Reduced-Rank Signature–Receiver Adaptation

Yakun Sun and Michael L. Honig

***Abstract*— Interference in code-division multiple access (CDMA) and multi-antenna systems can be avoided by choosing a signature (in space and/or time), which lies in the direction of least interference plus noise. If the interference statistics are unknown *a priori*, then the signature can be adaptively estimated with a training sequence. We present an iterative scheme for joint signature–receiver adaptation with an adaptive reduced-rank Multi-Stage Wiener Filter (MSWF) at the receiver. We establish convergence of the iterative signature–receiver optimization scheme for a single user with fixed interference, and show that the limiting performance for any filter rank $D \geq 2$ is the same as that obtained with a full-rank receiver. To reduce feedback requirements, we also consider joint signature–receiver adaptation with a reduced-rank signature, which is confined to a randomly chosen subspace. Numerical results are presented for both single- and multi-user (group) adaptation, which show that reduced-rank signature-receiver estimation can achieve near-optimal performance with relatively little training and low complexity.**

***Index Terms*— Signature optimization, reduced-rank, CDMA, adaptive receiver, interference avoidance.**

I. INTRODUCTION

ONE of the main limitations on the capacity and performance of wireless networks is interference. In addition to signal processing techniques at the receiver, which can suppress interference, it is also possible to design the transmitted waveform to avoid interference at the intended receiver. Interference mitigation and avoidance becomes especially important in ad-hoc and peer-to-peer Direct Sequence (DS)-Spread Spectrum (SS) networks, where power control cannot be used to solve the near-far problem. Techniques for interference avoidance can also be applied to multi-antenna, or multi-input/multi-output (MIMO) channels. Whereas signatures for DS-SS are defined in time, for a MIMO channel the combining coefficients across transmitter antennas form a set of spatial signatures.

Interference avoidance can be accomplished by choosing a signature (in space and/or time), which lies in the direction of least interference plus noise. An adaptive scheme to optimize the signature of a single DS-SS user has been presented in [1], and group signature optimization for the multiple access channel is discussed in [2], [3], [4], [5]. It has been shown in [6] that, in the absence of multipath, the optimal (sum-capacity achieving) signature sequences achieve the Welch Bound Equality. Given an arbitrary set of sequences with additive white Gaussian noise channels, it is shown in [3] that the optimal signatures can be obtained through iterative updates of signatures and receivers.

The optimal signature for a particular data stream can be computed at the receiver given knowledge of the channel and the interference-plus-noise covariance matrix. (This applies to peer-to-peer networks as well as cellular.) In practice, the optimal signature must be estimated with limited training, which degrades performance. The amount of training may be dictated by the availability of overhead capacity, and by the channel (and interference) coherence time. Hence we seek adaptive algorithms for signature estimation, which require minimal training.*

In this paper, we consider a synchronous multi-access channel without multipath. This is mainly for convenience. The signature adaptation scheme presented can also be applied with asynchronous transmissions, and can be modified to account for multipath, provided that the receiver knows the channel. Furthermore, the adaptive algorithm for a single user can be applied in more general networking scenarios (e.g., peer-to-peer). With a full-rank receiver, it has been shown in [7] that for the multi-access channel, iterative signature–receiver optimization in an asynchronous CDMA system provides the same capacity as that obtained in a synchronous system.

We consider an iterative scheme for joint signature–receiver optimization in which the receiver and transmitted signature for a particular user are alternately optimized to maximize the received Signal-to-Interference Plus Noise Ratio (SINR) until convergence. In contrast with previous work, here we assume a reduced-rank receiver, based on the Multi-Stage Wiener Filter (MSWF) [8], which achieves full-rank performance with relatively small rank, and requires less training than a full-rank linear filter in an adaptive mode [9], [10], [11]. We show that the optimal signature and associated output SINR with the reduced-rank filter are the same as those with the full-rank Minimum Mean Squared Error (MMSE) filter provided that the rank of the reduced-rank filter is greater than one. We also establish the convergence of the iterative algorithm to this fixed point. We then study the performance of a joint reduced-rank receiver and signature optimization with limited training via simulation.

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*In what follows, signature and/or receiver "optimization" assumes that the channel and interference-plus-noise covariance matrix are known, whereas signature and/or receiver "adaptation" refers to the scenario in which the receiver filter and/or signature(s) are estimated from a training sequence.

Reduced-rank *signature* optimization is also considered in which the signature is constrained to lie in a random subspace. Reducing the subspace dimension reduces the number of parameters to estimate (e.g., a one-dimensional signature subspace corresponds to conventional power control), and can therefore reduce feedback requirements, but limits the degrees of freedom available to avoid interference [13]. Our results show that with limited training, the combined reduced-rank receiver and reduced-rank signature can perform better than using the reduced-rank technique at either the receiver or the transmitter. Rank reduction also implies lower complexity. As the amount of training increases, reduced- and full-rank receivers give the same performance.

In addition to signature adaptation in the presence of fixed (random) interference, we also consider *group* signature adaptation with adaptive reduced-rank receivers for the multi-access channel. That is, all users adapt their signatures. We first show that when the users update their signatures successively, the set of signatures converges to an optimal set (i.e., that achieves the Welch Bound Equality). Numerical results with simultaneous signature updates across users again show that signature adaptation with reduced-rank receivers (and signatures) can perform substantially better than the corresponding full-rank estimation with limited training.

In the next section we present the system model and the class of reduced-rank receivers considered. The iterative algorithm for computing the jointly optimal signature and reduced-rank receiver is presented in Section III, and an adaptive version of the algorithm is presented in Section IV. Group adaptation is discussed in Section V. Joint adaptation of a reduced-rank signature with a reduced-rank receiver is discussed in Section VI, and conclusions are presented in Section VII.

