Adaptive Multiuser Decision Feedback Demodulation for GSM

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Abstract

Demodulation of multiple users in a cellular TDMA system is considered where the receiver is situated at a random location within a cell. Our approach is to combine an adaptive multiuser decision-feedback demodulator (DFD) with multiple antennas. The DFD contains (matrix) finite-impulse response feedforward and and feedback filters to mitigate both intersymbol and multiple-access interference. The MMSE filter coefficients are specified, and adaptive estimation algorithms are presented which rely on training data from each user. An iterative estimation technique is proposed for alleviating the deleterious effects of error propagation. Simulation results show that by cancelling a relatively strong user, the DFD offers a significant improvement in performance for weak users relative to the linear MMSE receiver.

1 Introduction

We consider adaptive space-time multiuser demodulation of digital signals in a cellular environment with limited side information. The communication system of interest is the forward link of a cellular Time-Division Multiple-Access (TDMA) network (in particular, Global System for Mobile telecommunications (GSM)). The receiver is assumed to be randomly located within a particular cell, and has no prior knowledge of users' spatial signatures or channels. Since the co-channel users to be demodulated are in different cells, there is a significant near-far problem which causes a large power imbalance among the received signals.

The approach we take is to use multiuser decision-feedback demodulation combined with spatial diversity. Decision-feedback equalization for multiple-input/multiple output (MIMO) channels was presented in [1], and applied to multiuser detection for DS-CDMA in [2, 3, 4].

The emphasis of our study is on an *adaptive* multiuser decision-feedback demodulator (DFD), which promises superior performance to an adaptive linear Minimum Mean-Square Error (MMSE) receiver, at the cost of a moderate increase in complexity. The feedforward filter of the adaptive DFD consists of an adaptive *linear* MMSE filter followed by an adaptive whitening filter. For the channel model considered, both multiple-access and intersymbol interference are present. (That is, all users are subject to slow frequency-selective fading.) The optimal infinite-impulse-response (IIR) feedforward and feedback filters [3] are approximated as finite-impulse-response (FIR) filters [5, 6]. Embedding the received vectors in a higher-dimensional space leads to a convenient method for estimating the optimal (matrix) filter coefficients.

Since the near-far problem is likely for the communication scenario considered, proper ordering of users (strongest to weakest) for successive cancellation is important.

We propose an iterative algorithm for refining the uncoded bit estimates in order to alleviate error propagation. The algorithm relies on the observation that bit estimates from the DFD are generally more reliable than those from the linear front-end. Hence, we may use these refined (uncoded) bit estimates to generate refined interference estimates. Ideally, this process is repeated until the resulting uncoded bit estimates do not change from one iteration to the next.

In the next section, we present the communication system model. In Section 3, we present the MMSE-DFD, along with the embedded representation of the filters for frequency-selective fading channels. This is followed, in Section 4, by a discussion of adaptive Least-Squares (LS) algorithms for estimating the filter coefficients. In Section 5, we present the iterative technique for mitigating error propagation. Simulation results are presented in Section 6, and conclusions are presented in Section 7.

2 System Model

We assume that the receiver is randomly located in a cell and attempts to demodulate the desired user within that cell, in addition to co-channel users in other cells. The other-cell users are likely to be located much further away from the receiver than the adjacent base station, creating a large disparity in received powers. GMSK modulation is assumed, which is a nonlinear technique. However, it has been shown that a GMSK signal can be accurately approximated with the standard linear baseband signal [7, 8]

$$s_k(t) = \sum_i b_k(i) g(t - iT - \tau_k) \tag{1}$$

for user k, where $b_k(i)$ is the i^{th} transmitted bit for user k, $g(t)$ is the transmitted pulse shape, T is the symbol period, and τ_k is the random delay associated with the k^{th} user. In a frequency-selective fading environment, the received signal at the m^{th} antenna is given by

$$r_m(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} h_{m,k}^{(l)}(t) s_k(t - \tau_m^{(l)}) + n_m(t) \tag{2}$$

where $h_{m,k}^{(l)}(t)$ is the complex fading coefficient associated with user k and path l, $n_m(t)$ is additive white complex-Gaussian noise with variance σ^2, $\tau_m^{(l)}$ is the delay associated with path l on antenna m, L is the number of paths, and K is the number of users. We assume independent Rayleigh fading on each antenna element. In the case of flat fading, the spatial signature for a particular user is the vector of complex fade coefficients across the antennas. All channels are assumed to be constant throughout each packet, but are statistically independent from packet to packet.

