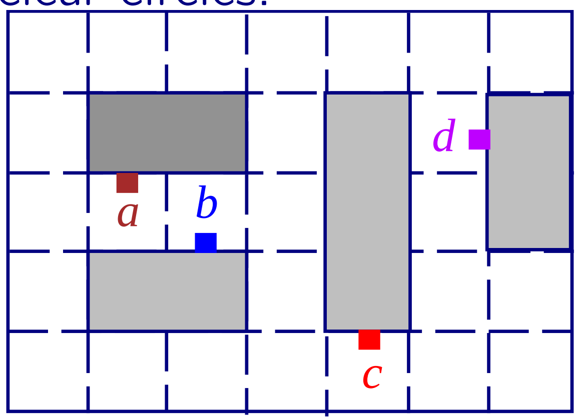
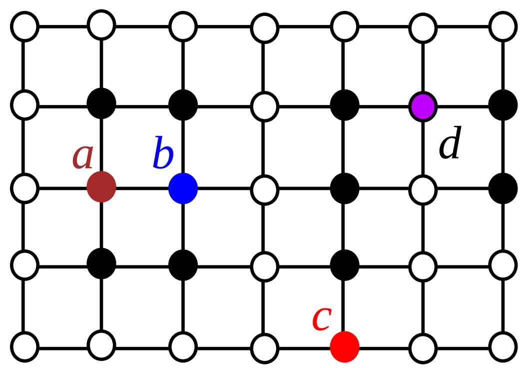
Graph Models for Global Routing: Grid Graph

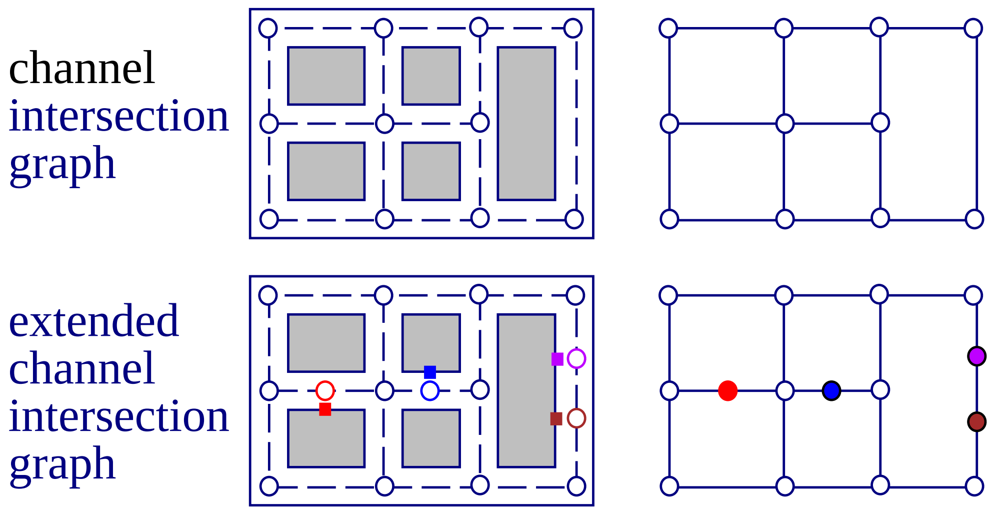
- Each cell is represented by a vertex.
- Two vertices are joined by an edge if the corresponding cells are adjacent to each other.
- The occupied cells are represented as filled circles, whereas the others are as clear circles.