II. System Model and Reduced-Rank Receiver

In what follows, we will refer to a CDMA system with K synchronous users and processing gain N. (In the presence of multipath this is analogous to a MIMO system with K transmit antennas and N receive antennas.) The $N \times 1$ received vector at time i is given by

$$\mathbf{r}(i) = \sum_{k=1}^{K} \mathbf{s}_k A_k b_k(i) + \mathbf{n}(i) = \mathbf{S}\mathbf{A}\mathbf{b}(i) + \mathbf{n}(i) \tag{1}$$

where $\mathbf{s}_k$ is the normalized signature for user k ($\|\mathbf{s}_k\| = 1$), and $\mathbf{S} = [\mathbf{s}_1 \ \ \mathbf{s}_2 \ \ \cdots \ \ \mathbf{s}_K]$ is the $N \times K$ signature matrix. At time i, the transmitted symbol is $b_k(i)$ with $E[|b_k(i)|^2] = 1$, A_k is the amplitude of each symbol, and $\mathbf{A}$ is the diagonal amplitude matrix. The transmitted signal is corrupted by complex white Gaussian noise $\mathbf{n}(i)$ with covariance $\sigma^2\mathbf{I}$.

We assume a linear receiver. The optimal (MMSE) receiver for user k is then given by

$$\mathbf{c}_k = \mathbf{R}^{-1}\mathbf{s}_k = \left(\mathbf{S}|\mathbf{A}|^2\mathbf{S}^\dagger + \sigma^2\mathbf{I}\right)^{-1}\mathbf{s}_k \tag{2}$$

where $(\cdot)^\dagger$ denotes conjugate transpose. To reduce both the amount of training needed to estimate the MMSE receiver in an adaptive mode, and the complexity of the adaptive filter, we consider a reduced-rank receiver filter in which the received vector $\mathbf{r}(i)$ is projected onto a lower dimensional subspace. Both the filtering and the filter estimation take place in the subspace. Suppose the D-dimensional subspace, $D < N$, is spanned by the columns of the $N \times D$ matrix $\mathbf{S}_D$. The projected received vector is given by

$$\tilde{\mathbf{r}}(i) = \mathbf{S}_D^\dagger \mathbf{r}(i) \tag{3}$$

The output of the reduced-rank linear receiver is $\hat{b}_k(i) = \tilde{\mathbf{c}}_k^\dagger \tilde{\mathbf{r}}(i)$, and the reduced-rank filter, which minimizes $\mathsf{MSE} = E\left[|b_k(i) - \hat{b}_k(i)|^2\right]$, is given by

$$\tilde{\mathbf{c}}_k = (\mathbf{S}_D^\dagger \mathbf{R}\mathbf{S}_D)^{-1}\mathbf{S}_D^\dagger \mathbf{s}_k. \tag{4}$$

We also have that

$$\mathsf{SINR} = \frac{|\tilde{\mathbf{c}}_k^\dagger \mathbf{s}_k|^2}{\tilde{\mathbf{c}}_k^\dagger \mathbf{R}_\mathrm{I}\tilde{\mathbf{c}}_k} = \frac{1}{\mathsf{MSE}} - 1, \tag{5}$$

where the interference-plus-noise covariance matrix $\mathbf{R}_I = \mathbf{R} - |A_k|^2\mathbf{s}_k\mathbf{s}_k^\dagger$, so that minimizing the MSE is equivalent to maximizing the SINR.

We consider the reduced-rank MSWF [8], which corresponds to the projection matrix [9], [10]

$$\mathbf{S}_D = [\mathbf{s}_k \ \ \mathbf{R}\mathbf{s}_k \ \ \cdots \ \ \mathbf{R}^{D-1}\mathbf{s}_k] \tag{6}$$

That is, the columns of $\mathbf{S}_D$ span a D-dimensional Krylov space. A computationally efficient rank–recursive algorithm can be used to compute the rank-D filter from the rank-$(D-1)$ filter [14].

III. Signature Optimization with Reduced-Rank Receiver

Here we present an iterative algorithm for optimizing the signature for user k with a reduced-rank receiver assuming stationary interference (i.e., other signatures are fixed) with known covariance matrix. With a full-rank MMSE receiver the received SINR is given by $\mathsf{SINR} = \mathbf{s}_k^\dagger \mathbf{R}_\mathrm{I}^{-1}\mathbf{s}_k$. To maximize the SINR, the optimal signature is therefore chosen to be the eigenvector of $\mathbf{R}_\mathrm{I}$ associated with the smallest eigenvalue. This optimal signature can be computed at the receiver and relayed back to the transmitter.

With a reduced-rank receiver, computation of the optimal signature is not as straightforward, since the projection matrix in (6) depends on the signature. However, we note that for any fixed receiver $\mathbf{c}_k$, the optimal normalized signature (i.e., that minimizes MSE, or maximizes SINR in the presence of fixed interference) is $\mathbf{s}_k = \mathbf{c}_k/\|\mathbf{c}_k\|$. Hence we propose the following iterative scheme to optimize the signature and reduced-rank receiver jointly. In each iteration, the transmitter sends a packet containing training information. The receiver computes the filter from the training and relays the normalized filter to the transmitter as the signature for the next iteration. The iterations stop when the desired accuracy is achieved. In what follows the superscript denotes the iteration number. The initial signature $\mathbf{s}_k^{(0)}$ is chosen randomly, and ε is a small positive constant, which determines the accuracy of the calculation.

