) \log\left(\frac{h}{\lambda_p}\right) dh. \tag{14}$$

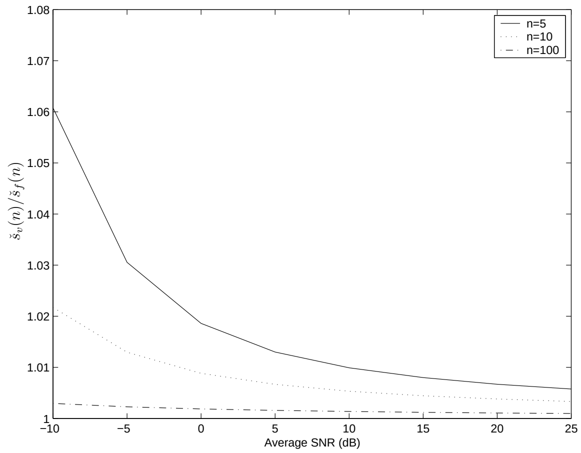

Fig. 3. Ratio of $\check{s}_v(n)$ to $\check{s}_f(n)$ as a function of the average received SNR in a Rayleigh fading channel with $n = 5, 10$ and 100 users.

The next proposition states that once again, the optimal variable-rate sum throughput is asymptotically equivalent to that obtained with a fixed-rate system.

Proposition 5: Let $\{p_n\}$ be a non-negative, decreasing sequence of probabilities such that $p_n \to 0$ as $n \to \infty$. If $f_H(h)$ has a well-behaved tail, then $\bar{s}_v(p_n, n) \eqsim \bar{s}_f(p_n, n)$.

The proof is given in Appendix H. For finite n, using an optimal (water-filling) power allocation will have some advantage over a fixed-rate scheme. The main advantage is that the fixed-rate scheme requires most of the power for "poor" channel states, while the optimal power allocation can save this power for better channel states. However, as n increases, the channel threshold $\bar{F}_H^{-1}(p_n)$ will also increase and in both cases a user will only transmit when the channel is "good". Intuitively, this explains why asymptotically there is no difference in these two schemes. Figure 3 shows the ratio of $\check{s}_v(n) \equiv \check{s}_v(\frac{1}{n}, n)$ to $\check{s}_f(n)$ for different values of n as a function of the average SNR ($h_0\bar{P}$) in a Rayleigh fading channel with $R(\gamma)$ given by (2). It can be seen that the ratio is decreasing with both the number of users and the SNR; even for a small number of users (i.e. $n = 5$) and small SNR the ratio is very close to 1.

D. Asymmetric Model

So far we have been considering a symmetric system where each user's fading was identically distributed and each user received the same average throughput. In this section, we will relax these assumptions and look at some simple asymmetric models. To begin consider a model where there are two classes of users. Class 1 has n_1 users and each user has the channel distribution $\bar{F}_{H_1}(h)$. Class 2 has n_2 users and each user has the channel distribution $\bar{F}_{H_2}(h)$. Again, each user has independent fading. We also allow these two classes to have different priorities, which is modeled by allowing one class of users to transmit with a higher probability than the other. Specifically we constrain the ratio of the transmission probabilities to satisfy $p_1 = \alpha p_2$. Without loss of generality we assume that $\alpha \geq 1$, i.e., class 1 has the higher priority. As in the symmetric model, each class $i = 1, 2$ will choose a channel threshold, $h'_i = \bar{F}^{-1}_{H_i}(p_i)$, and only transmit when their channel gain exceeds this threshold.

Here we focus on a fixed-rate algorithm with a short-term power constraint of $\check{P}_i$ for each class i. Similar ideas apply to the other settings. In this case, for a given transmission probability, each class i can transmit at rate $R\left(\check{P}_i \bar{F}^{-1}_{H_i}(p_i)\right)$, whenever $h \geq h'_i$. The sum throughput for all class 1 users is then given by

$$\check{s}^1_f(p_1, p_2, n_1, n_2) = n_1 p_1 (1 - p_1)^{n_1 - 1} (1 - p_2)^{n_2} R\left(\check{P}_1 \bar{F}^{-1}_{H_1}(p_1)\right). \tag{15}$$

Likewise, the sum throughput for class 2 users is

$$\check{s}^2_f(p_1, p_2, n_1, n_2) = n_2 p_2 (1 - p_2)^{n_2 - 1} (1 - p_1)^{n_1} R\left(\check{P}_2 \bar{F}^{-1}_{H_2}(p_2)\right). \tag{16}$$

Once again, we want to characterize how the total throughput scales as the number of users increases. In this case, we consider increasing n_1 and n_2 while keeping their ratio fixed, i.e. $n_2 = \beta n_1$ for some $\beta > 0$. With this assumption, letting $n = n_1$ and $p = p_1$, the total throughput can be written as

$$\begin{aligned} \check{s}^{(2)}_f(p, n) &= \check{s}^1_f(p, \alpha p, n, \beta n) + \check{s}^2_f(p, \alpha p, n, \beta n) \\ &= np(1-p)^{n-1}(1-\alpha p)^{\beta n} R\left(\check{P}_1 \bar{F}^{-1}_{H_1}(p)\right) \\ &\quad + \alpha\beta p(1-p)^n (1-\alpha p)^{\beta n - 1} R\left(\check{P}_2 \bar{F}^{-1}_{H_2}(\alpha p)\right). \end{aligned} \tag{17}$$

To analyze the asymptotic performance we also make the following assumption about the fading distributions of the two classes:

Definition 2: Two fading densities $f_{H_1}(h)$ and $f_{H_2}(h)$ on $[0, \infty)$ have ***similar tails*** if they both have well-behaved tails and there exists some constant $c > 0$ such that

$$\lim_{h \to \infty} \frac{f_{H_1}(h)}{f_{H_2}(h)} = c.$$

This definition requires the tails of the the two fading distributions to be asymptotically equivalent. For example, this will be true if both distributions correspond to Rayleigh fading with different means. Moreover, if two densities have similar tails, then as the next lemma states, the rates $R(\check{P}_1 \bar{F}^{-1}_{H_1}(p))$ and $R(\check{P}_2 \bar{F}^{-1}_{H_2}(\alpha p))$ will also be asymptotically equivalent as $p \to 0$.

Lemma 5: If $f_{H_1}(h)$ and $f_{H_2}(h)$ have similar tails, then for any $\alpha > 0$

$$\lim_{p \to 0^+} \frac{R(\check{P}_1 \bar{F}^{-1}_{H_1}(p))}{R(\check{P}_2 \bar{F}^{-1}_{H_2}(\alpha p))} = 1.$$

The proof is given in Appendix I. Another property we will use for two distributions with similar tails is:

Lemma 6: Given that $f_{H_1}(h)$ and $f_{H_2}(h)$ have similar tails, let $f_{\tilde{H}}(h)$ be a fading density with distribution, $F_{\tilde{H}}(h) = F_{H_1}(h)F_{H_2}(h)$. For $i = 1, 2$, $f_{\tilde{H}}(h)$ and $f_{H_i}(h)$ also have similar tails.

The proof is given in Appendix J. Notice that $f_{\tilde{H}}(h)$ is the density of the maximum of two independent random variables with distributions $f_{H_1}(h)$ and $f_{H_2}(h)$. This lemma states that if both of these random variables have similar tails, then they will each have similar tails with their maximum.

Let $p^*(n)$ be the value of p which maximizes the total throughput in (17) as a function of n. The following proposition generalizes Proposition 3 to this setting.

Proposition 6: If $f_{H_1}(h)$ and $f_{H_2}(h)$ have similar tails, then as $n \to \infty$,

$$\check{s}_f^{(2)}(p^*(n), n) \doteqdot \check{s}_f^{(2)}\left(\tfrac{1}{(1+\alpha\beta)n}, n\right).$$

In other words, for this asymmetric model, it is asymptotically optimal to set $p_1 = \frac{1}{(1+\alpha\beta)n} = \frac{1}{n_1+\alpha n_2}$ and $p_2 = \alpha p_1$. Note that if $\alpha = 1$, i.e. both classes have the same priority, then just as in the symmetric case, it is asymptotically optimal for both classes to transmit with a probability of 1 over the total number of users. The proof of this Proposition is given in Appendix K. The main idea in this proof is to consider a symmetric system, where every user has a fading density given by $f_{\tilde{H}}(h)$ as defined in Lemma 6. We then use our previous results for a symmetric system along with lemmas 5 and 6 to derive the desired results.

Using Proposition 6, it can be seen that total throughput for class 1 users satisfies

$$\check{s}_f^1(p^*(n), \alpha p^*(n), n, \beta n) \doteqdot \frac{1}{e(1+\alpha\beta)} R\left(\check{P}_1 \bar{F}_{H_1}^{-1}\left(\frac{1}{(1+\alpha\beta)n}\right)\right).$$

Likewise for class 2,

$$\check{s}_f^2(p^*(n), \alpha p^*(n), n, \beta n) \doteqdot \frac{\alpha\beta}{e(1+\alpha\beta)} R\left(\check{P}_2 \bar{F}_{H_2}^{-1}\left(\frac{1}{(1+\alpha\beta)n}\right)\right).$$

The total throughput for both classes satisfies

$$\check{s}_f^{(2)}(p^*(n), n) \doteqdot \frac{1}{e} R\left(\check{P}_1 \bar{F}_{H_1}^{-1}\left(\frac{1}{(1+\alpha\beta)n}\right)\right) \doteqdot \frac{1}{e} R\left(\check{P}_2 \bar{F}_{H_2}^{-1}\left(\frac{1}{(1+\alpha\beta)n}\right)\right),$$

where the last step follows from the similar tail property.

This results can easily be extended to $k > 2$ classes, where each class $i = 1, \ldots, k$ has a different channel distribution $\bar{F}_{H_i}(h)$ and different short-term power constraint $\check{P}_i$. Let the transmission probability of each class $i > 1$ be constrained to satisfy $p_i = \alpha_i p_1$, and let the number of users in class i, satisfy $n_i = \beta_i n_1$. Denote by $\check{s}_f^{(k)}(p, n)$ the total throughput of all k classes, as a function of $p = p_1$ and $n = n_1$, and let $p^*(n)$ be the probability which maximizes this throughput for a given n.

Corollary 2: If $\{f_{H_i}(h)\}_{i=1}^k$ are a family of fading densities where each pair have similar tails, then as $n \to \infty$,

$$\check{s}_f^{(k)}(p^*(n), n) \asymp \check{s}_f^{(k)}\left(\frac{1}{(1+\sum_{i=2}^k \alpha_i\beta_i)n}\right).$$

The proof of this follows the same argument as in Proposition 6 and so is omitted. From this corollary, it follows that the total throughput for each class i using the optimal transmission probability $\alpha_i p^*(n)$ is strongly asymptotically equivalent to

$$\frac{\alpha_i\beta_i}{e(1+\sum_{i=1}^k \alpha_i\beta_i)} R\left(\check{P}_i \bar{F}_H^{-1}\left(\frac{1}{(1+\sum_{i=2}^k \alpha_i\beta_i)n}\right)\right).$$

E. Performance Comparisons

We conclude this section by comparing the performance of several other protocols to the performance of channel-aware ALOHA in a symmetric Rayleigh fading channel with average channel gain h_0, when $R(\gamma)$ is given by (2). For simplicity, we focus on the case of a fixed-rate policy with a short-term power constraint; similar results hold for variable rate protocols and long-term power constraints. Recall in this case the throughput of the channel aware Aloha protocol grows like $\frac{1}{e}R(\check{P}h_0\log(n))$ as a function of n for Rayleigh fading channels.

The first alternative we consider is a slotted ALOHA system where there is no fading and the channel between each user and the transmitter has a constant gain of h_0. In this case, given a short-term power constraint of $\check{P}$, the maximum rate a user can send at when it transmits is $R(h_0\check{P})$ independent of the number of users. The sum throughput is then maximized by choosing $p = 1/n$, yielding

$$\check{s}_{nf}(n) := \left(1 - \tfrac{1}{n}\right)^{n-1} R(h_0\check{P}).$$

As $n \to \infty$, this decreases and approaches the constant value of $\frac{1}{e}R(h_0\check{P})$, while the throughput of channel aware ALOHA grows unbounded with n.