When the received signal is sampled at the symbol rate, (2) may be expressed in vector form as

$$\mathbf{r}(i) = \sum_{l=1}^{L_c} \mathbf{H}^{(l)} \mathbf{b}(i - l + 1) + \mathbf{n}(i) \tag{3}$$

where the k^{th} column of the matrix

$$\mathbf{H}^{(l)} = \begin{bmatrix} h_{1,1}^{(l)} & h_{1,2}^{(l)} & \cdots & h_{1,K}^{(l)} \\ h_{2,1}^{(l)} & h_{2,2}^{(l)} & \cdots & h_{1,K}^{(l)} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M,1}^{(l)} & h_{M,2}^{(l)} & \cdots & h_{M,K}^{(l)} \end{bmatrix} \tag{4}$$

is the spatial signature for user k, path l, and L_c represents the overall channel order. In this case, $\mathbf{H}^{(l)}$ incorporates both the channel response and the intersymbol interference due to the Gaussian pulse shaping filters.

3 Multiuser MMSE-DFD

A block diagram of the multiuser DFD is shown in Figure 1. The soft output at time i is

$$\mathbf{x}(i) = \mathbf{F}\mathbf{r}(i) - \mathbf{B}\widehat{\mathbf{b}}(i) \tag{5}$$

where $\mathbf{F}$ and $\mathbf{B}$ are respectively the feedforward and feedback filters, $\mathbf{B}$ is lower diagonal (with zeros along the diagonal), and $\widehat{\mathbf{b}}(i)$ is the output of the decision device. The performance criterion used to optimize the receiver is Minimum Mean Squared Error (MMSE), defined as

$$\min_{\mathbf{F},\mathbf{B}} \mathcal{E} = \min_{\mathbf{F},\mathbf{B}} E\left[\|\mathbf{b}(i) - \mathbf{x}(i)\|^2\right] \tag{6}$$

For tractability, it is typically assumed that $\widehat{\mathbf{b}}(i) = \mathbf{b}(i)$ (i.e., no error propagation), in which case the optimal filters are [1]

$$\mathbf{F} = (\mathbf{I} + \mathbf{B})\mathbf{P}^{\dagger}\mathbf{R}^{-1} \tag{7}$$

where

$$\mathbf{P} = E[\mathbf{r}(i)\mathbf{b}^{\dagger}(i)], \qquad \mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^{\dagger}(i)] \tag{8}$$

and

$$\mathbf{B} = \mathbf{L} - \mathbf{I} \tag{9}$$

where $\mathbf{L}$ is a lower-diagonal matrix which satisfies

$$\mathbf{L}^{\dagger}\mathbf{D}\mathbf{L} = \mathbf{P}^{\dagger}\mathbf{P} + \sigma^2\mathbf{I} \tag{10}$$

where $\mathbf{D}$ is diagonal, and the matrix in (10) is the error covariance matrix associated with the linear MMSE receiver. The users are demodulated successively as

$$\begin{aligned} \hat{b}_1(i) &= \operatorname{sgn}\left[\operatorname{Re}\left\{z_1(i)\right\}\right] \\ \hat{b}_2(i) &= \operatorname{sgn}\left[\operatorname{Re}\left\{z_2(i) - \mathbf{B}_{2,1}\hat{b}_1(i)\right\}\right] \\ &\vdots \\ \hat{b}_K(i) &= \operatorname{sgn}\left[\operatorname{Re}\left\{z_K(i) - \sum_{k=1}^{K-1}\mathbf{B}_{K,K-k}\hat{b}_k(i)\right\}\right] \end{aligned} \tag{11}$$

where

$$z_k(i) = \mathbf{L}\mathbf{C}_k^\dagger \mathbf{r}(i) \tag{12}$$

is the output of the feedforward filter for user k.