Algorithm 1 Iterative signature-receiver optimization

Initialize: Choose random $\mathbf{s}_k^{(0)}$
repeat
for $n = 0, 1, 2, \cdots$

$$\mathbf{S}_D^{(n)} = [\mathbf{s}_k^{(n)}\ \mathbf{R}^{(n)}\mathbf{s}_k^{(n)}\ \cdots\ \mathbf{R}^{(n)D-1}\mathbf{s}_k^{(n)}] \quad (7)$$

$$\mathbf{c}_k^{(n)} = \mathbf{S}_D^{(n)}\tilde{\mathbf{c}}^{(n)} = \mathbf{S}_D^{(n)}\left(\mathbf{S}_D^{(n)\dagger}\mathbf{R}^{(n)}\mathbf{S}_D^{(n)}\right)^{-1}\mathbf{S}_D^{(n)\dagger}\mathbf{s}_k^{(n)} \quad (8)$$

$$\mathbf{s}_k^{(n+1)} = \frac{\mathbf{c}_k^{(n)}}{\|\mathbf{c}_k^{(n)}\|} \quad (9)$$

until $\|\mathbf{s}_k^{(n+1)} - \mathbf{s}_k^{(n)}\| < \varepsilon$

We first consider a rank-1 (matched filter) receiver. In that case $\mathbf{S}_D^{(n)} = \mathbf{s}_k^{(n)}$, and

$$\begin{aligned}\mathbf{c}_k^{(n)} &= \mathbf{s}_k^{(n)}\left(\mathbf{s}_k^{(n)\dagger}\mathbf{R}^{(n)}\mathbf{s}_k^{(n)}\right)^{-1}\mathbf{s}_k^{(n)\dagger}\mathbf{s}_k^{(n)} \\ &= \frac{1}{\mathbf{s}_k^{(n)\dagger}\mathbf{R}^{(n)}\mathbf{s}_k^{(n)}}\mathbf{s}_k^{(n)} \\ \mathbf{s}_k^{(n+1)} &= \mathbf{s}_k^{(n)}\end{aligned}$$

which means the initial signature is a fixed point of the algorithm, and the performance does not improve with iteration n. In contrast, for $D \geq 2$, the following two theorems state that the only stable fixed point of the iterative algorithm corresponds to the optimal signature with the full-rank MMSE filter.

Theorem 1: For any initial signature $\mathbf{s}_k^{(0)}$ with $\|\mathbf{s}_k^{(0)}\| = 1$, the iterative signature-receiver algorithm, given by Algorithm 1, converges to a fixed point, where $\mathbf{s}_k^{(n)}$ converges to an eigenvector of $\mathbf{R}_{\mathrm{I}}$.

The proof is given in the appendix. For $D = N$, convergence of the iterative algorithm to a fixed point follows directly from the fact that the MMSE must decrease monotonically with iterations n. However, showing this for $D < N$ is not as straightforward since the subspace in which the reduced-rank receiver filter resides depends on $\mathbf{s}_k^{(n)}$, which changes with n.

Although Theorem 1 establishes the convergence of Algorithm 1, the signature can converge to any eigenvector of $\mathbf{R}_{\mathrm{I}}$, whereas the optimal signature, which maximizes the received SINR, is the eigenvector corresponding to the smallest eigenvalue of $\mathbf{R}_{\mathrm{I}}$. The following theorem implies that only the optimal signature corresponds to a *stable* fixed point of the iterative algorithm. That is, at any other fixed point, an arbitrary small perturbation (e.g., caused by adaptation in a noisy environment) will cause the algorithm to move the signature towards the optimal signature.

Suppose the eigenvectors of $\mathbf{R}_{\mathrm{I}}$ are ordered as $\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_N$, which correspond to the eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$, and the subspace spanned by $\mathbf{u}_m, \cdots, \mathbf{u}_n$ is denoted as $\mathcal{U}_{m:n}$. Let $\mathsf{SINR}_D(\mathbf{x})$ be the received SINR for the rank-D receiver as a function of the signature for the desired user $\mathbf{x}$.

Theorem 2: Given the signature $\mathbf{u}_n$, for any $\lambda_i < \lambda_n$ and $\mathbf{u} \in \mathcal{U}_{n+1:N}$, $\mathsf{SINR}_D(\mathbf{u}_n) < \mathsf{SINR}_D\left(\frac{\mathbf{u}_n+\epsilon\mathbf{u}}{\|\mathbf{u}_n+\epsilon\mathbf{u}\|}\right)$ for any $\epsilon \neq 0$.

The proof is given in the appendix. Theorem 2 states that the received reduced-rank SINR increases if the signature moves along any direction in $\mathcal{U}_{n+1:N}$. This implies that the only stable fixed-point signature for the iterative algorithm is the eigenvector corresponding to the smallest eigenvalue. Hence the optimal signature with a reduced-rank receiver is the same as that with a full-rank receiver, and the output SINRs are also the same.

IV. Signature–Receiver Adaptation

We now consider the performance of joint signature–receiver *adaptation*, in which the jointly optimal reduced-rank receiver and signature are estimated from a finite training sequence. Given T training symbols, the Least Squares (LS) estimate of the full-rank MMSE receiver is

$$\hat{\mathbf{c}}_k = \hat{\mathbf{R}}^{-1}\hat{\mathbf{s}}_k \quad (10)$$

and the LS estimate of the reduced-rank receiver is

$$\hat{\tilde{\mathbf{c}}}_k = (\hat{\mathbf{S}}_D^\dagger\hat{\mathbf{R}}\hat{\mathbf{S}}_D)^{-1}\hat{\mathbf{S}}_D^\dagger\hat{\mathbf{s}}_k \quad (11)$$

where $\hat{\mathbf{R}}$ and $\hat{\mathbf{s}}_k$ are estimated from the training sequence [9], [11].[†] The receiver computes the jointly optimal signature and receiver according to (8)-(9), and then relays the signature back to the transmitter, which transmits the packet.