The next alternative we consider is an ALOHA system with Rayleigh fading, where the users do not base their transmissions on the channel state. When a user transmits, it does so at a fixed-rate for a given backlog n. However, with Rayleigh fading, a user would not be able to transmit at any fixed-rate R as the channel gain approaches zero and still satisfy the short-term power constraint. To accommodate this, we assume that each user's transmission is only successful when its channel is above a threshold h_{min}. The difference here is that this threshold will not change with the number of users, and the users will transmit regardless of the threshold (this is reasonable, for example if the user's do not have any CSI). The choice of h_{min} subject to a short-term power constraint which maximizes the average throughput is

$$h_{min} = \arg\max_h \left\{ R(\check{P}h)e^{-\frac{h}{h_0}} \right\}. \tag{18}$$

The sum throughput in this case is again maximized by choosing $p = 1/n$, yielding

$$\check{s}_{nc}(n) := \left(1 - \frac{1}{n}\right)^{n-1} R(\check{P}h_{min})e^{-\frac{h_{min}}{h_0}}.$$

Again this will be decreasing with n. In this case, the throughput approaches an asymptote of $\frac{1}{e}R(\check{P}h_{min})e^{-\frac{h_{min}}{h_0}}$. Furthermore, it can be shown that $h_{min} < h_0$, and so $\check{s}_{nc}(n) < \check{s}_{nf}(n)$. This can be interpreted as saying that if the transmitters can not exploit fading, then the fading reduces the throughput of an ALOHA system over that achieved in a non-faded channel.

The last case we consider is a TDM system in a Rayleigh fading channel, where each user is assigned a fixed time-slot in a TDM frame. During each time slot only one user can transmit. As in the second case above, we assume users transmit at a constant transmission rate R, and the transmission is only successful when the channel gain is larger than h_{min} in (18). The sum throughput in this case is given by $\check{s}_{TDM}(n) := R(\check{P}h_{min})e^{-\frac{h_{min}}{h_0}}$, which is a constant, independent of n.

Figure 4 shows an example of the sum throughput as a function of n in all fours cases. Notice that for small values of n, the TDM approach has a higher throughput than the channel aware ALOHA system. As n grows, however, the ALOHA approach quickly achieves higher throughputs, despite the fact that collisions occur. This is interesting as in a wire-line channels, a TDM approach is always preferable to any random access technique for a backlogged system. However in this wireless setting, the channel-aware ALOHA system has a higher throughput when enough users are present to provide sufficient multiuser diversity.

Let $\hat{n}$ denote the minimum number of users required for the throughput of the channel-aware ALOHA system to be greater than or equal to the throughput of the TDM scheme. For a fixed-rate system with transmission probability $\frac{1}{n}$, $\hat{n}$ is the smallest n such that $\check{S}_{TDM}(n) \leq \check{s}_f(n)$. Figure 5 shows $\hat{n}$ for a Rayleigh fading channel as a function of the average SNR. It can be seen that at low SNR only a few users are needed for the channel aware ALOHA to outperform TDM. This number increases with the average SNR. Also shown in Fig. 5 is the same quantity in a Nakagami fading channel with parameter $m = 2$ under a short-term power constraint and a Rayleigh fading channel with a long-term power constraint. Under a long-term power constraint $\hat{n}$ increases slower than in the same channel under a short-term constraint. This suggests that the channel aware ALOHA system benefits more from the ability to allocate power than the TDM system In the Nakagami fading model, $\hat{n}$ is larger than in the Rayleigh model. The fading in a Nakagami channel with $m = 2$ has a smaller variance than in a Rayleigh channel. A more variable fading environment is beneficial for opportunistic transmission schemes such as channel-aware ALOHA.

IV. Comparisons with Centralized Schedulers

In this section, we compare the performance of the channel aware ALOHA protocol to that achieved by a centralized scheduler with perfect knowledge of every user's channel

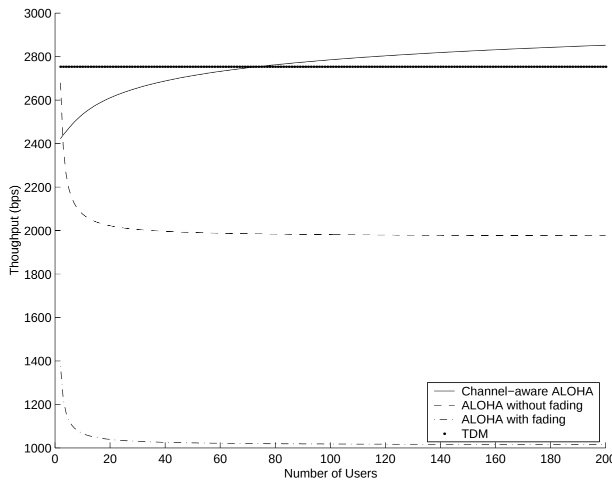

Fig. 4. Comparison of the sum throughput versus the number of users for fixed-rate systems under a short-term power constraint. The parameter used here are $W = 1$KHz, $\check{P} = 40$W, $N_0 = 0.001$W/Hz and $h_0 = 1$.

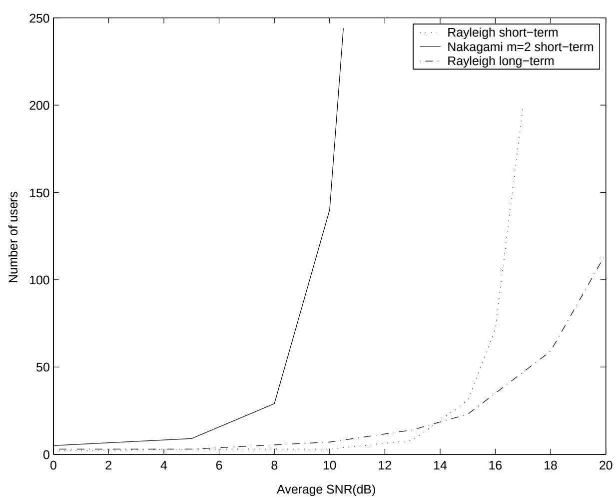

Fig. 5. The number of users required for channel-aware ALOHA to outperform TDM as a function of the average SNR.

gain. We again consider a backlogged symmetric system. From Section III, we know that with an appropriate transmission probability the throughput of the channel aware ALOHA will increase with the number of backlogged users as does the maximum sum throughput in a centralized system. In this section we compare the rate at which the throughput increases in these two systems.

We consider centralized schedulers that are restricted to scheduling one user per time-slot. With this restriction, the centralized scheduling policy that maximizes the sum throughput would always schedule the user with the largest channel gain in each time-slot.[14] Assuming a short-term power constraint of $\check{P}$ and variable rate transmissions, the average sum throughput achieved by such a policy with n users is

$$\check{s}_{ct}(n) := \mathbb{E}_{\mathbf{H}}\left(R(\check{P}\max_{i=1,\dots,n} H_i)\right). \tag{19}$$

The next proposition compares this to the throughput $\check{s}_v(\frac{1}{n}, n)$ of a variable-rate channel aware ALOHA protocol with transmission probability $p = \frac{1}{n}$ (cf. (11)).

Proposition 7: For all n, $\check{s}_v(\frac{1}{n}, n) > (1 - \frac{1}{n})^{n-1}\check{s}_{ct}(n)$.

The proof is given in Appendix L. It is based on showing that the channel gain conditioned on a success in the ALOHA system is stochastically larger than the maximum unconditional channel gain in the centralized system. This implies that the throughput in the ALOHA system averaged only over the successful slots will be greater than the throughput in the centralized system; however, the actual throughput in the ALOHA system will be decreased by the probability of collision which is $(1 - \frac{1}{n})^{n-1}$. We note that this proof does not require any assumptions about the tail of the fading distribution, nor does $R(\gamma)$ need to have zero asymptotic elasticity.

Since $(1 - \frac{1}{n})^{n-1} \to 1/e$ as $n \to \infty$, an immediate corollary of Proposition 7 is that $\check{s}_{ct}(n) \asymp \check{s}_v(\frac{1}{n}, n)$, i.e., the throughput of the distributed system increases asymptotically at the same rate as the optimal centralized system. From the previous section, it follows that $\check{s}_f(p^*(n), n)$ and $\check{s}_f(n)$ are also asymptotically equivalent to $\check{s}_{ct}(n)$.

Note that Proposition 7 is not just valid asymptotically, but holds for all finite n. For each n consider the ratio

$$r_f(n) = \frac{\tilde{s}_v(\frac{1}{n}, n)}{\tilde{s}_{ct}(n)},$$

which can be viewed as a measure of the loss in throughput of the medium access control protocol over a centralized scheduler. Proposition 7 states that

$$r_f(n) < \left(1 - \frac{1}{n}\right)^{n-1}$$

[14] As noted previously, with a long-term power constraint the restriction to scheduling one use per time-slot is not needed, since the optimal solution from [19] without this restriction will have this property.

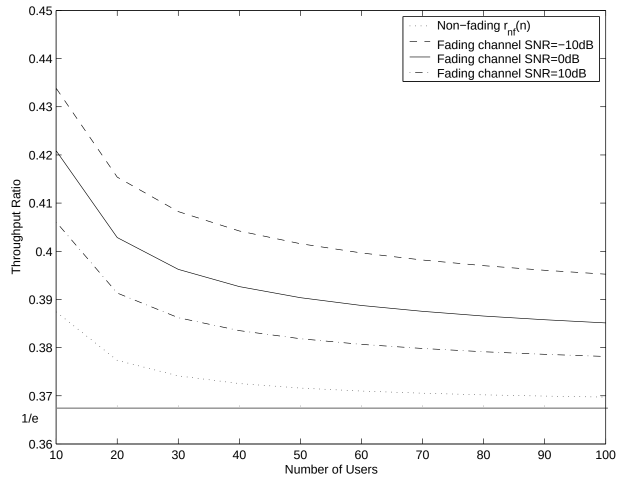

Fig. 6. The ratios $r_{nf}(n)$ and $r_f(n)$, for several different average SNRs, as a function of n. The asymptote of $\frac{1}{e}$ is also shown.

for all n. In a channel without fading, we can also define the ratio of the throughput when using a standard ALOHA protocol, compared to a centralized scheduler. In this case the centralized scheduler will have a normalized throughput of 1 transmission per slot independent of n. Hence, the throughput ratio will simply be

$$r_{nf}(n) = \left(1 - \frac{1}{n}\right)^{n-1}.$$

Therefore, for all finite n, $r_f(n) < r_{nf}(n)$. In other words, the penalty for lack of coordination is smaller in a fading channel for any finite n. Figure 6 shows $r_{nf}(n)$ and $r_f(n)$ for several different average SNRs, as a function of n in a Rayleigh fading channel, with $R(\gamma)$ given by (2). As expected $r_{nf}(n) < r_f(n)$ for all n, with the difference decreasing as the SNR increases.

A similar result to Proposition 7 applies for a system with a long-term power constraint. In this case, the average sum throughput with a centralized scheduler is given by

$$\bar{s}_{ct}(n) := \mathbb{E}_{\mathbf{H}} \left(R(P(\mathbf{H}) \max_{i=1,\ldots,n} H_i) \right), \tag{20}$$

where $P(\mathbf{H})$ denotes the optimal power allocation which satisfies the long-term power constraints of $\bar{P}$ for each user. For a symmetric model with independent fading, it can be seen that the optimal power allocation will only be a function of $\max_{i=1,\ldots,n} H_i$. When $R(\gamma)$ is given by (2), this will again be a water-filling allocation. Likewise, for the distributed system, let $\bar{s}_v(\frac{1}{n}, n)$ denote the optimal variable-rate throughput when each user transmits with probability $1/n$ using the optimal power allocation $P(H)$ when it transmits.[15] In this case we have:

Proposition 8: For all n, $\bar{s}_v(\frac{1}{n}, n) > (1 - \frac{1}{n})^{n-1}\bar{s}_{ct}(n)$.