Multipath introduces interference across time in addition to interference across users. The ideal MMSE-DFD in this case has matrix IIR feedforward and feedback filters with transfer functions $\mathbf{F}(z)$ and $\mathbf{B}(z)$ [1]. Here we instead assume Finite-Impulse Response (FIR) filters, which we optimize according to the MMSE criterion. To do this we define the stacked symbol vector

$$\widetilde{\mathbf{b}}(i) = \begin{bmatrix} b_1(i-L_c+1) & \cdots & b_K(i-L_c+1) & \cdots & b_1(i) & \cdots & b_K(i) \end{bmatrix}^T \tag{13}$$

where the users are ordered according to received power, and across time. In what follows, all variables with a tilde are stacked variables. The intersymbol interference can be treated as interference from separate, virtual users. The cost function then becomes

$$\min_{\widetilde{\mathbf{F}},\widetilde{\mathbf{B}}} \mathcal{E} = \min_{\widetilde{\mathbf{F}},\widetilde{\mathbf{B}}} E\left(\left\| \mathbf{b}(i) - \left(\widetilde{\mathbf{F}}\widetilde{\mathbf{r}}(i) - \widetilde{\mathbf{B}}\widetilde{\mathbf{b}}(i) \right) \right\|^2 \right) \tag{14}$$

where

$$\widetilde{\mathbf{F}} = \begin{bmatrix} \mathbf{F}_{L_c-1} & \mathbf{F}_{L_c-2} \dots & \mathbf{F}_0 \end{bmatrix} \tag{15}$$

$$\widetilde{\mathbf{B}} = \begin{bmatrix} \mathbf{B}_{L_c-1} & \mathbf{B}_{L_c-2} & \dots & \mathbf{B}_0 \end{bmatrix} \tag{16}$$

and $\mathbf{F}_n$ and $\mathbf{B}_n$ represent the feedforward and feedback filters at sampling instance n, respectively. The stacked received vector is given by

$$\widetilde{\mathbf{r}}(i) = \widetilde{\mathbf{H}}\widetilde{\mathbf{b}}(i) + \widetilde{\mathbf{n}}(i) \tag{17}$$

where $\widetilde{\mathbf{H}}$ represent the overall response of the communication channel.

As before, the solutions to the higher dimensional minimization problem are

$$\mathcal{F} = (\mathbf{I}+\mathbf{B})\mathbf{P}^\dagger(\widetilde{\mathbf{H}}^\dagger\widetilde{\mathbf{H}} + \sigma^2\mathbf{I})^{-1}, \qquad \mathcal{B} = \mathbf{L} - \mathbf{I} \tag{18}$$

where

$$\mathbf{L}^\dagger\mathbf{D}\mathbf{L} = (\widetilde{\mathbf{H}}^\dagger\widetilde{\mathbf{H}} + \sigma^2\mathbf{I}) \tag{19}$$

The solutions to the original (lower-dimensional) problem are then determined as follows:

$$\mathcal{B} = \begin{bmatrix} \mathcal{B}_1 & \mathbf{0} & \mathbf{0} & \dots \\ \mathcal{B}_2 & \mathcal{B}_3 & \mathbf{0} & \dots \\ & \vdots & \ddots & \\ \mathbf{B}_{L_c-1} & \mathbf{B}_{L_c-2} & \dots & \mathbf{B}_0 \end{bmatrix} \} \, \widetilde{\mathbf{B}} \qquad \mathcal{F} = \begin{bmatrix} \mathcal{F}_1 & \mathbf{0} & \mathbf{0} & \dots \\ \mathcal{F}_2 & \mathcal{F}_3 & \mathbf{0} & \dots \\ & \vdots & \ddots & \\ \mathbf{F}_{L_c-1} & \mathbf{F}_{L_c-2} & \dots & \mathbf{F}_0 \end{bmatrix} \} \, \widetilde{\mathbf{F}} \tag{20}$$

The users are then demodulated successively as

$$\begin{aligned} \hat{b}_1(i) &= \operatorname{sgn}\left[z_1(i) - \sum_{j=0}^{L_c-1} \mathbf{B}_j(1,1)\hat{b}_1(i-j) \right] \\ &\vdots \\ \hat{b}_K(i) &= \operatorname{sgn}\left[z_K(i) - \sum_{j=0}^{L_c-1}\sum_{k=1}^{K} \mathbf{B}_j(K,k)\hat{b}_k(i-j) \right] \end{aligned} \tag{21}$$

In this way, interference from stronger users generated at times $i - L_c + 1, \ldots, i$ is cancelled. Note, however, that the dimension of the signal space has increased, and hence more data are required to estimate $\widetilde{\mathbf{H}}$. Also, errors propagate in time and across users.