For the first iteration of (8)-(9) we estimate the covariance matrix and signature as

$$\hat{\mathbf{R}}^{(1)} = \frac{1}{T}\sum_{i=1}^{T}\mathbf{r}^{(1)}(i)\mathbf{r}^{(1)\dagger}(i) \quad (12)$$

$$\hat{\mathbf{s}}_k^{(1)} = \frac{1}{T}\sum_{i=1}^{T}\mathbf{r}^{(1)}(i)b_k^{(1)*}(i) \quad (13)$$

In subsequent iterations, we can use the estimated signature for the desired user to refine the estimate of the covariance matrix in (8). That is, letting $\mathbf{r}_{\mathrm{I}}^{(n)}(i) = \mathbf{r}^{(n)}(i) - A_k\mathbf{s}_k^{(n)}b_k^{(n)}(i)$, the received covariance matrix can be estimated as

$$\hat{\mathbf{R}}^{(n)} = \frac{1}{T}\sum_{i=1}^{T}\mathbf{r}_{\mathrm{I}}^{(n)}(i)\mathbf{r}_{\mathrm{I}}^{(n)\dagger}(i) + |A_k|^2\mathbf{s}_k^{(n)}\mathbf{s}_k^{(n)\dagger} \quad (14)$$

for $n \geq 2$. The adaptive version of the iterative signature–receiver algorithm is obtained by replacing the covariance matrix in (8)-(9) by the estimated matrix. Numerical experiments show that the estimate (14) gives better performance than (12) when used throughout the iterations.

We now present some numerical results, which illustrate the performance of joint signature–receiver adaptation with a reduced-rank receiver. We assume a synchronous CDMA system with 16 equal-power users, processing gain $N = 24$, and background SNR = 10 dB. The receiver estimates the covariance matrix using (12) in the first iteration and (14) in subsequent iterations, and (8) and (9) are used to update the signature in each transmission. (Note that the optimal signature is orthogonal to the interfering signatures.) Figure 1 shows

[†]We focus on adaptation with training, as opposed to blind adaptation, since an adaptive blind LS algorithm generally converges more slowly, and is more complex with an unknown channel.

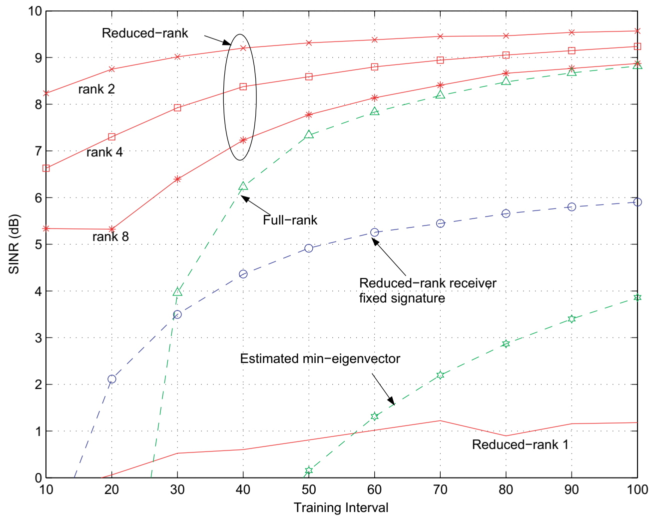

Fig. 1. SINR vs training interval for single-user joint signature–receiver adaptation.

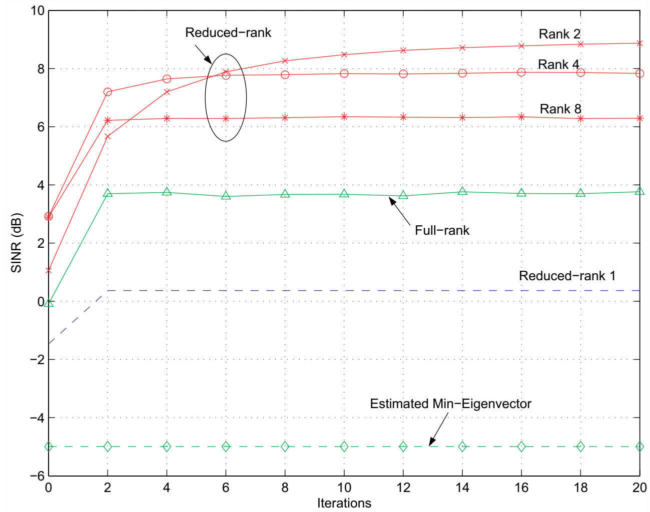

Fig. 2. SINR vs number of iterations with 30 training symbols.

SINR versus training for an adaptive user in the presence of fixed interference. In each case, the iterative algorithm is run until the relative increase in received SINR for the adapting user is less than 10^{-3}. Here and in what follows, the SINR is averaged over initial random signature assignments.[‡]

For comparison, the curve in Figure 1 labeled "estimated min-eigenvector" shows the SINR when the signature is the eigenvector associated with the minimal eigenvalue of the *estimated* interference plus noise covariance matrix. This adaptive scheme was proposed in [13], and is non-iterative, since the estimated signature does not explicitly depend on the particular receiver. Although this algorithm does not require multiple iterations, it requires substantially more training than the iterative algorithm. Figure 1 also shows the SINR with an adaptive reduced-rank receiver and *fixed* signatures. Those results correspond to the optimal rank, i.e., the rank is chosen to maximize the SINR for each realization. (Adaptive rank selection techniques are discussed in [9], [12].)

With limited training, the rank-2, 4, and 8 receivers significantly outperform the full-rank receiver. As the training increases, the performance gain diminishes, and the SINR for both reduced– and full–rank receivers converge to the single-user bound (10 dB), i.e., the performance with orthogonal signatures and matched filters. With little training ($T < 30$), the reduced-rank receiver with fixed signature outperforms the full-rank receiver with signature adaptation. The rank-1 receiver performs relatively poorly since the signature does not converge to the optimal signature.