The proof is given in Appendix M.

A. First order constants

We have shown that the sum throughput of the optimal distributed system is increasing at the same rate as the throughput of the optimal centralized system, but we have not specified the first order constants. Propositions 7 and 8 bound this constant to be greater than $1/e$, but do not show that this bound is tight. In this section, we show that this bound is indeed tight with an additional assumption on the tail of the fading distribution. In the case of a short-term power constraint, this means that $\check{s}_f(n) \asymp \frac{1}{e}s_{ct}(n)$. Combining this with our previous results it follows that $\check{s}_v(p^*(n), n)$ and $\check{s}_v(\frac{1}{n}, n)$ are also strongly asymptotically equivalent to $\frac{1}{e}s_{ct}(n)$. In terms of the ratio $r_f(n)$ defined above, this implies that as $n \to \infty$, $r_f(n) \to \frac{1}{e}$, the same as the limit of $r_{nf}(n)$ for the non-faded system. Referring to Figure 6, this means that each curve is asymptotically converging to $\frac{1}{e}$.

To characterize the first order constant, we use the following result from extreme order statistics:

Lemma 7 ([11]): Let $\{Z_i\}_{i=1}^n$ be i.i.d. non-negative random variables with a complimentary distribution function $\bar{F}_Z(\cdot)$ and p.d.f. $f_Z(\cdot)$ satisfying $\bar{F}_Z(z) > 0$ for all $z \in [0, \infty)$, $\bar{F}_Z(z)$ is twice differentiable for all z, and $\lim_{z\to\infty} \frac{d}{dz}\left[\frac{\bar{F}_Z(z)}{f_Z(z)}\right] = 0$. Then as $n \to \infty$,

$$\Pr\left(\max_{1\le i\le n} z_i - l_n)a_n \le u\right) \to \exp(-e^{-u})$$

uniformly in u, where l_n is given by $\bar{F}_Z(l_n) = 1/n$, and $a_n = nf_Z(l_n)$.

In other words, the given conditions are sufficient to ensure that with suitable normalization the distribution function of the maximum will converge to a Gumble distribution ($\exp(-e^{-u})$). We will apply this result to analyze the throughput growth with a centralized scheduler. To do this, we will assume that the fading density $f_H(h)$ satisfies the condition:

$$\lim_{h\to\infty} \frac{d}{dh}\left[\frac{\bar{F}_H(h)}{f_H(h)}\right] = 0. \tag{21}$$

This will be true for all common fading models, such as Ricean and Rayleigh fading. The next lemma shows that the set of fading distributions which satisfy this condition are nearly the same as those which have a well-behaved tail.

Lemma 8: If a fading distribution satisfies (21), then it has a well-behaved tail. Conversely, if a fading distribution has a well-behaved tail and the limit in (21) exists, then the limit will be zero.

[15]Note the optimal power allocation will be different in the centralized and distributed systems.

The proof is a simple consequence of L'Hospital's rule.

Note that the sum throughput with a centralized scheduler can be written as

$$\check{s}_{ct}(n) = \mathbb{E}_{\mathbf{H}}(\max_{i=1,\ldots,n} R(\check{P}H_i)).$$

To analyze this throughput, we will apply Lemma 7 to the random variables $Z_i = R(\check{P}H_i)$. The next lemma states that if the random variables H_i satisfy condition (21), then the random variables Z_i will satisfy a similar condition.

Lemma 9: If the fading density, $f_H(h)$ satisfies (21), then the density $f_Z(z)$ of the random variables $Z_i = R(\check{P}H_i)$ satisfies:

$$\lim_{z\to\infty} \frac{d}{dz}\left[\frac{\bar{F}_Z(z)}{f_Z(z)}\right] = 0. \tag{22}$$

The proof is given in Appendix N.

Proposition 9: If the fading density satisfies (21), then $\check{s}_f(n) \asymp \frac{1}{e}\check{s}_{ct}(n)$.

The proof of this is given in Appendix O.

Under an average constraint, consider a centralized system where each user transmits a variable rate but uses a "flat" power allocation of $n\bar{P}$, i.e. when a users is scheduled, its transmission rate is $R(n\bar{P}H)$. Let $\bar{s}_{ct,flat}(n)$ denote the sum throughput of this system. Using the same argument as in the proof of Proposition 9 we have:

Corollary 3: If the fading density satisfies (21), then $\bar{s}_f(n) \asymp \frac{1}{e}\bar{s}_{ct,flat}(n)$.

Clearly, $\bar{s}_{ct,flat}(n) \leq \bar{s}_{ct}(n)$ and from Proposition 5, when $R(\gamma)$ is given by (2), $\bar{s}_v(\frac{1}{n}, n) \asymp \bar{s}_f(n)$. Thus from this corollary and Proposition 8, it follows that $\bar{s}_{ct,flat}(n) \asymp \bar{s}_{ct}(n)$ and so,

Corollary 4: If the fading density satisfies (21), and $R(\gamma)$ is given by (2) then $\bar{s}_f(n) \asymp \frac{1}{e}\bar{s}_{ct}(n)$.

In other words, when $R(\gamma)$ is given by (2), then under a long-term power constraint, the ratio of the throughputs of the optimal centralized and distributed schemes again converges to $1/e$.

V. Random arrivals

In the previous sections we assumed that all nodes are always backlogged. In this section, we relax this assumption and briefly examine a simple model where packets randomly arrive with total arrival rate λ. Specifically, we consider an infinite user model, where it is assumed that each new packet arrives to a new user [7]. Such a model is reasonable for a system with a large number of users, each with a small arrival rate. We still assume that the number of backlogged users n in each slot is known. This is more questionable now that the number of backlogged users is varying with time. Practically, the backlog would have to be estimated using an algorithm such as the Pseudo-Bayesian algorithm [7].

We still consider an approach where users base their transmission on whether their channel gain is above a threshold h'; however, now we allow this threshold to depend on the backlog. Specifically we assume that

$$h'(n) = \begin{cases} h_{min} & \text{for } \bar{F}_H^{-1}(\frac{1}{n}) < h_{min} \\ \bar{F}_H^{-1}(\frac{1}{n}) & \text{for } \bar{F}_H^{-1}(\frac{1}{n}) \geq h_{min}. \end{cases} \tag{23}$$

Here h_{min} is a minimum threshold above which the user will transmit regardless of the backlog.[16]

We consider a symmetric, fixed-rate model with a short-term power constraint of $\check{P}$. Given that n users are backlogged, each user will transmit at rate $R(\check{P}h'(n))$ if successful. As n increases, the transmission rate $R(\check{P}h'(n))$ will also increase. If all packets have a fixed length of L bits, then the time needed to transmit a packet is $L/R(\check{P}h'(n))$, which will decrease as n increases. We consider a slotted-time model, where the length of time-slots vary with the backlog according to this relationship. Packet arrivals are assumed to be independent in each time-slot with an expected arrival rate of $\lambda L/R(\check{P}h'(n))$. In this section, we still assume that the channel variation is memoryless between slots. Since the slot sizes are variable, this may seem to be a questionable assumption. However as discussed in [28], this may be reasonable for a fixed rate model. The main idea is that for a fixed-rate model, the key parameter is the probability that the channel exceeds the transmission threshold in each slot. For many channel models the correlation in this threshold crossing probability will increase with the slot length, but decrease with the threshold level. Over a limited range these two effects can balance out making the i.i.d. assumption reasonable. A more detailed discussion of this assumption and extensions to other channel models can be found in [28]; here, we simply take this as an idealized model to convey the basic ideas.

Given the above assumptions, we consider over what range of arrival rates, λ, the system is stable. The following proposition states that if $R(\check{P}h'(n))$ is unbounded (as in the Rayleigh fading model), then the system will be stable for any total arrival rate. However, for high enough arrival rates this requires a prohibitively high diversity gain and the underlying physical model becomes unrealistic.

Proposition 10: If $R(\check{P}h'(n))$ is unbounded, then the infinite user, channel-aware ALOHA system is stable for any total arrive rate λ.

The proof is given in Appendix P. Figure 7 illustrates the basic idea behind this result. This figure shows both the total arrival rate and departure rate normalized in units of packets per time-slot, as a function of the backlog. The parameters used in the figure are $\lambda = 0.6$ packets/second, $L = 1000$bits/packets, $W = 1$kHz and $\frac{\check{P}h_0}{N_0 W} = 1$. For small backlogs the normalized arrival rate is larger than the departure rate, and so the backlog will tend to increase. Eventually, for high enough backlogs, the arrival rate will drop

[16]For a fixed rate system, a threshold $h_{min} > 0$ is needed for similar reasons as in the TDM system from Section III-E.

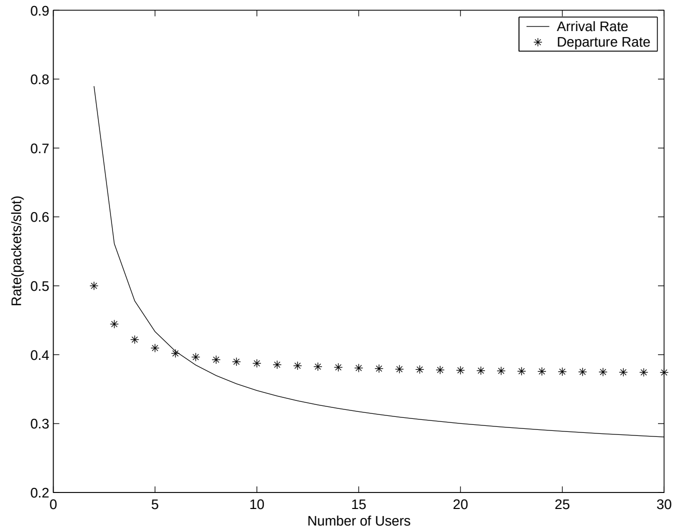

Fig. 7. Stability of the channel-aware ALOHA.

below the departure rate; the system will stabilize around the point where these curves cross. As the arrival rate increases, the system will stabilize around a larger backlog; this is because more users are needed to provide the multiuser diversity gain necessary to stabilize the system. The higher backlog results in a larger delay. This is illustrated in Figure 8. This figure shows simulation results of the delay for a system with a finite number of user for various total arrival rates, λ. In the simulations, each user has a queue and arriving packets are queued before transmission. Packet arrivals are modeled as a Poisson process. For each curve the total arrival rate is fixed as the number of users varies. Notice that for a given arrival rate, the delay decreases as the number of users increase; this is due to the increased multiuser diversity.

We note that Proposition 10 does not imply that a system with a finite number of users is stable for any arrival rate. For example, consider a system with n users and symmetric traffic. If the total arrival rate λ satisfies $\lambda > \check{s}_f(n)$, the system will be unstable. What this result does say is that if any arrival rate is spread across enough users, then the system can be stabilized.

VI. Conclusions and future work

In this paper, we presented a distributed protocol, channel-aware ALOHA, for exploiting multi-user diversity in a fading multiple access channel without a centralized controller. For a backlogged model, we characterized the throughput scaling for such a system under both long-term and short-term power constraints. The total throughput was shown to scale at the same rate as in an optimal centralized system, with an asymptotic

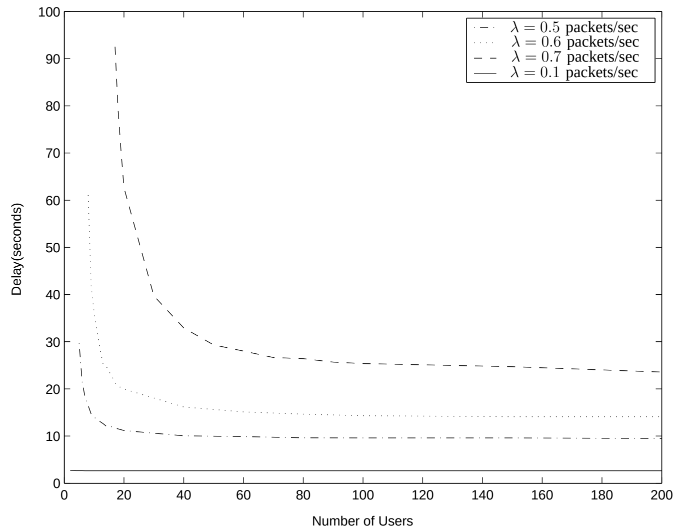

Fig. 8. Average delay versus the number of users with Poisson arrivals and various arrival rates.

ratio of $1/e$. Moreover, with sufficient users, the throughput of this approach exceeds that of a static TDM approach without contention. This shows that even with backlogged users, such a contention-based approach may be useful in a fading environment. We have also shown that there is little advantage to be gained in such a system from allocating transmission power and rate based on the channel state. Finally, in the random arrival case, this ALOHA system was shown to be stable for any arrival rate in an infinite user model, but at the expense of large backlogs.