4 Multiuser LS-DFD

In this section we present three variants of LS algorithms that can be used to estimate the MMSE-DFD coefficients.

Decision-Directed LS

We wish to minimize the LS cost function, i.e.,

$$\min_{\widehat{\mathbf{F}},\widehat{\mathbf{B}}} \mathcal{E} = \min_{\widehat{\mathbf{F}},\widehat{\mathbf{B}}} \sum_{i=1}^{B} \|\mathbf{b}(i) - (\widehat{\mathbf{F}}\mathbf{r}(i) - \widehat{\mathbf{B}}\widehat{\mathbf{b}}(i))\|^2 \tag{22}$$

where B is the number of bits in a packet. Minimization of this cost function requires either a training sequence or estimates of the transmitted symbols $\mathbf{b}(i)$ from each user. Assuming $\widehat{\mathbf{b}} = \mathbf{b}$, it is easily shown that

$$\widehat{\mathbf{F}} = (\mathbf{I} + \widehat{\mathbf{B}})\widehat{\mathbf{P}}^{\dagger}\widehat{\mathbf{R}}^{-1}, \tag{23}$$

where

$$\widehat{\mathbf{P}} = \frac{1}{B}\sum_{i=1}^{B} \mathbf{r}(i)\mathbf{b}^{\dagger}(i), \qquad \widehat{\mathbf{R}} = \frac{1}{B}\sum_{i=1}^{B} \mathbf{r}(i)\mathbf{r}^{\dagger}(i) \tag{24}$$

Defining the estimation error

$$\zeta(i) = \mathbf{b}(i) - \widehat{\mathbf{P}}^{\dagger}\widehat{\mathbf{R}}^{-1}\mathbf{r}(i), \tag{25}$$

the sample error covariance at the output of the feedforward filter is given by

$$\mathcal{E} = \mathbf{L}\left(\frac{1}{B}\sum_{i=1}^{B} \zeta(i)\zeta^{\dagger}(i)\right)\mathbf{L}^{\dagger} \tag{26}$$

where the lower-diagonal matrix

$$\widehat{\mathbf{L}} = \mathbf{I} + \widehat{\mathbf{B}}. \tag{27}$$

To minimize the LS cost function, we choose $\mathbf{L}$ to whiten the sample error covariance matrix,

$$\widehat{\mathbf{L}}^{-\dagger}\widehat{\mathbf{D}}\widehat{\mathbf{L}}^{-1} = \frac{1}{B}\sum_{i=1}^{B} \zeta(i)\zeta^{\dagger}(i) \tag{28}$$

where $\mathbf{D}$ is diagonal. A decision-directed version of the preceding algorithm is obtained by using estimates from the linear LS filter:

$$\widehat{\mathbf{b}}(i) = \mathrm{sgn}\left[\mathrm{Re}\left\{\widehat{\mathbf{P}}^{\dagger}\widehat{\mathbf{R}}^{-1}\mathbf{r}(i)\right\}\right] \tag{29}$$

Channel Estimation

An alternative to the preceding LS approach is to estimate the channel parameters associated with the users, and then use these to estimate the MMSE filters. (This approach is taken in [4]). In this case, we select $\mathbf{L}$ to satisfy

$$\widehat{\mathbf{L}}^{-1}\widehat{\mathbf{D}}^{-1}\widehat{\mathbf{L}}^{-\dagger} = (\widehat{\mathbf{P}}^{\dagger}\widehat{\mathbf{P}} + \hat{\sigma}^2\mathbf{I})^{-1} \tag{30}$$

where $\widehat{\mathbf{P}}$ is given by (24), and $\hat{\sigma}$ is an estimate of the noise variance σ. The matrix in (30) is an estimate of the error covariance matrix in (10). The noise variance can be estimated as

$$\widehat{\sigma}^2 = \frac{1}{K}\,\mathrm{tr}(\widehat{\mathbf{R}} - \widehat{\mathbf{P}}\widehat{\mathbf{P}}^{\dagger}) \tag{31}$$