Figure 2 shows SINR versus number of iterations with 30 training symbols for the algorithms used to generate Figure 1. Iteration $n = 0$ corresponds to a random signature and the reduced-rank LS estimate given by (11)-(13). The performance of the rank-1 receiver remains constant after the first iteration,[§] whereas the SINRs for rank-2, 4, and 8 receivers increase with n. The rank-2 receiver eventually performs best, but takes more iterations to converge than the rank-4 and 8 receivers. Hence in this case there is a trade-off between delay, due to iterations, and performance. The full-rank SINR essentially converges after only two iterations, but requires significantly more training to perform as well as the reduced-rank receivers. Although the estimated optimal (min-eigenvector) signature does not require multiple iterations, the performance is significantly worse than the iterative schemes, even taking into account the total training period after multiple iterations. These results indicate that joint signature–receiver adaptation can provide a substantial improvement in SINR with modest training overhead.

[‡]The speed of convergence is generally insensitive to the initial choice of sequence unless the sequence happens to be close to the optimal sequence.

[§]This is because the signature is estimated for $n = 0$, and is explicitly known for $n > 0$.

V. Group Adaptation

The iterative algorithm (8)-(9) can also be applied to a group of signatures. For example, in a synchronous CDMA system, a group of users can update their signatures in successive order [3]. That is, each iteration in the signature–receiver algorithm contains K sub-iterations corresponding to successive updates by each user. At the n^{th} iteration, after the jth signature update, the signature matrix is given by $\mathbf{S}^{(n)}(j) = \left[\mathbf{s}_1^{(n)}, \cdots, \mathbf{s}_{j-1}^{(n)}, \frac{\mathbf{c}_j^{(n)}}{\|\mathbf{c}_j^{(n)}\|}, \mathbf{s}_{j+1}^{(n)}, \cdots, \mathbf{s}_K^{(n)}\right]$, and $\mathbf{S}^{(n)} = \mathbf{S}^{(n)}(K)$. Alternatively, during each iteration the users might update their signatures *simultaneously*. (This may be more appropriate in a MIMO channel with synchronized transmitted data streams.) In that case, the signature matrix at the n^{th} iteration is given by $\mathbf{S}^{(n)} = \left[\frac{\mathbf{c}_1^{(n)}}{\|\mathbf{c}_1^{(n)}\|}, \cdots, \frac{\mathbf{c}_K^{(n)}}{\|\mathbf{c}_K^{(n)}\|}\right]$.

We first consider the convergence of iterative group optimization with optimal reduced–rank receivers (i.e., infinite training). The following theorem states that the set of signatures with successive iterative group optimization converges to an optimal signature set. We have not proved the analogous result for simultaneous group optimization, but illustrate convergence for simultaneous group adaptation (versus the training interval) through numerical examples.

Theorem 3: Given an initial set of signature sequences $\{\mathbf{s}_k^{(0)}\}$ with $\|\mathbf{s}_k^{(0)}\| = 1$ for $1 \leq k \leq K$, and where the

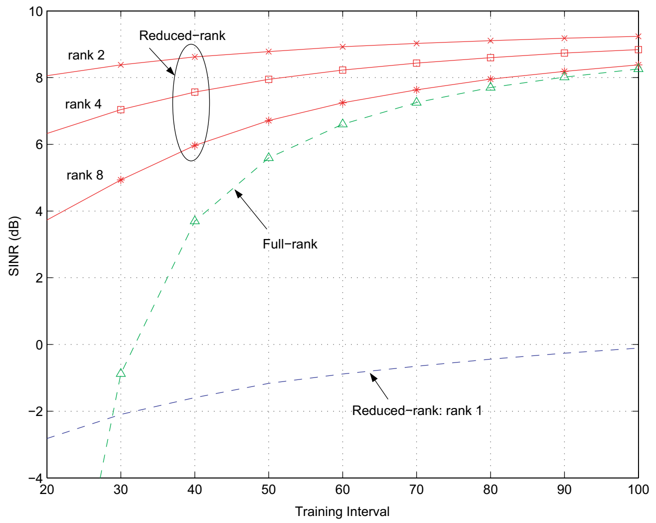

Fig. 3. Average SINR vs training with simultaneous group adaptation.

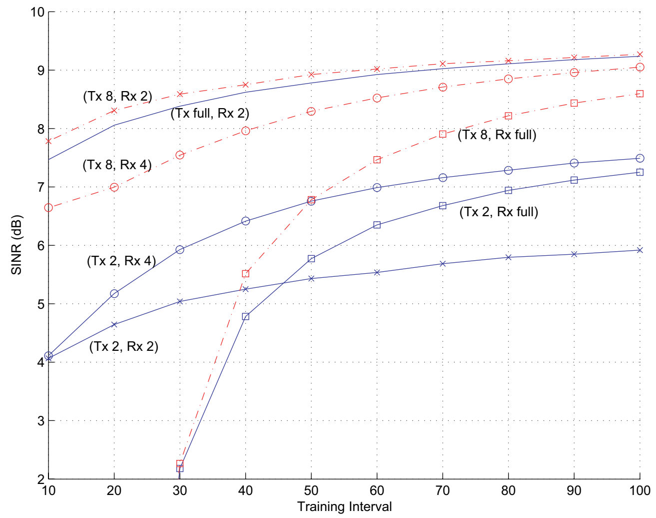

Fig. 4. Average SINR vs training for simultaneous group adaptation with reduced-rank signatures and reduced-rank receivers. (Tx a, Rx b) indicates that the transmitter signature has rank a and the receiver has rank b.

corresponding signature matrix $\mathbf{S}^{(0)}$ is full-rank, the signature set with iterative successive group optimization converges to a fixed point, where $\mathbf{s}_k^{(n)}$, $1 \le k \le K$ achieve the Welch Bound Equality.

The proof is given in the appendix.