As we have noted, in practice one may be able to implement more sophisticated random access protocols as well as utilize more sophisticated physical layer processing (i.e. to enable multiuser reception). Such approaches will naturally improve the performance over that obtained by channel aware ALOHA. However, we note that under a long-term power constraint, our results suggest that any such technique can not improve the order of the asymptotic growth rate, but could increase the constant of $1/e$. This is because under a long-term power constraint, we are comparing to the capacity achieving scheduling policy. Under a short-term power constraint, however, the order could improve with multi-user reception. This is because with a short-term power constraint a centralized system that used joint decoding, will have a capacity that scales like $\Theta(\log(n))$ instead of the $\Theta(\log(\log(n)))$ achieved with scheduling, assuming Rayleigh fading.

In related work, [30], we have considered a splitting protocol for this setting, which can reduce the contention loss at the expense of an increase in overhead. We are also considering extensions of this model to parallel channel models, as in an OFDM system.

APPENDIX A

Proof of Proposition 1: The growth rate of

$$\bar{s}_f(n) = \left(1 - \tfrac{1}{n}\right)^{n-1} R\left(\frac{\overline{P}}{\int_{\bar{F}_H^{-1}(\frac{1}{n})}^{\infty} f_H(h)\frac{1}{h}\,dh}\right) \tag{24}$$

with n depends on the behavior of $\int_{\bar{F}_H^{-1}(\frac{1}{n})}^{\infty} f_H(h)\frac{1}{h}\,dh$. This quantity is upper-bounded by

$$\begin{aligned}\int_{\bar{F}_H^{-1}(\frac{1}{n})}^{\infty} f_H(h)\frac{1}{h}\,dh &< \int_{\bar{F}_H^{-1}(\frac{1}{n})}^{\infty} f_H(h)\frac{1}{\bar{F}_H^{-1}\left(\frac{1}{n}\right)}\,dh \\ &< \frac{1}{n\bar{F}_H^{-1}(\frac{1}{n})},\end{aligned} \tag{25}$$

where we have used that $f_H(h) = -\frac{d}{dh}\bar{F}_H(h)$. Since $R(\gamma)$ is increasing in γ, it follows that $\bar{s}_f(n) \geq \left(1 - \frac{1}{n}\right)^{n-1} R\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)$, and so

$$\lim_{n\to\infty} \frac{\bar{s}_f(n)}{R\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)} > \frac{1}{e}. \tag{26}$$

Next, we lower bound $\int_{\bar{F}_H^{-1}(\frac{1}{n})}^{\infty} f_H(h)\frac{1}{h}\,dh$. Since $f_H(h)$ has a well-behaved tail, for any $\delta > 0$, there exists a $\tilde{h} > 0$, such that for all $h > \tilde{h}$, $\frac{\bar{F}_H(h)}{hf_H(h)} < \delta$. It follows that for all $h > \tilde{h}$,

$$f_H(h) > \tfrac{1}{\delta+1}\left(\tfrac{\bar{F}_H(h)}{h} + f_H(h)\right).$$

Therefore, for large enough values of n,

$$\begin{aligned}\int_{\bar{F}_H^{-1}(\frac{1}{n})}^{\infty} f_H(h)\frac{1}{h}\,dh &> \frac{1}{\delta+1}\int_{\bar{F}_H^{-1}(\frac{1}{n})}^{\infty}\left(\frac{\bar{F}_H(h)}{h^2} + \frac{f_H(h)}{h}\right)dh \\ &= \left(\frac{1}{\delta+1}\right)\left(\frac{1}{n\bar{F}_H^{-1}(\frac{1}{n})}\right).\end{aligned}$$

Substituting this into (24) and using the monotonicity of $R(\cdot)$ yields that for n large enough,

$$\bar{s}_f(n) < \left(1 - \tfrac{1}{n}\right)^{n-1} R\left((\delta+1)\bar{P}n\bar{F}_H^{-1}\left(\tfrac{1}{n}\right)\right). \tag{27}$$

Next note that since R is convex, for any $\gamma \geq 0$,

$$\begin{aligned}R((\delta+1)\gamma) &\leq R(\gamma) + \delta\gamma R'(\gamma) \\ &\leq (\delta+1)R(\gamma),\end{aligned}$$

where the second step follows because $R(\gamma)-\gamma R'(\gamma) > R(0) = 0$. Combining this with (27), we have for large enough n,

$$\bar{s}_f(n) < \left(1-\tfrac{1}{n}\right)^{n-1}(\delta+1)R\left(\bar{P}n\bar{F}_H^{-1}\left(\tfrac{1}{n}\right)\right),$$

and so

$$\lim_{n\to\infty}\frac{\bar{s}_f(n)}{R\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)} < \lim_{n\to\infty}(\delta+1)\left(1-\tfrac{1}{n}\right)^{n-1} = \frac{\delta+1}{e}. \tag{28}$$

Since, δ can be arbitrarily small, from (26) and (28), it follows that $\bar{s}_f(n) \precsim \frac{1}{e}R\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)$ as desired. ■

APPENDIX B

Proof of Lemma 2: For each n, let $\tilde{p}(n) = \frac{\tilde{\alpha}(n)}{n}$, so that $\tilde{\alpha}(n)$ will be the value of α in $[0,n]$ that maximizes

$$\alpha\left(1-\tfrac{\alpha}{n}\right)^{n-1}R\left(\bar{P}\bar{F}_H^{-1}\left(\tfrac{\alpha}{n}\right)\tfrac{n}{\alpha}\right). \tag{29}$$

This is the product of two terms. The first, $\alpha\left(1-\frac{\alpha}{n}\right)^{n-1}$, is maximized by choosing $\alpha = 1$; the remaining term is decreasing in α. Therefore, for any n, it must be that $\tilde{\alpha}(n) \le 1$. To complete the proof, we show that for n large enough, $\tilde{\alpha}(n) \ge \check{\alpha}$, for some $\check{\alpha} > 0$.

Since $\ln(\cdot)$ is monotonic, $\tilde{\alpha}(n)$ will also maximize

$$Z(n,\alpha) := \ln\left(R\left(\bar{P}\bar{F}_H^{-1}\left(\tfrac{\alpha}{n}\right)\tfrac{n}{\alpha}\right)\right) + \ln(\alpha) + (n-1)\ln\left(1-\tfrac{\alpha}{n}\right).$$

Differentiating the first term on the right with respect to α yields

$$\begin{aligned}&\frac{d}{d\alpha}\ln\left(R\left(\bar{P}\bar{F}_H^{-1}\left(\frac{\alpha}{n}\right)\frac{n}{\alpha}\right)\right)\\ &= \frac{R'\left(\bar{P}\bar{F}_H^{-1}\left(\frac{\alpha}{n}\right)\frac{n}{\alpha}\right)}{R\left(\bar{P}\bar{F}_H^{-1}\left(\frac{\alpha}{n}\right)\frac{n}{\alpha}\right)}\cdot\bar{P}\left[\frac{n}{\alpha}\left(\frac{-1}{f_H\left(\bar{F}_H^{-1}\left(\frac{\alpha}{n}\right)\right)}\right)\frac{1}{n} + \frac{-n}{\alpha^2}\bar{F}_H^{-1}\left(\frac{\alpha}{n}\right)\right],\end{aligned}$$

where we have applied the inverse function theorem to $\frac{d}{d\alpha}F_H^{-1}\left(\frac{\alpha}{n}\right)$. Rearranging terms, we have

$$\begin{aligned}&\frac{d}{d\alpha}\ln\left(R\left(\bar{P}\bar{F}_H^{-1}\left(\frac{\alpha}{n}\right)\frac{n}{\alpha}\right)\right)\\ &= -1\left(\frac{R'(x)x}{R(x)}\right)\left(\frac{\bar{F}_H(y)}{yf_H(y)}+1\right)\left(\frac{1}{\alpha}\right),\end{aligned}$$

where $x = \bar{P}\bar{F}_H^{-1}\left(\frac{\alpha}{n}\right)\frac{n}{\alpha}$ and $y = \bar{F}_H^{-1}\left(\frac{\alpha}{n}\right)$. Note that as $\frac{\alpha}{n} \to 0$, then $x \to \infty$ and $y \to \infty$. Since the asymptotic elasticity of $R(\gamma)$ is zero and $f_H(h)$ has a well-behaved tail, it follows that for any $\delta \in (0,1)$ and $\bar{\alpha} \in (0,1)$, there exists $N > 0$, such that for all $n \ge N$ and $\alpha \le \bar{\alpha}$,

$$\frac{d}{d\alpha}\ln\left(R\left(\bar{P}\bar{F}_H^{-1}\left(\tfrac{\alpha}{n}\right)\tfrac{n}{\alpha}\right)\right) \ge -\delta\left(\tfrac{1}{\alpha}\right).$$

Hence, for all $n \geq N$ and $\alpha \leq \bar{\alpha}$,

$$\begin{aligned}\frac{d}{d\alpha}Z(n,\alpha) &\geq (1-\delta)\frac{1}{\alpha} - \left(\frac{1-\frac{1}{n}}{1-\frac{\alpha}{n}}\right)\\ &\geq (1-\delta)\frac{1}{\alpha} - \left(\frac{1}{1-\frac{\bar{\alpha}}{N}}\right).\end{aligned}$$

And so, there must exist some $\check{\alpha} \in (0, \bar{\alpha})$ such that for all $n \geq N$ and $\alpha \leq \check{\alpha}$, $\frac{d}{d\alpha}Z(n,\alpha) > 0$. Therefore, since $\tilde{\alpha}(n)$ maximizes $Z(n,\alpha)$, it must be that for all $n \geq N$, $\tilde{\alpha}(n) \geq \check{\alpha}$. ■

APPENDIX C

Proof of Lemma 3: By assumption $\bar{F}_H^{-1}(h)$ is a positive and strictly decreasing function on $[0,1]$. Let

$$G(y) := \int_y^1 \bar{F}_H^{-1}(h)\,dh,$$

so that $\frac{d}{dy}G(y) = -\bar{F}_H^{-1}(y)$. Note that since H is a non-negative random variable, $\lim_{y\to 0^+} G(y) = \mathbb{E}H$. Likewise, let

$$G^\alpha(y) := \int_y^{1/\alpha} \bar{F}_H^{-1}(\alpha h)\,dh = \alpha^{-1}\int_{\alpha y}^1 \bar{F}_H^{-1}(z)\,dz,$$

so that $\frac{d}{dy}G^\alpha(y) = -\bar{F}_H^{-1}(\alpha y)$. In this case, we have $\lim_{y\to 0^+} G^\alpha(y) = \alpha^{-1}\mathbb{E}H$.