Blind Estimation

In the absence of training bits or reliable symbol estimates, it is possible to compute a blind estimate of the whitening filter. Namely, expanding the error covariance matrix gives

$$\begin{aligned}\frac{1}{B}\sum_{i=1}^{B}\zeta(i)\zeta^{\dagger}(i) = \frac{1}{B}\sum_{i=1}^{B}\Big[&\widehat{\mathbf{b}}(i)\widehat{\mathbf{b}}^{\dagger}(i) - \widehat{\mathbf{b}}(i)\mathbf{r}^{\dagger}(i)\widehat{\mathbf{R}}^{-1}\widehat{\mathbf{P}} - \\ &\widehat{\mathbf{P}}^{\dagger}\widehat{\mathbf{R}}^{-1}\mathbf{r}(i)\widehat{\mathbf{b}}^{\dagger}(i) + \widehat{\mathbf{P}}^{\dagger}\widehat{\mathbf{R}}^{-1}\mathbf{r}(i)\mathbf{r}^{\dagger}(i)\widehat{\mathbf{R}}^{-1}\widehat{\mathbf{P}}\Big]\end{aligned} \tag{32}$$

Since the bits are assumed to be i.i.d, the first term in (32) may be approximated by

$$\frac{1}{B}\sum_{i=1}^{B}\widehat{\mathbf{b}}(i)\widehat{\mathbf{b}}^{\dagger}(i) \approx \mathbf{I}. \tag{33}$$

Substituting this approximation and the estimate for $\widehat{\mathbf{P}}$ in (24) into (32) gives

$$\frac{1}{B}\sum_{i=1}^{B}\zeta(i)\zeta^{\dagger}(i) \approx \mathbf{I} - \frac{1}{B}\sum_{i=1}^{B}\mathbf{y}(i)\mathbf{y}^{\dagger}(i) \tag{34}$$

where

$$\mathbf{y}(i) = \widehat{\mathbf{P}}^{\dagger}\widehat{\mathbf{R}}^{-1}\mathbf{r}(i) \tag{35}$$

In a stationary environment this estimate converges to the error covariance matrix as $B \to \infty$. The matrix $\widehat{\mathbf{L}}$ is then selected to satisfy

$$\widehat{\mathbf{L}}^{-\dagger}\widehat{\mathbf{D}}\widehat{\mathbf{L}}^{-1} = \mathbf{I} - \frac{1}{B}\sum_{i=1}^{B}\mathbf{y}(i)\mathbf{y}^{\dagger}(i) \tag{36}$$

To summarize, we have presented three different estimators for the MMSE-DFD. All converge to the MMSE solution as the number of data samples increases to infinity. A comparison of these techniques is given in Section 6.

5 Iterative Estimation

The main disadvantage of the DFD is its sensitivity to error propagation. Performance can degrade substantially when the feedback error rate is high. To improve performance, we propose an iterative technique in which the symbol estimates at the output of the decision device are repeatedly fed back through the feedback filter $\mathbf{B}$. Specifically, letting m denote the iteration, we have

$$\widehat{\mathbf{b}}^{(m+1)}(i) = \operatorname{sgn}\left[\operatorname{Re}\left\{\mathbf{F}\mathbf{r}(i) - \mathbf{B}\widehat{\mathbf{b}}^{(m)}(i)\right\}\right] \tag{37}$$

where the initial value $\widehat{\mathbf{b}}^{(1)}(i)$ can be taken from the output of the front-end linear adaptive filter. Convergence occurs when the estimated bits do not change from one iteration to the next. Convergence analysis is difficult due to the presence of the nonlinear decision device. Problems with convergence, however, have not been observed in simulations.