We now present simulation results that illustrate the performance of simultaneous group adaptation with reduced–rank receivers. Figure 3 shows average SINR versus training with simultaneous group adaptation. The system parameters are the same as in previous figures. We assume that all users have the same receiver rank and change their signatures simultaneously at each iteration. The performance improvements offered by rank-2, 4, and 8 receivers, relative to the full-rank receiver, are similar to those observed in the single-user case. Again, the signature for the rank-1 receiver does not converge to the optimal signature.

VI. Reduced-Rank Signature

To reduce the amount of feedback required for joint signature–receiver adaptation, a reduced-rank *signature* adaptation scheme was presented in [13]. Namely, the signature is constrained to lie in a lower dimensional subspace. The reduced-rank signature for user k can therefore be expressed as

$$\mathbf{s}_k = \mathbf{F}_k \underline{\alpha}_k \tag{15}$$

where $\mathbf{F}_k$ is an $N \times D_p$ matrix, the columns of which are the basis vectors for the D_p-dimensional subspace, D_p is the signature "rank" (assuming $\mathbf{F}_k$ is full-rank), and $\underline{\alpha}_k$ is a $D_p \times 1$ vector of combining coefficients. A reduced-rank signature reduces the number of coefficients (and hence feedback bits) needed to represent the signature, but also reduces the degrees of freedom available to avoid interference.

We now consider reduced-rank signature adaptation in combination with a reduced-rank receiver. To constrain the signature to the D_p-dimensional subspace defined by $\mathbf{F}_k$, we replace the signature update in (9) by

$$\mathbf{s}_k^{(n+1)} = \frac{\mathbf{F}_k(\mathbf{F}_k^\dagger \mathbf{F}_k)^{-1}\mathbf{F}_k^\dagger \mathbf{c}_k^{(n)}}{\|\mathbf{F}_k(\mathbf{F}_k^\dagger \mathbf{F}_k)^{-1}\mathbf{F}_k^\dagger \mathbf{c}_k^{(n)}\|} \tag{16}$$

In what follows, we assume that $\mathbf{F}_k$ is a random unitary matrix, i.e., the signature is constrained to lie in a random subspace. This corresponds to the scenario in which the interfering signatures are isotropically distributed.

Figure 4 shows average SINR versus training for simultaneous group adaptation with reduced-rank signatures and reduced-rank receivers. The system parameters are the same as in the preceding figures. Curves are shown for transmitter ranks 2 and 8, and receiver ranks 2, 4, and 24 (full-rank). The performance of full-rank signatures with a rank-2 receiver, taken from Figure 3, is also presented for comparison. For a fixed receiver rank, the performance generally improves with signature rank, although with a rank-2 receiver, the rank-8 signature performs slightly better than the full-rank signature. Again, the reduced-rank receiver gives full-rank performance with sufficient training. With limited training, the reduced-rank receiver performs much better than the full-rank receiver.

VII. Conclusions

We have presented an iterative algorithm for joint optimization of signatures and reduced-rank MSWF receivers in a synchronous CDMA system without multipath. The signature converges to the optimal signature associated with the full-rank MMSE receiver provided that the receiver rank $D \ge 2$. Furthermore, the reduced-rank receiver converges to the matched filter, which is the MMSE receiver. When the receiver and signature are estimated from training, numerical results show that signature adaptation with a reduced-rank receiver provides a substantial performance gain relative to signature adaptation with the full-rank receiver.

We also considered reduced-rank *signature* adaptation with a reduced-rank receiver. This reduces the number of signature coefficients, which must be estimated at the receiver and relayed back to the transmitter. Numerical results show that with limited training, this scheme can perform slightly better than full-rank signature adaptation. Also, reduced-rank signature adaptation with a reduced-rank receiver again converges faster than reduced-rank signature adaptation with a full-rank receiver.

Numerical results have also been presented for group signature–receiver adaptation, in which all users adapt signatures simultaneously. With limited training a low complexity ($D = 2$) reduced-rank receiver again performs much better than a full-rank receiver.

The iterative signature–receiver optimization presented here can be easily modified for MIMO channels, and for CDMA with multipath. (See [13], which discusses full-rank receiver–signature adaptation in the presence of multipath.) Asynchronous CDMA requires no modification, although convergence analysis of the iterative algorithm for group adaptation must consider the shifted signatures corresponding to the user delays [7].

Finally, we have assumed that the signatures are relayed to the transmitter over a high-rate feedback channel, and have ignored the effect of signature quantization. Although there has been some work on signature quantization with limited feedback (e.g., see [15]), the performance with limited training for signature estimation combined with limited feedback remains to be studied.

Appendix I
Proof of Theorem 1

To establish convergence of the iterative algorithm, we show that $\mathsf{MMSE}_D^{(n+1)} \le \mathsf{MMSE}_D^{(n)}$, where $\mathsf{MMSE}_D^{(n)}$ is the MMSE for a rank D filter at iteration n (with signature $\mathbf{c}^{(n)}/\|\mathbf{c}^{(n)}\|$).¶ Also, we show that if equality holds, then $\mathbf{s}^{(n)}$ must be an eigenvector of $\mathbf{R}_\mathrm{I}$.