Now, note that

$$\begin{aligned}\frac{\alpha G^\alpha(y) - G(y)}{y} &= \frac{1}{y}\int_{\alpha y}^{y} \bar{F}_H^{-1}(h)\,dh\\ &\geq \bar{F}_H^{-1}(\alpha y)(1-\alpha),\end{aligned}$$

where the last step follows because $F_H^{-1}(h)$ is decreasing. Therefore,

$$\lim_{y\to 0^+}\frac{\alpha G^\alpha(y) - G(y)}{y} \geq \lim_{y\to 0^+}\bar{F}_H^{-1}(\alpha y)(1-\alpha) = \infty.$$

Notice that as $y \to 0^+$, both $\alpha G^\alpha(y) - G(y) \to 0$ and $y \to 0$. Hence, L'Hospital's rule can be applied, yielding

$$\lim_{y\to 0^+}\frac{\alpha G^\alpha(y) - G(y)}{y} = \lim_{y\to 0^+} -\alpha\bar{F}_H^{-1}(\alpha y) + \bar{F}_H^{-1}(y).$$

Combining the above observations, we have that

$$\lim_{y\to 0^+} -\alpha\bar{F}_H^{-1}(\alpha y) + \bar{F}_H^{-1}(y) = \infty.$$

Therefore, for any $M > 0$, when y is small enough, $-\alpha\bar{F}_H^{-1}(\alpha y) + \bar{F}_H^{-1}(y) > M$, or equivalently,

$$\frac{\bar{F}_H^{-1}(\alpha y)}{\bar{F}_H^{-1}(y)} < \frac{1}{\alpha}\left(1 - \frac{M}{\bar{F}_H^{-1}(y)}\right).$$

Taking limits as $y \to 0^+$, the desired results follows. ■

APPENDIX D

Proof of Proposition 2: From Lemma 1 we have that

$$\bar{s}_f(p^*(n), n) \doteqdot n\tilde{p}(n)\left(1 - \tilde{p}(n)\right)^{n-1} R\left(\bar{P}\frac{\bar{F}_H^{-1}(\tilde{p}(n))}{\tilde{p}(n)}\right),$$

where $\tilde{p}(n)$ is given by (9). Likewise, $\bar{s}_f(n) \doteqdot \left(1 - \frac{1}{n}\right)^{n-1} R\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)$. Therefore, it is sufficient to show that

$$n\tilde{p}(n)\left(1 - \tilde{p}(n)\right)^{n-1} R\left(\bar{P}\frac{\bar{F}_H^{-1}(\tilde{p}(n))}{\tilde{p}(n)}\right) \doteqdot \left(1 - \frac{1}{n}\right)^{n-1} R\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right).$$

From the definition of $\tilde{p}(n)$ it follows that

$$\liminf_{n\to\infty} \frac{n\tilde{p}(n)\left(1 - \tilde{p}(n)\right)^{n-1} R\left(\bar{P}\frac{\bar{F}_H^{-1}(\tilde{p}(n))}{\tilde{p}(n)}\right)}{\left(1 - \frac{1}{n}\right)^{n-1} R\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)} \geq 1.$$

Also note that since $p = \frac{1}{n}$ maximizes $np(1-p)^{n-1}$, then

$$\limsup_{n\to\infty} \frac{n\tilde{p}(n)\left(1 - \tilde{p}(n)\right)^{n-1}}{\left(1 - \frac{1}{n}\right)^{n-1}} \leq 1.$$

To complete the proof, we show that

$$\limsup_{n\to\infty} \frac{R\left(\bar{P}\frac{\bar{F}_H^{-1}(\tilde{p}(n))}{\tilde{p}(n)}\right)}{R\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)} \leq 1.$$

First note that since $R(\cdot)$ is concave,

$$\begin{aligned} R\left(\bar{P}\frac{\bar{F}_H^{-1}(\tilde{p}(n))}{\tilde{p}(n)}\right) \leq{}& R\left(\bar{P}n\bar{F}_H^{-1}\left(\tfrac{1}{n}\right)\right) \\ &+ R'\left(\bar{P}n\bar{F}_H^{-1}\left(\tfrac{1}{n}\right)\right)\bar{P}\left(\frac{\bar{F}_H^{-1}(\tilde{p}(n))}{\tilde{p}(n)} - n\bar{F}_H^{-1}\left(\tfrac{1}{n}\right)\right). \end{aligned} \tag{30}$$

Using the mean value theorem, there exists a $x_n \in [\tilde{p}(n), \frac{1}{n}]$ such that

$$\frac{\bar{F}_H^{-1}(\tilde{p}(n))}{\tilde{p}(n)} - n\bar{F}_H^{-1}\left(\tfrac{1}{n}\right) = \frac{d}{dp}\left.\frac{\bar{F}_H^{-1}(p)}{p}\right|_{p=x_n}\left(\tilde{p}(n) - \frac{1}{n}\right)$$

$$= \frac{\bar{F}_H^{-1}(x_n)}{x_n^2}\left(1 + \frac{\bar{F}_H(y_n)}{y_n f_H(y_n)}\right)\left(\tfrac{1}{n} - \tilde{p}(n)\right),$$

where $y_n := \bar{F}_H^{-1}(x_n)$. Let $Z_n := 1 + \frac{\bar{F}_H(y_n)}{y_n f_H(y_n)}$ and note that as $n \to \infty$, $Z_n \to 1$, since f_H has a well-behaved tail. Substituting this into (30), we have

$$\begin{aligned} R\left(\bar{P}\frac{\bar{F}_H^{-1}(\tilde{p}(n))}{\tilde{p}(n)}\right) &\le R\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right) \\ &\quad + R'\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)\bar{P}\frac{\bar{F}_H^{-1}(x_n)}{x_n^2}Z_n\left(\frac{1}{n} - \tilde{p}(n)\right). \end{aligned} \tag{31}$$

From Lemma 2, for n large enough, $x_n \ge \tilde{p}(n) \ge \frac{\breve{\alpha}}{n}$. Therefore, for large enough n, the second term on the right-hand side of (31) is upperbounded by

$$R'\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)\bar{P}n\bar{F}_H^{-1}\left(\tfrac{1}{n}\right)\left(\frac{\bar{F}_H^{-1}\left(\frac{\breve{\alpha}}{n}\right)}{\bar{F}_H^{-1}\left(\frac{1}{n}\right)}\right)(Z_n)\left(\frac{1-\breve{\alpha}}{\breve{\alpha}^2}\right).$$

Substituting this bound into (31), dividing by $R\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)$, and taking limits yields

$$\begin{aligned} &\limsup_{n\to\infty}\frac{R\left(\bar{P}\frac{\bar{F}_H^{-1}(\tilde{p}(n))}{\tilde{p}(n)}\right)}{R\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)} \\ &\le 1 + \left(\frac{1-\breve{\alpha}}{\breve{\alpha}^2}\right)\limsup_{n\to\infty}\left(\frac{R'\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)}{R\left(\bar{P}n\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)}\right)\left(\frac{\bar{F}_H^{-1}\left(\frac{\breve{\alpha}}{n}\right)}{\bar{F}_H^{-1}\left(\frac{1}{n}\right)}\right) \\ &= 1. \end{aligned}$$

The last step follows from Lemma 3 and the fact that $R(\gamma)$ has zero asymptotic elasticity. ■

APPENDIX E

Proof of Lemma 4: We use a similar argument as in Appendix B; the key difference here is that $R(\cdot)$ has a different argument. In this case, for each n, let $p^*(n) = \frac{\tilde{\alpha}(n)}{n}$, so that $\tilde{\alpha}(n)$ will be the value of α in $[0, n]$ which maximizes

$$\alpha\left(1 - \tfrac{\alpha}{n}\right)^{n-1} R\left(\check{P}\bar{F}_H^{-1}\left(\tfrac{\alpha}{n}\right)\right). \tag{32}$$

By the same argument as in Appendix B, for any n, it must be that $\tilde{\alpha}(n) \le 1$. To complete the proof, we show that for n large enough, $\tilde{\alpha}(n) \ge \breve{\alpha}$, for some $\breve{\alpha} > 0$. We do this by showing that the derivative of

$$Z(n,\alpha) \equiv \ln\left(R\left(\check{P}\bar{F}_H^{-1}\left(\tfrac{\alpha}{n}\right)\right)\right) + \ln(\alpha) + (n-1)\ln\left(1 - \tfrac{\alpha}{n}\right)$$

is strictly positive for n large enough and α small enough. Differentiating the first term on the right with respect to α using the inverse function and rearranging terms, we have

$$\frac{d}{d\alpha}\ln\left(R\left(\check{P}\bar{F}_H^{-1}\left(\frac{\alpha}{n}\right)\right)\right) = -1\left(\frac{R'(x)x}{R(x)}\right)\left(\frac{\bar{F}_H(y)}{yf_H(y)}\right)\left(\frac{1}{\alpha}\right),$$

where $x = \check{P}\bar{F}_H^{-1}\left(\frac{\alpha}{n}\right)$ and $y = F_H^{-1}\left(\frac{\alpha}{n}\right)$. As $\frac{\alpha}{n} \to 0$, then $x \to \infty$ and $y \to \infty$. Since the asymptotic elasticity of $R(\gamma)$ is zero and $f_H(h)$ has a well-behaved tail, it follows that for any $\delta \in (0,1)$ and $\bar{\alpha} \in (0,1)$, there exists a $N > 0$ such that for all $n \geq N$ and $\alpha \leq \bar{\alpha}$, $\frac{d}{d\alpha}\ln\left(R\left(\bar{P}\bar{F}_H^{-1}\left(\frac{\alpha}{n}\right)\frac{n}{\alpha}\right)\right) \geq -\delta\left(\frac{1}{\alpha}\right)$. The remainder of the proof is exactly the same as in Appendix B. ∎

APPENDIX F

Proof of Proposition 3: This proof follows a similar argument as in Appendix D. Since $p^*(n)$ is optimal, we clearly have

$$\liminf_{n\to\infty}\frac{\check{s}_f(p^*(n),n)}{\check{s}_f(n)} = \liminf_{n\to\infty}\frac{np^*(n)(1-p^*(n))^{n-1}R\left(\check{P}\bar{F}_H^{-1}(p^*(n))\right)}{(1-\frac{1}{n})^{n-1}R\left(\check{P}\bar{F}_H^{-1}(\frac{1}{n})\right)} \geq 1.$$

Also, as in Appendix D,

$$\limsup_{n\to\infty}\frac{np^*(n)\left(1-p^*(n)\right)^{n-1}}{\left(1-\frac{1}{n}\right)^{n-1}} \leq 1.$$

We complete the proof by showing that

$$\limsup_{n\to\infty}\frac{R\left(\check{P}\bar{F}_H^{-1}(p^*(n))\right)}{R\left(\check{P}\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)} \leq 1.$$

Since $R(\cdot)$ is concave,

$$R\left(\check{P}\bar{F}_H^{-1}(p^*(n))\right) \leq R\left(\check{P}\bar{F}_H^{-1}(\tfrac{1}{n})\right) + R'\left(\check{P}\bar{F}_H^{-1}(\tfrac{1}{n})\right)\check{P}\left(\bar{F}_H^{-1}(p^*(n)) - \bar{F}_H^{-1}(\tfrac{1}{n})\right). \quad (33)$$

Using the mean value theorem, there exists a $x_n \in [p^*(n), \frac{1}{n}]$ such that

$$\bar{F}_H^{-1}(p^*(n)) - \bar{F}_H^{-1}(\tfrac{1}{n}) = \frac{-1}{f_H(\bar{F}_H^{-1}(x_n))}\left(p^*(n) - \tfrac{1}{n}\right).$$

Substituting this into (33), we have

$$R\left(\check{P}\bar{F}_H^{-1}(p^*(n))\right) \leq R\left(\check{P}\bar{F}_H^{-1}(\tfrac{1}{n})\right) + R'\left(\check{P}\bar{F}_H^{-1}(\tfrac{1}{n})\right)\frac{\check{P}}{f_H(\bar{F}_H^{-1}(x_n))}\left(\tfrac{1}{n} - p^*(n)\right). \quad (34)$$

From Lemma 4, there exists a $\check{\alpha} > 0$ such that for n large enough, $\frac{1}{n} \geq x_n \geq \tilde{p}(n) \geq \frac{\check{\alpha}}{n}$. Therefore, for large enough n, from (34) we have

$$\frac{R\left(\check{P}\bar{F}_H^{-1}(p^*(n))\right)}{R\left(\check{P}\bar{F}_H^{-1}\left(\frac{1}{n}\right)\right)} \leq 1 + \left(\frac{R'\left(\check{P}\bar{F}_H^{-1}(\frac{1}{n})\right)\check{P}\bar{F}_H^{-1}(\frac{1}{n})}{R\left(\check{P}\bar{F}_H^{-1}(\frac{1}{n})\right)}\right)\left(\frac{\bar{F}_H(z_n)}{f_H(z_n)z_n}\right) \times \left(\frac{\bar{F}_H^{-1}(\frac{\check{\alpha}}{n})}{\bar{F}_H^{-1}(\frac{1}{n})}\right)\left(\frac{1}{\check{\alpha}} - 1\right), \tag{35}$$

where $z_n = \bar{F}_H^{-1}(x_n)$. Since $R(\cdot)$ has zero asymptotic elasticity, $f_H(h)$ has a well-behaved tail, and using Lemma 3, it can be seen that the right-hand side of (35) converges to 1 as $n \to \infty$, yielding the desired result. ■

APPENDIX G

Proof of Proposition 4: From their definitions it can be seen that showing $\check{s}_f(p_n, n) \asymp \check{s}_v(p_n, n)$ is equivalent to showing that

$$\lim_{n\to\infty} \frac{\frac{1}{p_n}\int_{\bar{F}_H^{-1}(p_n)}^{\infty} f_H(h)R(\check{P}h)\,dh}{R(\check{P}\bar{F}_H^{-1}(p_n))} = 1.$$

Clearly, the $\liminf$ of this ratio is greater than or equal to 1. So, the proof will be complete by showing that the $\limsup$ of this ratio is no greater than one.