6 Numerical Results

The results in this section assume a fixed number of users, where one user is relatively strong (inside the cell), and the rest are weak (outside the cell). The modulation scheme is GMSK, where we use the linear model presented in [7]. The time slot is a normal burst, as defined by the GSM standard, and the receiver is synchronous with respect to the users. Since the linearized GMSK pulse shaping filter is partial response, $\mathbf{r}(i)$ contains both intersymbol and multiple-access interference. All users experience a frequency-selective Rayleigh fading channel with three rays one symbol interval apart. The fading coefficients associated with the paths and antennas are assumed to be independent, and are constant for the duration of the packet.

Figure 2 shows uncoded error rate vs. SNR for the MMSE-DFD and linear receivers. In this case, there are two users, two antennas, and the observation window spans 5 symbols. The strong user is 10 dB stronger than the weak user. (That is, the SNR for user 1 shown on the x-axis is $\mathrm{SNR}_1 = \mathrm{SNR}_2 + 10\mathrm{dB}$). The figure shows that the MMSE-DFD offers a significant improvement for the weak user, relative to linear detection, since the MMSE-DFD cancels interference from the strong user, in addition to ISI. The strong user also exhibits some improvement due to cancellation of ISI.

Figure 3 illustrates the performance of the MMSE-DFD in a rank-deficient system with two antennas and three users. The two weaker users are received with the same SNR, while the strongest user is 10 dB stronger than the weak users. The two weak users have identical performance with the linear MMSE receiver. Again, the MMSE-DFD offers a significant performance improvement for the weak users, since the strong user is effectively cancelled. Note that the performance of the last user is better than that of the second user since more interferers are cancelled.

Figure 4 illustrates the performance of the iterative estimation scheme described in Section 5. The scenario simulated is the same as that presented in Figure 3. These results show that iterative updating offers a substantial performance improvement at high SNRs. It is also interesting to note that the relative performance of the users changes when iterative estimation is introduced. This is because without iteration, error propagation degrades the performance of user 3 relative to user 2. This error propagation is reduced with iterative estimation.

Finally, Figure 5 shows error rate as a function of training data for the three adaptive algorithms discussed in Section 4. In this case, the decision-directed LS-DFD converges relatively quickly compared with the other techniques.

7 Conclusions

We have studied the application of the MMSE-DFD receiver with multiple antennas to multiuser demodulation of GMSK signals. It has been shown that for the cases considered, where there is a large power imbalance among received users, the MMSE-DFD offers a significant performance improvement for the weak users relative to the MMSE linear receiver. Adaptive estimation algorithms have been presented with complexity comparable to that of analogous linear LS algorithms. (An additional whitening operation is required for the DFD.) The filter structure considered includes memory, which is needed to mitigate the effects of multipath-induced ISI.

The main disadvantage of the DFD relative to a linear detector is error propagation. Results show that when the SNR for users being cancelled is low (say, less than 5 dB), error propagation is likely to increase the error rates of successive users relative to linear detection. An iterative approach for alleviating this problem has been proposed, and works well at moderate received SNRs (> 8 dB).

As the number of DFD coefficients increases (due to both ISI and more users), more training data is needed to obtain an accurate estimate of the MMSE coefficients. This is a potential problem for the application considered, in which relatively little training data is available. Future efforts will include consideration of other adaptive techniques, and a comparison of the MMSE-DFD with maximum-likelihood detection.

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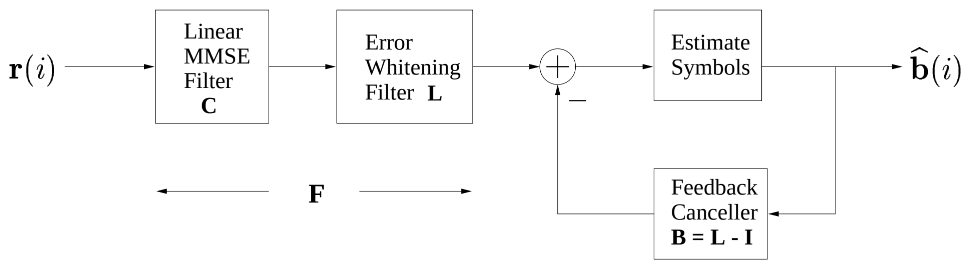