First, we observe that $\mathsf{MMSE}_D^{(n)} \ge \mathsf{MSE}^{(n+1)}$, where $\mathsf{MSE}^{(n+1)}$ is the MSE with the reduced-rank receiver $\mathbf{c}^{(n)}$ (given $\mathbf{s}^{(n)}$) and updated signature $\mathbf{s}^{(n+1)}$. This is because for a given receiver $\mathbf{c}^{(n)}$ and any normalized signature $\mathbf{s}$, the $\mathsf{MSE} = \left(1 - \mathbf{s}^\dagger \mathbf{c}^{(n)}\right)^2 + \mathbf{c}^{(n)\dagger}\mathbf{R}_\mathrm{I}\mathbf{c}^{(n)}$ is minimized by choosing the signature to be $\mathbf{s}^{(n+1)} = \frac{\mathbf{c}^{(n)}}{\|\mathbf{c}^{(n)}\|}$, which gives

$$\mathsf{MSE}^{(n+1)} = 1 - 2\|\mathbf{c}^{(n)}\| + \mathbf{c}^{(n)\dagger}\mathbf{R}^{(n+1)}\mathbf{c}^{(n)}. \tag{17}$$

Furthermore, with a rank-D MMSE receiver $\mathbf{c}^{(n+1)}$, the MMSE decreases with D, i.e., $\mathsf{MMSE}_D^{(n+1)} \le \mathsf{MMSE}_1^{(n+1)}$. Now

$$\left(\|\mathbf{c}^{(n)}\| - \mathbf{c}^{(n)\dagger}\mathbf{R}^{(n+1)}\mathbf{c}^{(n)}\right)^2 \ge 0$$

implies that

$$\frac{\|\mathbf{c}^{(n)}\|^2}{\mathbf{c}^{(n)\dagger}\mathbf{R}^{(n+1)}\mathbf{c}^{(n)}} + \mathbf{c}^{(n)\dagger}\mathbf{R}^{(n+1)}\mathbf{c}^{(n)} \ge 2\|\mathbf{c}^{(n)}\|.$$

so that

$$\mathsf{MMSE}_1^{(n+1)} = 1 - \frac{\|\mathbf{c}^{(n)}\|^2}{\mathbf{c}^{(n)\dagger}\mathbf{R}^{(n+1)}\mathbf{c}^{(n)}} \le \mathsf{MSE}^{(n+1)}$$

Therefore

$$\mathsf{MMSE}_D^{(n+1)} \underset{(a)}{\le} \mathsf{MMSE}_1^{(n+1)} \underset{(b)}{\le} \mathsf{MSE}^{(n+1)} \underset{(c)}{\le} \mathsf{MMSE}_D^{(n)}.$$

¶We emphasize that $\mathsf{MMSE}_D^{(n)}$ is *not* the MMSE obtained from Algorithm 1 at iteration n, since D does not correspond to the receiver rank used in preceding iterations.

Equality (b) holds only when $\|\mathbf{c}^{(n)}\| = \mathbf{c}^{(n)\dagger}\mathbf{R}^{(n+1)}\mathbf{c}^{(n)}$, and some manipulation gives $\|\mathbf{c}^{(n)}\| = \mathbf{s}^{(n)\dagger}\mathbf{c}^{(n)}$. From the Cauchy inequality we have

$$\begin{aligned}\|\mathbf{c}^{(n)}\|^2 &= 1 \cdot \|\mathbf{c}^{(n)}\|^2 = \left(\mathbf{s}^{(n)\dagger}\mathbf{s}^{(n)}\right)\left(\mathbf{c}^{(n)\dagger}\mathbf{c}^{(n)}\right) \\ &\ge \left(\mathbf{s}^{(n)\dagger}\mathbf{c}^{(n)}\right)^2,\end{aligned}$$

and since equality holds, we must have

$$\mathbf{s}^{(n)} \overset{(b)}{=} \kappa\mathbf{c}^{(n)} = \frac{\mathbf{c}^{(n)}}{\|\mathbf{c}^{(n)}\|} \overset{(c)}{=} \mathbf{s}^{(n+1)} \tag{18}$$

which establishes convergences.

Equality (b) also implies that starting with an arbitrary initial signature, the receiver filter converges to a rank-1, or matched filter (equality (a)). That is, the D-dimensional subspace, in which the receiver filter is constrained to exist, collapses to a one-dimensional subspace, which implies that the signature $\mathbf{s}^{(n)}$ is an eigenvector of $\mathbf{R}_\mathrm{I}$.

Appendix II
Proof of Theorem 2

When the signature is an eigenvector $\mathbf{u}_n$, the Krylov subspace degenerates to a one-dimensional subspace spanned by $\mathbf{u}_n$. Hence the SINR with a rank-D receiver,

$$\mathsf{SINR}_D(\mathbf{u}_n) = \mathsf{SINR}_1(\mathbf{u}_n) = \frac{\mathbf{u}_n^\dagger\mathbf{u}_n}{\mathbf{u}_n^\dagger\mathbf{R}_\mathrm{I}\mathbf{u}_n} = \frac{1}{\lambda_n}$$

independent of D. Since $\mathbf{u} \in \mathcal{U}_{n+1:N}$, we have

$$\mathbf{u} = \sum_{i=n+1}^{N} a_i\mathbf{u}_i$$

where $\exists a_j \ne 0$, for $n+1 \le j \le N$. Let $\mathbf{s} = \frac{\mathbf{u}_n + \epsilon\mathbf{u}}{\|\mathbf{u}_n + \epsilon\mathbf{u}\|}$. With a fixed signature, the received SINR increases with D. Hence $\mathsf{SINR}_D(\mathbf{s}) \ge \mathsf{SINR}_1(\mathbf{s})$, and

$$\begin{aligned}\mathsf{SINR}_1(\mathbf{s}) &= \frac{\left(\mathbf{s}^\dagger\mathbf{c}\right)^2}{\mathbf{c}^\dagger\mathbf{R}\mathbf{c} - \left(\mathbf{s}^\dagger\mathbf{c}\right)^2} \\ &= \frac{1}{\frac{1}{\mathbf{s}^\dagger\mathbf{S}_1(\mathbf{S}_1^\dagger\mathbf{R}\mathbf{S}_1)^{-1}\mathbf{S}_1^\dagger\mathbf{s}} - 1} \\ &= \frac{1}{\mathbf{s}^\dagger\mathbf{R}\mathbf{s} - 1} \\ &= \frac{1 + \epsilon^2\sum_{i=n+1}^{N}|a_i|^2}{\lambda_n + \epsilon^2\sum_{i=n+1}^{N}|a_i|^2\lambda_i} > \frac{1}{\lambda_n}\end{aligned}$$

Therefore $\mathsf{SINR}_D(\mathbf{u}_n) < \mathsf{SINR}_D\left(\frac{\mathbf{u}_n + \epsilon\mathbf{u}}{\|\mathbf{u}_n + \epsilon\mathbf{u}\|}\right)$.