Since $R(\cdot)$ is concave, for all h and p_n,

$$R(\check{P}h) \leq R(\check{P}\bar{F}_H^{-1}(p_n)) + R'(\check{P}\bar{F}_H^{-1}(p_n))\left(\check{P}(h - \bar{F}_H^{-1}(p_n))\right).$$

It follows that

$$\begin{aligned}\frac{\frac{1}{p_n}\int_{\bar{F}_H^{-1}}^{\infty} f_H(h)R(\check{P}h)\,dh}{R(\check{P}\bar{F}_H^{-1}(p_n))} &\leq 1 + \frac{R'(\check{P}\bar{F}_H^{-1}(p_n))}{p_n R(\check{P}\bar{F}_H^{-1}(p_n))}\check{P}\int_{\bar{F}_H^{-1}(p_n)}^{\infty}(h - \bar{F}_H^{-1}(p_n))f_H(h)\,dh \\ &= 1 + \left(\frac{R'(\check{P}x_n)\check{P}x_n}{R(\check{P}x_n)}\right)\left(\frac{\int_{x_n}^{\infty} f_H(h)(h - x_n)\,dh}{x_n\bar{F}_H(x_n)}\right),\end{aligned} \tag{36}$$

where $x_n = \bar{F}_H^{-1}(p_n)$. The last term on the right in (36) is the product of two terms. The first term goes to zero as $n \to \infty$ because $R(\cdot)$ has zero asymptotic elasticity. We show that the second term also goes to zero as $n \to \infty$. This term can be written as

$$\frac{\int_{x_n}^{\infty} f_H(h)(h - x_n)\,dh}{x_n\bar{F}_H(x_n)} = \frac{\int_{x_n}^{\infty} f_H(h)h\,dh}{x_n\bar{F}_H(x_n)} - 1. \tag{37}$$

Since H has a finite mean, as $n \to \infty$, both $\int_{x_n}^{\infty} f_H(h)h\,dh$ and $x_n\bar{F}_H(x_n)$ must go to zero. Hence, L'Hospital's rule can be applied yielding,

$$\begin{aligned}\lim_{n\to\infty}\frac{\int_{x_n}^{\infty} f_H(h)h\,dh}{x_n\bar{F}_H(x_n)} &= \lim_{n\to\infty}\frac{-f_H(x_n)x_n}{-f_H(x_n)x_n + \bar{F}_H(x_n)}. \\ &= 1\end{aligned}$$

The last step follows since $f_H(h)$ has a well-behaved tail. Substituting this into (37), the desired result follows. ■

APPENDIX H

Proof of Proposition 5: From Lemma 1, it follows that for $R(\gamma)$ given by (2),

$$\bar{s}_f(p_n,n) \asymp np_n(1-p_n)^{n-1}\log\left(\frac{\bar{P}\bar{F}_H^{-1}(p_n)}{p_n}\right).$$

Likewise, for n large enough $\bar{s}_v(p,n)$ is given by (14). Hence, it is sufficient to show that

$$\lim_{n\to\infty}\frac{\frac{1}{p_n}\int_{\bar{F}_H^{-1}(p_n)}^{\infty} f_H(h)\log\left(\frac{h}{\lambda_{p_n}}\right)\,dh}{\log\left(\frac{\bar{P}\bar{F}_H^{-1}(p_n)}{p_n}\right)} = 1,$$

where λ_{p_n} is given by (13). Since the optimal variable rate policy will always have a greater throughput than the corresponding fixed rate policy, it follows that the $\liminf$ of this ratio must be no less than 1. To complete the proof, we show that the $\limsup$ of this ratio is no greater than 1.

Using the concavity of $\log(\cdot)$, it can be shown that

$$\frac{\frac{1}{p_n}\int_{\bar{F}_H^{-1}(p_n)}^{\infty} f_H(h)\log\left(\frac{h}{\lambda_{p_n}}\right)\,dh}{\log\left(\frac{\bar{P}\bar{F}_H^{-1}(p_n)}{p_n}\right)} \le 1 + \frac{\int_{\bar{F}_H^{-1}(p_n)}^{\infty} f_H(h)\left(\frac{h}{\lambda_{p_n}} - \frac{\bar{P}\bar{F}_H^{-1}(p_n)}{p_n}\right)\,dh}{\bar{P}\bar{F}_H^{-1}(p_n)\log\left(\frac{\bar{P}\bar{F}_H^{-1}(p_n)}{p_n}\right)}.$$

We next show that the second term on the right goes to zero as $n\to\infty$. Using the definition of λ_{p_n}, this term can be written as

$$\begin{aligned}
&\frac{\int_{\bar{F}_H^{-1}(p_n)}^{\infty} f_H(h)\left(\frac{h}{\lambda_{p_n}} - \frac{\bar{P}\bar{F}_H^{-1}(p_n)}{p_n}\right)\,dh}{\bar{P}\bar{F}_H^{-1}(p_n)\log\left(\frac{\bar{P}\bar{F}_H^{-1}(p_n)}{p_n}\right)} \\
&= \left(\frac{\int_{\bar{F}_H^{-1}(p_n)}^{\infty} f_H(h)h\,dh}{p_n\bar{F}_H^{-1}(p_n)}\right)\left(\frac{\bar{P}+\int_{\bar{F}_H^{-1}(p_n)}^{\infty}\frac{1}{h}f_H(h)\,dh}{\log\left(\frac{\bar{P}\bar{F}_H^{-1}(p_n)}{p_n}\right)}\right) - \frac{1}{\log\left(\frac{\bar{P}\bar{F}_H^{-1}(p_n)}{p_n}\right)}.
\end{aligned} \tag{38}$$

As $n\to\infty$, the quantity

$$\frac{\int_{\bar{F}_H^{-1}(p_n)}^{\infty} f_H(h)h\,dh}{p_n\bar{F}_H^{-1}(p_n)} \to 0$$

as shown in Appendix G. It can be seen that the other terms on the right-hand side of (38) also go to zero as $n\to\infty$. Combining these observations, the desired result follows. ■

APPENDIX I

Proof of Lemma 5: First note that if $f_{H_1}(h)$ and $f_{H_2}(h)$ have similar tails, then clearly,

$$\lim_{p\to 0^+} \frac{\bar{F}_{H_1}^{-1}(p)}{\bar{F}_{H_2}^{-1}(p)} = \frac{1}{c}.$$

Combining this with Lemma 3, it follows that for any $\alpha > 0$, there exists some constant $c_2 > 0$ such that,

$$\limsup_{p\to 0^+} \frac{\bar{F}_{H_1}^{-1}(p)}{\bar{F}_{H_2}^{-1}(\alpha p)} \leq c_2. \tag{39}$$

Since $R(\cdot)$ is concave, using a first order Taylor expansion around $\check{P}_2\bar{F}_{H_2}^{-1}(\alpha p)$ we have,

$$\frac{R(\check{P}_1\bar{F}_{H_1}^{-1}(p))}{R(\check{P}_2\bar{F}_{H_2}^{-1}(\alpha p))} \leq 1 + \frac{R'(\check{P}_2\bar{F}_{H_2}^{-1}(\alpha p))\check{P}_2\bar{F}_{H_2}^{-1}(\alpha p)}{R(\check{P}_2\bar{F}_{H_2}^{-1}(\alpha p))}\left(\frac{\check{P}_1\bar{F}_{H_1}^{-1}(p)}{\check{P}_2\bar{F}_{H_2}^{-1}(\alpha p)} - 1\right).$$

From the asymptotic elasticity of $R(\cdot)$ and (39), it follows that

$$\limsup_{p\to 0^+} \frac{R(\check{P}_1\bar{F}_{H_1}^{-1}(p))}{R(\check{P}_2\bar{F}_{H_2}^{-1}(\alpha p))} \leq 1.$$

Switching the roles of $\check{P}_1\bar{F}_{H_1}^{-1}(p)$ and $\check{P}_2\bar{F}_{H_2}^{-1}(\alpha p)$, the same result follows. Therefore,

$$\lim_{p\to 0^+} \frac{R(\check{P}_1\bar{F}_{H_1}^{-1}(p))}{R(\check{P}_2\bar{F}_{H_2}^{-1}(\alpha p))} = 1,$$

as desired. ■

APPENDIX J

Proof of Lemma 6: First we show that $f_{\tilde{H}}(h)$ has a well-behaved tail. From its definition, it follows that $f_{\tilde{H}}(h) = f_{H_1}(h)F_{H_2}(h)+F_{H_1}(h)f_{H_2}(h)$, and $\bar{F}_{\tilde{H}}(h) = \bar{F}_{H_1}(h)F_{H_2}(h)+\bar{F}_{H_2}(h)$. Using these two expressions, we have

$$\begin{aligned}\lim_{h\to\infty} \frac{\bar{F}_{\tilde{H}}(h)}{hf_{\tilde{H}}(h)} &= \lim_{h\to\infty} \frac{\frac{\bar{F}_{H_1}(h)}{hf_{H_1}(h)}F_{H_2}(h)}{F_{H_2}(h) + F_{H_1}(h)\frac{f_{H_2}(h)}{f_{H_1}(h)}} + \frac{\frac{\bar{F}_{H_2}(h)}{hf_{H_2}(h)}}{\frac{f_{H_2}(h)}{f_{H_1}(h)}F_{H_2}(h) + F_{H_1}(h)} \\ &= 0,\end{aligned}$$

where the last step follows since $f_{H_1}(h)$ and $f_{H_2}(h)$ have similar tails (and therefore both have well-behaved tails). Hence, $f_{\tilde{H}}(h)$ has a well-behaved tail.