Figure 1: Decision-Feedback Demodulator Block Diagram

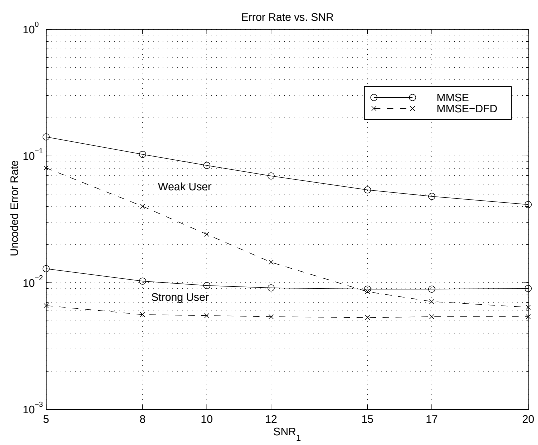

Figure 2: Uncoded error rate for a 2-user, 3-ray multipath GSM channel with 2 antenna elements.

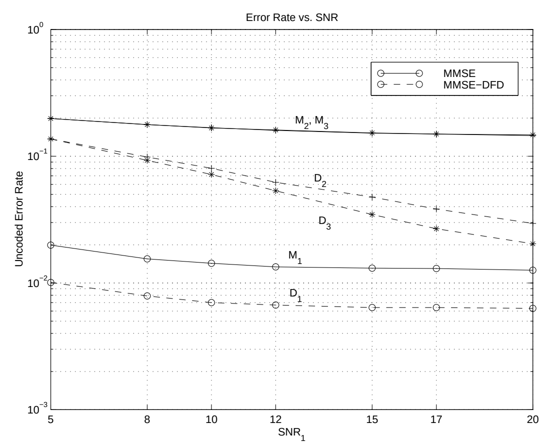

Figure 3: Uncoded error rate for a rank-deficient system: 3 users, 2 antenna elements, 3-ray multipath GSM channel. $\mathrm{M}_k, \mathrm{D}_k$ = Performance of user k under linear MMSE and MMSE-DFD receivers, respectively.

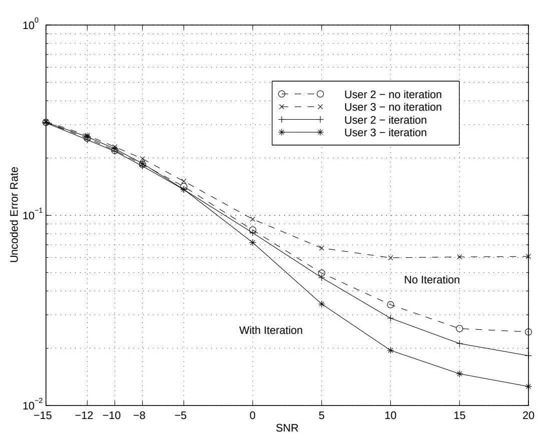

Figure 4: Uncoded error rate vs. $\mathrm{SNR}_{2,3}$ with iterative feedback update, 3 users (weak users shown), 3-ray multipath GSM channel with 2 antenna elements.

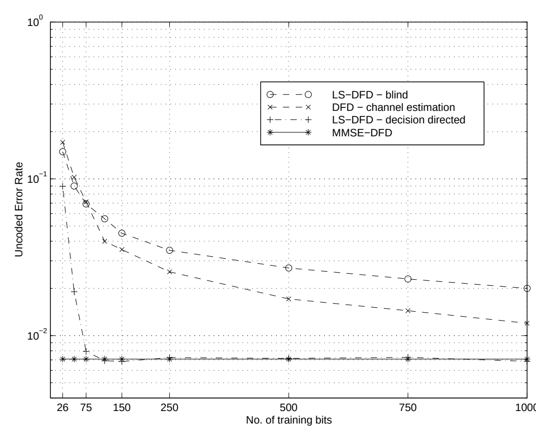

Figure 5: Uncoded error rate convergence for various adaptive DFD algorithms, 2 users (weak user shown), 3-ray multipath GSM channel, 2 antenna elements, $\mathrm{SNR}_2 = 2$ dB.