Appendix III
Proof of Theorem 3

As in [3], [7], we use the total squared correlation (TSC) as a metric to study the convergence of successive group optimization. We have

$$TSC = \sum_{i=1}^{K}\sum_{j=1}^{K}|\mathbf{s}_i^\dagger\mathbf{s}_j|^2 = trace\left[(\mathbf{S}\mathbf{S}^\dagger)^2\right] \tag{19}$$

In [7] it is shown that minimizing TSC is equivalent to maximizing the sum capacity. To prove convergence, we show that the TSC is nonincreasing after each signature update. Suppose that user 1 replaces the signature $\mathbf{s}_1$ with $\bar{\mathbf{s}}_1 = \frac{\mathbf{c}_1}{\|\mathbf{c}_1\|}$. For convenience, we omit the superscript denoting iteration in this section. Letting $\mathbf{S}_1 = [\mathbf{s}_2, \cdots, \mathbf{s}_K]$, we have that

$$\begin{aligned} TSC &= trace\left[(\mathbf{S}_1\mathbf{S}_1^\dagger + \mathbf{s}_1\mathbf{s}_1^\dagger)^2\right] \\ &= trace\left[(\mathbf{S}_1\mathbf{S}_1^\dagger)^2\right] + 2\mathbf{s}_1^\dagger\mathbf{S}_1\mathbf{S}_1^\dagger\mathbf{s}_1 + 1 \end{aligned}$$

To show that the TSC is nonincreasing, we must show that $\mathbf{s}_1^\dagger\mathbf{S}_1\mathbf{S}_1^\dagger\mathbf{s}_1 \geq \bar{\mathbf{s}}_1^\dagger\mathbf{S}_1\mathbf{S}_1\bar{\mathbf{s}}_1$, or

$$\mathbf{s}_1^\dagger\mathbf{R}\mathbf{s}_1 - 1 - \sigma^2 \geq \bar{\mathbf{s}}_1^\dagger\mathbf{R}\bar{\mathbf{s}}_1 - |\bar{\mathbf{s}}_1^\dagger\mathbf{s}_1|^2 - \sigma^2 \quad (20)$$

Reordering (20) gives

$$\|\mathbf{c}_1\|^2(\mathbf{s}_1^\dagger\mathbf{R}\mathbf{s}_1 - 1) \geq \mathbf{c}_1^\dagger\mathbf{R}\mathbf{c}_1 - |\mathbf{c}_1^\dagger\mathbf{s}_1|^2 \quad (21)$$

From the Cauchy-Schwarz Inequality, we have

$$1 \cdot \|\mathbf{c}_1\|^2 = (\mathbf{s}_1^\dagger\mathbf{s}_1)(\mathbf{c}_1^\dagger\mathbf{c}_1) \geq |\mathbf{s}_1^\dagger\mathbf{c}_1|^2 \quad (22)$$

where the equality holds iff $\mathbf{c}_1 = \kappa\mathbf{s}_1$.

Clearly, $\mathbf{s}_1^\dagger\mathbf{R}\mathbf{s}_1 > 1$. Therefore, in order to show (21), we only need to show

$$|\mathbf{c}_1^\dagger\mathbf{s}_1|^2(\mathbf{s}_1^\dagger\mathbf{R}\mathbf{s}_1 - 1) \geq \mathbf{c}_1^\dagger\mathbf{R}\mathbf{c}_1 - |\mathbf{c}_1^\dagger\mathbf{s}_1|^2. \quad (23)$$

From (4) we have $\mathbf{c}_1^\dagger\mathbf{R}\mathbf{c}_1 = \mathbf{s}_1^\dagger\mathbf{S}_D(\mathbf{s}_D^\dagger\mathbf{R}\mathbf{S}_D)^{-1}\mathbf{S}_D^\dagger\mathbf{R}\mathbf{S}_D(\mathbf{S}_D^\dagger\mathbf{R}\mathbf{S}_D)^{-1}\mathbf{S}_D^\dagger\mathbf{s}_1 = \mathbf{c}_1^\dagger\mathbf{s}_1$, which is a real number. Hence (23) becomes

$$(\mathbf{c}_1^\dagger\mathbf{s}_1)(\mathbf{s}_1^\dagger\mathbf{R}\mathbf{s}_1 - 1) \geq 1 - \mathbf{c}_1^\dagger\mathbf{s}_1 \quad (24)$$

or

$$(\mathbf{c}_1^\dagger\mathbf{s}_1)(\mathbf{s}_1^\dagger\mathbf{R}\mathbf{s}_1) \geq 1. \quad (25)$$

Letting MMSE_D denote the MMSE for the rank-D receiver, we have $\mathsf{MMSE}_D = 1 - \mathbf{c}_1^\dagger\mathbf{s}_1 \leq \mathsf{MMSE}_1 = 1 - \frac{1}{\mathbf{s}_1^\dagger\mathbf{R}\mathbf{s}_1}$, which establishes (25). Hence the TSC converges, and the fixed point corresponds to equality in (25), which holds when the rank-D signal subspace collapses into a rank-1 subspace. Consequently, the signature $\mathbf{s}_1$ at the fixed point must be an eigenvector of $\mathbf{R}$. The preceding argument holds for each user, hence all signatures must converge. Having established this, we can now apply the proofs of Theorems 3 and 4 in [3] (see also [7]) to show that the set of signatures at the fixed point achieve the Welch Bound Equality.

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