Finally, note that

$$\begin{aligned}\lim_{h\to\infty} \frac{f_{\tilde{H}}(h)}{f_{H_1}(h)} &= \lim_{h\to\infty} F_{H_2}(h) + F_{H_1}(h)\frac{f_{H_2}(h)}{f_{H_1}(h)} \\ &= 1 + 1/c,\end{aligned}$$

where we have again used that $f_{H_1}(h)$ and $f_{H_2}(h)$ have similar tails. Therefore, $f_{H_1}(h)$ and $f_{\tilde{H}}(h)$ have similar tails. The same argument can be applied for $f_{H_2}(h)$. ■

Appendix K

Proof of Proposition 6: Let $\check{P}_{max} = \max(\check{P}_1, \check{P}_2)$. Consider a symmetric model with $(1+\alpha\beta)n$ users[17], where each user has a short-term power constraint of $\check{P}_{max}$ and fading distribution $f_{\tilde{H}}(h)$, as defined in Lemma 6. If each user transmits with probability p with a fixed-rate, then the sum throughput for this symmetric model is

$$\check{s}_f^s(p,n) = (1+\alpha\beta)np(1-p)^{(1+\alpha\beta)n-1}R\left(\check{P}_{max}\bar{F}_{\tilde{H}}^{-1}(p)\right). \tag{40}$$

First, we show that for all n and p, $\check{s}_f^{(2)}(p,n) \leq \check{s}_f^s(p,n)$, i.e. the throughput of this symmetric system upper bounds the throughput of the asymmetric system with the same parameters. To see this, first note that by definition for $i = 1,2$, $\bar{F}_{\tilde{H}}(h) \geq \bar{F}_{H_i}(h)$. Therefore, since the complementary distribution is strictly decreasing, we have for $i = 1,2$, $\bar{F}_{\tilde{H}}^{-1}(p) \geq \bar{F}_{H_i}^{-1}(p)$, for any $p \in [0,1]$. It follows that,

$$R(\check{P}_{max}\bar{F}_{\tilde{H}}^{-1}(p)) \geq R(\check{P}_i\bar{F}_{H_i}^{-1}(p)),\ i = 1,2.$$

Using this we have,

$$\begin{aligned}
\check{s}_f^{(2)}(p,n) &\leq \left[np(1-p)^{n-1}(1-\alpha p)^{\beta n} + \alpha\beta np(1-p)^n(1-\alpha p)^{\beta n-1}\right]R(\check{P}_{max}\bar{F}_{\tilde{H}}^{-1}(p)) \\
&\leq (1+\alpha\beta)np(1-p)^{(1+\alpha\beta)n-1}R(\check{P}_{max}\bar{F}_{\tilde{H}}^{-1}(p)) \\
&= \check{s}_f^s(p,n).
\end{aligned}$$

Here we have used that since $\alpha \geq 1$ and $0 < p < 1$, then $(1-p)^\alpha > 1-\alpha p$.

Let $\tilde{p}(n)$ be the optimal transmission probability for the symmetric system, i.e. the probability that maximizes $\check{s}_f^s(p,n)$ in (40). For all n, it follows that

$$\tilde{s}_f^{(2)}\left(\tfrac{1}{(1+\alpha\beta)n}, n\right) \leq \tilde{s}_f^{(2)}(p^*(n),n) \leq \tilde{s}_f^s(p^*(n),n)) \leq \tilde{s}_f^s(\tilde{p}(n),n).$$

Hence, it is sufficient to prove that $\tilde{s}_f^{(2)}(\frac{1}{(1+\alpha\beta)n},n) \asymp \tilde{s}_f^s(\tilde{p}(n),n)$. Furthermore, from Lemma 6, $f_{\tilde{H}}(h)$ has a well-behaved tail and so Proposition 3 applies, i.e. $\tilde{s}_f^s(\tilde{p}(n),n) \asymp \tilde{s}_f^s(\frac{1}{(1+\alpha\beta)n},n)$. Therefore, it is sufficient to show that $\tilde{s}_f^{(2)}(\frac{1}{(1+\alpha\beta)n},n) \asymp \tilde{s}_f^s(\frac{1}{(1+\alpha\beta)n},n)$. From Lemma 6, $f_{\tilde{H}}(h)$ and $f_{H_i}(h)$ have similar tails for $i = 1,2$. Thus, Lemma 5 applies. Using this, it can be shown that

$$\lim_{n\to\infty}\frac{\tilde{s}_f^{(2)}\left(\frac{1}{(1+\alpha\beta)n},n\right)}{\tilde{s}_f^s\left(\frac{1}{(1+\alpha\beta)n},n\right)} = 1,$$

as desired. ∎

[17] Of course, for some values of n, $1+\alpha\beta$ may not be an integer; however, this does not effect our analysis.

APPENDIX L

Proof of Proposition 7: Note that

$$\check{s}_v(\tfrac{1}{n}, n) = (1 - \tfrac{1}{n})^{n-1}\mathbb{E}_H\left(R(\check{P}H)|H > \bar{F}_H^{-1}(\tfrac{1}{n})\right).$$

Thus, the desired result is equivalent to showing that

$$\mathbb{E}_H\left(R(\check{P}H)|H > \bar{F}_H^{-1}(\tfrac{1}{n})\right) > \mathbb{E}_{H_{max,n}}\left(R(\check{P}H_{max,n})\right), \tag{41}$$

where $H_{max,n} = \max_{i=1,\dots,n}\{H_i\}$. For this, we use the following stochastic ordering result [31]: for two random variables, X and Y, if $\bar{F}_X(a) \geq \bar{F}_Y(a)$ for all a, then $E[g(X)] \geq E[g(Y)]$ for all increasing functions g.

Let X be the channel gain H conditioned on a transmission attempt occurring in the ALOHA system, so that for any $h > \bar{F}_H^{-1}(\frac{1}{n})$,

$$\Pr(X > h) = \Pr(H > h|H > \bar{F}_H^{-1}(\tfrac{1}{n})) = \frac{\bar{F}_H(h)}{\bar{F}_H(\bar{F}_H^{-1}(\frac{1}{n}))},$$

and so

$$\bar{F}_X(h) = \begin{cases} n\bar{F}_H(h), & \text{for all} \quad h > \bar{F}_H^{-1}(\frac{1}{n}), \\ 1, & \text{otherwise.} \end{cases}$$

Let $Y = H_{max,n}$, so that $\bar{F}_Y(h) = 1 - (1 - \bar{F}_H(h))^n$. For all $h < \bar{F}_H^{-1}(\frac{1}{n})$, clearly $\bar{F}_Y(h) \leq \bar{F}_X(h) = 1$. For all $h > \bar{F}_H^{-1}(\frac{1}{n})$, using that $nz > 1 - (1 - z)^n$ for all $0 < z < 1$, we have $\bar{F}_X(h) > \bar{F}_Y(h)$. By assumption, $R(\gamma)$ is monotonically increasing; thus, applying the above result, with $g(x) = R(\check{P}x)$, (41) follows. ■

APPENDIX M

Proof of Proposition 8: The difficulty here, as opposed to proof of Prop. 7, is that with a long-term power constraint the centralized and the distributed systems may use different power allocations. Let $P^*(h)$ denote the optimal centralized power allocation that each user employs as a function of its own channel gain. In a symmetric system, this power allocation will satisfy

$$\int_0^\infty P^*(h)f_H(h)(1 - \bar{F}_H(h))^{n-1}\,dh = \bar{P},$$

where $f_H(h)(1 - \bar{F}_H(h))^{n-1}$ represents the probability that a user has the best channel gain and its channel gain is h.[18] The optimal sum throughput can then be written as

$$\bar{s}_{ct}(n) = n\int_0^\infty R(P^*(h)h)f_H(h)(1 - \bar{F}_H(h))^{n-1}\,dh. \tag{42}$$

[18]Note when $R(\gamma)$ is given by (2), then each user will use a water-filling power allocation over the channel with distribution $f_H(h)(1 - \bar{F}_H(h))^{n-1}$.

If the users in the distributed system use power allocation $P^*(h)$ when they transmit, then by a similar argument as in Appendix L it can be shown that

$$\int_{\bar{F}_H^{-1}(\frac{1}{n})}^{\infty} P^*(h) f_H(h)\, dh > \bar{P},$$

i.e., this power allocation is not feasible in the distributed system. Instead we consider a sub-optimal distributed system, where for every channel gain each user transmits probabilistically with probability $(1-\bar{F}_H(h))^{n-1}$, using power $P^*(h)$. This distributed system will meet the average power constraint, and each user will still transmit with probability

$$p \leq \int_0^{\infty} f_H(h)(1-\bar{F}_H(h))^{n-1}\, dh = \tfrac{1}{n},$$

where equality would hold if $P^*(h) > 0$ for all h. The throughput of this system is thus lower bounded by

$$\bar{s}_p(n) \geq n\left(1-\tfrac{1}{n}\right)^{n-1} \int_0^{\infty} R(P^*(h)h) f_H(h)(1-\bar{F}_H(h))^{n-1}. \tag{43}$$

Clearly, this system will have a lower throughput than in a system where the users transmit only when their channels exceed $\bar{F}_H^{-1}(\frac{1}{n})$ and use the optimal power allocation, i.e. $\bar{s}_v(\frac{1}{n}, n) > \bar{s}_p(n)$. This follows because the two systems will have the same probability of success, but the second system will have a higher throughput when successful. Combining this with (42) and (43), the desired results follows. ■

Appendix N

Proof of lemma 9: The complementary distribution function of $Z = R(\check{P}H)$ is given by

$$\bar{F}_Z(z) = \bar{F}_H\left(\frac{R^{-1}(z)}{\check{P}}\right). \tag{44}$$

Hence, using the inverse function theorem,

$$f_Z(z) = f_H\left(\frac{R^{-1}(z)}{\check{P}}\right)\frac{1}{\check{P}R'(R^{-1}(z))}. \tag{45}$$

Next note that

$$\begin{aligned}\frac{d}{dz}\left[\frac{\bar{F}_z(z)}{f_Z(z)}\right] &= -1 - \frac{\bar{F}_Z(z) f_Z'(z)}{(f_Z(z))^2} \\ &=: -1 - A(z).\end{aligned}$$

Therefore, to prove the lemma it is sufficient to show that $\lim_{z\to\infty} A(z) = -1$.

Using (44) and (45), $A(z)$ can be expressed as

$$A(z) = \frac{\bar{F}_H\left(\frac{R^{-1}(z)}{\check{P}}\right)\frac{d}{dz}\left[f_H\left(\frac{R^{-1}(z)}{\check{P}}\right)\frac{1}{\check{P}R'(R^{-1}(z))}\right]}{\left(f_H\left(\frac{R^{-1}(z)}{\check{P}}\right)\frac{1}{\check{P}R'(R^{-1}(z))}\right)^2}.$$

Changing variables to $y = \frac{R^{-1}(z)}{\check{P}}$ and simplifying we have:

$$\begin{aligned} A(y) = \frac{\bar{F}_H(y)f'_H(y)}{(f_H(y))^2} - & \frac{\bar{F}_H(y)R''(\check{P}y)\check{P}}{f_H(y)R'(\check{P}y)} \\ & =: B(y) - C(y). \end{aligned}$$

To complete the proof we show that $\lim_{y\to\infty} B(y) = -1$ and $\lim_{y\to\infty} C(y) = 0$.

First consider the $B(y)$ term. Note that

$$\frac{d}{dy}\left[\frac{\bar{F}_H(y)}{f_H(y)}\right] = -\left(B(y)+1\right).$$

By assumption, the left-hand side of this equality approaches zero; hence $\lim_{y\to\infty} B(y) = -1$, as desired.

Next, consider the $C(y)$ term. Note that

$$C(y) = \left(\frac{\bar{F}_H(y)}{yf_H(y)}\right)\left(\frac{R''(\check{P}y)\check{P}y}{R(\check{P}y)}\right).$$

From Lemma 8, $f_H(h)$ has a well-behaved tail, and so $\lim_{h\to\infty}\frac{\bar{F}_H(y)}{yf_H(y)} = 0$. Therefore, it is sufficient to show that $\frac{-R''(\gamma)\gamma}{R(\gamma)}$ is bounded as $\gamma\to\infty$. Since $\lim_{\gamma\to\infty} R(\gamma) = \infty$, then $\lim_{\gamma\to\infty} R'(\gamma)\gamma^2 = \infty$. Otherwise, $R(\gamma) = \int_0^\gamma R'(x)\,dx$ would be bounded. It follows that for large enough γ, $-\log(R'(\gamma)\gamma^2) < 0$. Equivalently, for large enough γ, $\frac{-\log(R'(\gamma))}{\log(\gamma)} < 2$. And so,

$$\lim_{\gamma\to\infty}\frac{-\log(R'(\gamma))}{\log(\gamma)} < 2,$$

assuming the limit exists. Applying L'Hospital's rule to this expression, we have that

$$\lim_{\gamma\to\infty}\frac{-\log(R'(\gamma))}{\log(\gamma)} = \lim_{\gamma\to\infty}\frac{-R''(\gamma)\gamma}{R'(\gamma)}.$$

Combining the above observations, it follows that $\frac{-R''(\gamma)\gamma}{R(\gamma)} < 2$ as $\gamma\to\infty$, and so $\lim_{y\to\infty} C(Y) = 0$ as desired. ■

APPENDIX O

Proof of Proposition 9: Since $f_H(h)$ satisfies (21), from Lemma 9, $f_Z(z)$ will satisfy (22). Using this and the assumed properties of $R(\gamma)$ and $f_H(h)$, it can be seen that the random variables $Z_i = R(\check{P}H_i)$ will satisfy the assumptions of Lemma 7.

Let $Z_{max,n} = \max_{i=1,\ldots,n} Z_i$. From Lemma 7, as $n \to \infty$,

$$\Pr\left((Z_{max,n} - l_n)a_n \le u\right) \to \exp(e^{-u})$$

uniformly in u. Here $l_n = R(\check{P}\bar{F}_H^{-1}(\frac{1}{n}))$ and

$$a_n = nf_Z(l_n) = \frac{f_H(y_n)}{\bar{F}_H(y_n)} \frac{1}{\check{P}R'(\check{P}y_n)},$$

where $y_n := \bar{F}_H^{-1}(\frac{1}{n})$.

It follows that as $n \to \infty$,

$$\mathbb{E}\left((Z_{max,n} - l_n)a_n\right) \to 1. \tag{46}$$

Note that

$$\frac{1}{l_n a_n} = \frac{\bar{F}_H(y_n)}{f_H(y_n)y_n} \frac{R'(\check{P}y_n)\check{P}y_n}{R(\check{P}y_n)}.$$

Therefore, using the asymptotic elasticity of $R(\gamma)$ and that $f_H(h)$ has a well-behaved tail, we have that $\lim_{n\to\infty} \frac{1}{a_n l_n} = 0$. Multiplying both sides of (46) by $\frac{1}{a_n l_n}$, it follows that

$$\lim_{n\to\infty} \frac{\mathbb{E}(Z_{max,n})}{l_n} = 1.$$

Equivalently, $\check{s}_{ct}(n) \eqsim R(\check{P}\bar{F}_H^{-1}(\frac{1}{n}))$. Comparing this to (8), the desired result follows. ■

APPENDIX P

Proof of Proposition 10: For $k = 1, 2, \ldots$, let $n(k)$ denote the backlog at the start of the kth time-slot. Given the memoryless assumption, $\{n(k)\}$ will be a Markov chain. To show that the system is stable, it is sufficient to show the following drift condition[7]: there exists some $D > 0$, $N > 0$ such that

$$E(n(k+1) - n(k)|n(k) = n) \le -D, \tag{47}$$

for all $n \ge N$.

Given that $n(t) = n$, each user will transmit with probability $1/n$ in each slot, therefore the departure rate in packets per time-slot is $(1 - \frac{1}{n})^{n-1}$. The arrival rate in packets per time-slot is $\lambda(n) = \lambda L/R(\check{P}h'(n))$. Thus we have,

$$E(n(k+1) - n(k)|n(k) = n) = -(1 - \frac{1}{n})^{n-1} + \lambda(n). \tag{48}$$

As n approaches to infinity, $\lambda(n)$ decreases to 0, while $(1 - \frac{1}{n})^{n-1}$ approaches to $1/e$. Therefore for any $\delta < 1/e$, an N can be found such that (47) is satisfied with $D = \frac{1}{e} - \delta$, and so the system is stable. ■

REFERENCES

[1] N. Abramson, "The ALOHA System - Another Alternative for Computer Communication," *Fall Joint Computer Conference*, 1970.

[2] S. Adireddy and L. Tong, "Exploiting Decentralized Channel State Information for Random Access," *IEEE Transactions on Information Theory,* Vol. 51, No. 2, pp. 537-561, Feb. 2005.

[3] R. Agrawal, A. Bendeckar, R. La, and V. Subramanian, "A Class and Channel-condition based Weighted Proportionally Fair Scheduler," *Proc. of ITC 2001* Salvador, Brazil, Dec. 2001.

[4] P. Bender., *et al.*, "CDMA/HDR: a bandwidth efficient high speed wireless data service for nomadic users," *IEEE Communications Magazine*, Vol. 38, No. 7, pp. 70-77, July 2000.

[5] R. Berry and R. Gallager, "Communication over fading channels with delay constraints," *IEEE Trans. on Information Theory*, vol. 48, pp. 1135–1149, May 2002.

[6] R. Berry and E. Yeh, "Cross-layer Wireless Resource Allocation," *IEEE Signal Processing Magazine,* vol. 21, No. 5, pp. 59-68, Sept. 2004.

[7] D. Bertsekas and R. Gallager, *Data Networks*, 2nd Ed., Prentice Hall, 1992.

[8] Bettesh and S. Shamai, "Optimal power and rate control for fading channels," in *Proc. IEEE Vehicular Technology Conference, Spring 2001*, (Rhodes, Greece), May 6-9 2001.

[9] G. Caire, G. Taricco, and E. Biglieri, "Optimum power control over fading channels," *IEEE Trans. Inf. Th.*, vol. 45, pp. 1468–1489, July 1999.

[10] J. Cao and E. Yeh, "Optimal Multiaccess Communication vai Distributed Rate Splitting," *42nd Allerton conference on Communication, Control and Computing,* Monticello, IL, Sept. 2004.

[11] H. A. David, *Order Statistics*, Wiley, 1970.

[12] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power contro l algorithm and its convergence," *IEEE Transactions on Vehicular Technology*, vol. 42, no. 4, Nov. 1993.

[13] R. G. Gallager, *Information Theory and Reliable Communication*, John Wiley and Sons, NY, 1968.

[14] R. G. Gallager, "A Perspective on Multiaccess Channels," *IEEE Trans. on Information Theory,* vol. 31, pp. 124-142, March 1985.

[15] S. Ghez, S. Verdu and S. Schwartz, "Optimal Decentralized Control in the Random Access Multipacket Channel," *IEEE Trans. on Automatic Control*, vol. 34, pp. 1153-1163, Nov. 1989.

[16] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad-hoc wireless networks" *Proc. INFOCOM 2001*, Vol. 3, pp. 1360-1369, 2001.

[17] S. Hanly and D. Tse, "Multiaccess-fading Channels: Part I: Polymatroid Structure, Optimal Resource Allocation, and Throughput Capacities," *IEEE Trans. on Information Theory*, vol. 44, no. 7, pp. 2816-2831, Nov. 1998.

[18] R. S. Kennedy, *Fading Dispersive Communications Channels,* Wiley Interscience, New York, NY, 1969.

[19] R. Knopp and P. A. Humblet, "Information capacity and power control in single-cell multiuser communications," *Proc. IEEE ICC '95*, Seattle, WA, June 1995.

[20] D. Kramkov and W. Schachermayer, "The Asymptotic Elasticity of Utility functions and Optimal Investment in Incomplete Markets,"*Ann. Appl. Prob.*, vol. 9, no. 3, pp. 904-950, 1999.

[21] C. C. Lee, "Random Signal Levels for Channel Access in Packet Broadcast Networks," *IEEE JSAC*, vol. 5, pp. 1026-1034, July 1987.

[22] X. Liu, E. K. P. Chong, and N. B. Shroff, "Opportunistic Transmission Scheduling with Resource-Sharing Constraints in Wireless Networks," *IEEE JSAC*, vol. 19, pp. 2053-2064, Oct. 2001.

[23] Y. Liu and E. Knightly, "Opportunistic Fair Scheduling over Multiple Wireless Channels," *Proc. of 2003 IEEE INFOCOM*, San Francisco, CA, April 2003.

[24] W. Luo and A. Ephremides, "Power Levels and Packet Lengths in Random Multiple Access," *IEEE Trans. on Information Theory*, vol. 48, pp. 46-58, Jan. 2002.

[25] M. Medard, *et al.*, "Capacity of Time-slotted Aloha Systems," *Proc. of 2000 IEEE Inter. Symp. on Information Th.*, June 2000.

[26] M. Medard, *et al.*, "Capacity of Time-slotted ALOHA Packetized Mulitple-Access Systems Over the AWGN Channel," *IEEE Trans. on Wireless Communications,* Vol. 3, no. 2, pp. 489-499, March 2004.

[27] L. Ozarow, S. Shamai, and A. Wyner, "Information Theoretic Considerations for Cellular Mobil Radio," *IEEE Transactions on Vehicular Technology, Vol. 43, No. 2,* pp. 359–378, May 1994.

[28] X. Qin and R. Berry, "Exploiting Multiuser Diversity for Medium Access Control in Wireless Networks" *Proc. 2003 IEEE INFOCOM*, San Francisco, CA, Mar. 2003.

[29] X. Qin and R. Berry, ”Exploiting Multiuser Diversity in Wireless Aloha Networks” *39th Annual Allerton Conference on Communication, Control and Computing*, Monticello, IL, October 2001.

[30] X. Qin and R. Berry, “Opportunistic Splitting Algorithms For Wireless Networks,” *Proc. 2004 IEEE INFOCOM*, Hong Kong, Mar. 2003.

[31] S. Ross, *Stochastic Processes* pp. 405, 1996.

[32] S. Shamai and E. Telatar, “Some Information Theoretic Aspects of Decentralized Power Control in Multiple Access Fading Channels,” *1999 Information Theory and Networking Workshop*, June 1999.

[33] D. Tse and S. Hanly, “Multi-access fading channels: Part I: Polymatroid structure, optimal resource allocation and throughput capacities,” *IEEE Transactions on Information Theory*, vol. 44, no. 7, pp. 2796–2815, 1998.

[34] D. Tse, “Optimal Power Allocation over Parallel Gaussian Broadcast Channels,” *Proc. of International Symp. on Information Th.* Ulm, Germany, June 1997.

[35] S. Verdu, “Spectral Efficiency in the Wideband Regime,” *IEEE Transactions on Information Theory*, vol. 84, no. 6, pp. 1319-1343, June 2002.

[36] P. Viswanath, D. Tse, and V. Anantharam, “Asymptotically optimal waterfilling in vector mulitple access channels,” *IEEE Trans. on Information Theory*, vol. 47, no. 1, pp. 241-267, Jan 2001.

[37] P. Viswanath, D. Tse and R. Laroia, “Opportunistic Beamforming using Dumb Antennas”, *IEEE Trans. on Information Theory*, vol. 48, no. 6, pp. 1277-1294, June 2002.

[38] H. Wang and N. Mandayam, “Opportunistic Transmission under Energy and Delay Constraints,” *Proc. ISWC’02*, Victoria, BC, Canada, Sept. 2002.

[39] E. Yeh and A. Cohen, “Information theory, queueing, and resource allocation in multi-user fading communications,” in *Proc. of the Conference on Information Sciences and Systems*, (Princeton, NJ), 2004.

[40] Y. Yu and G. Giannakis, “Opportunistic Access in Slotted Aloha Adapted to Decentralized CSI,” Proc. of IEEE GLOBECOM 2004, pp. 3332-3336, Nov. 2004.

[41] Q. Zhao and L. Tong, “A Multi-Queue Service Room MAC Protocol for Wireless Networks with Multipacket Reception,” *IEEE/ACM Transactions on Networking*, vol. 11, no. 1, Feb. 2003.

[42] Q. Zhao and L. Tong, “Opportunistic Carrier Sensing for Energy-Efficient Information Retrieval in Sensor Networks,” *EURASIP Journal on Wireless Communications and Networking*, vol. 2, pp. 231-241, May 2005.

[43] TIA/EIA IS-856, “CDMA 2000: High rate packet data air interface specification,” Std., Nov. 2000.