Graph Model: Channel Intersection Graph

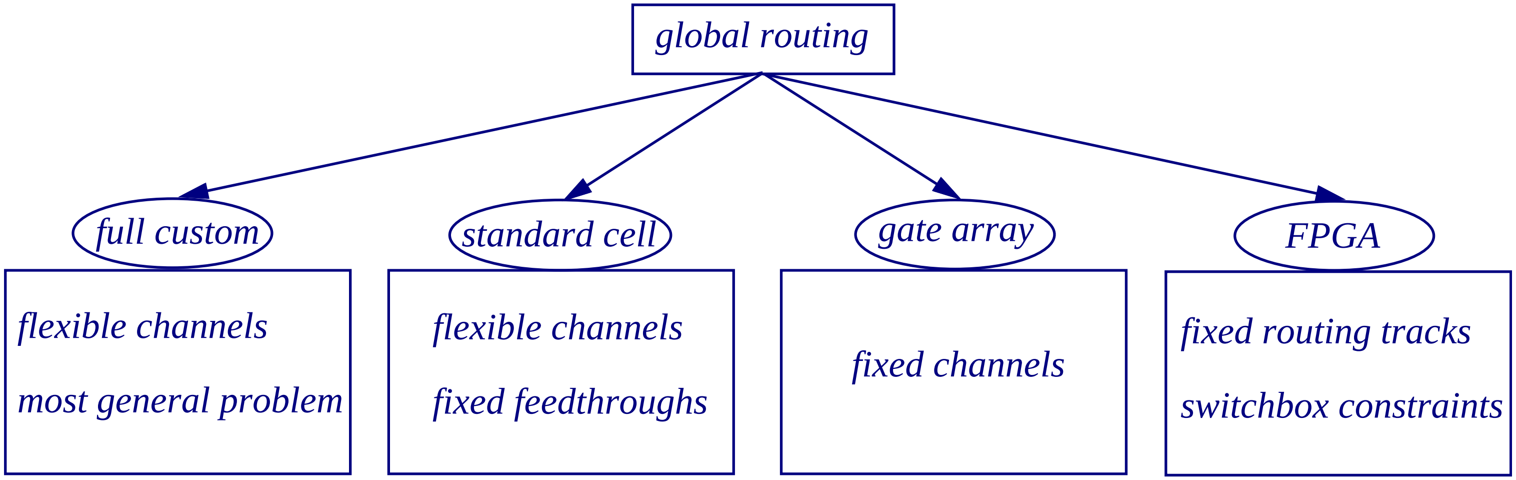
- Channels are represented as edges.
- Channel intersections are represented as vertices.
- Edge weight represents channel capacity.
- Extended channel intersection graph: terminals are also represented as vertices.



Global-Routing Problem

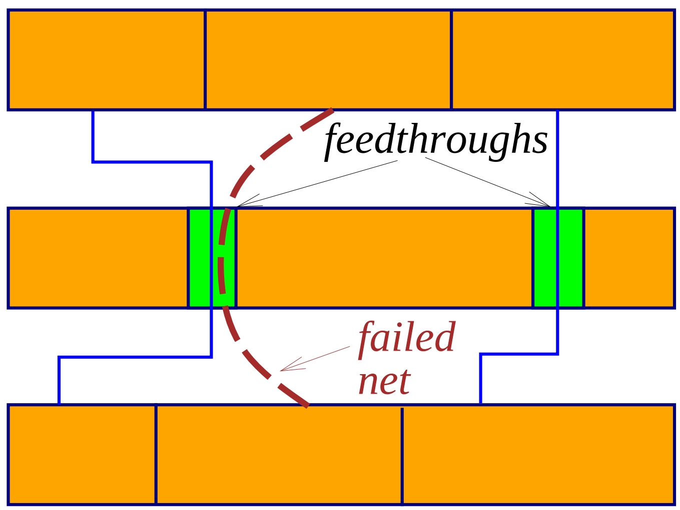
- Given a netlist N=$\{N_1, N_2, \ldots, N_n\}$, a routing graph $G = (V, E)$, find a Steiner tree T_i for each net N_i, $1 \le i \le n$, such that $U(e_j) \le c(e_j)$, $\forall e_j \in E$ and $\sum_{i=1}^{n} L(T_i)$ is minimized,
  where
  - $c(e_j)$: capacity of edge e_j;
  - $x_{ij} = 1$ if e_j is in T_i; $x_{ij} = 0$ otherwise;
  - $U(e_j) = \sum_{i=1}^{n} x_{ij}$: # of wires that pass through the channel corresponding to edge e_j;
  - $L(T_i)$: total wirelength of Steiner tree T_i.
- For high-performance, the maximum wirelength ($\max_{i=1}^{n} L(T_i)$) is minimized (or the longest path between two points in T_i is minimized).

Global Routing in different Design Styles



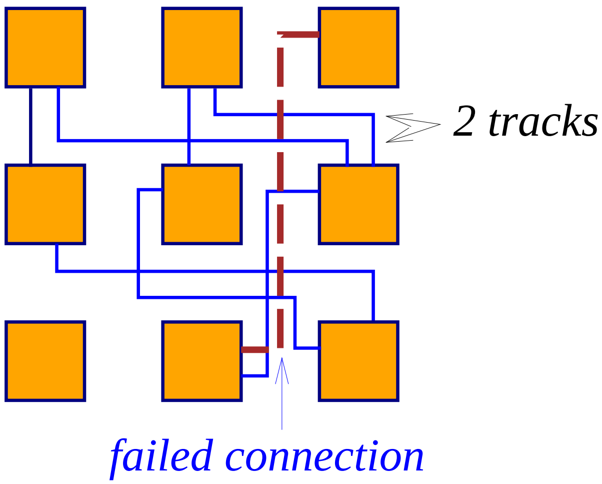
Global Routing in Standard Cell

- Objective
  - Minimize total channel height.
  - Assignment of **feedthrough**: Placement? Global routing?
- For high performance,
  - Minimize the maximum wire length.
  - Minimize the maximum path length.



Global Routing in Gate Array

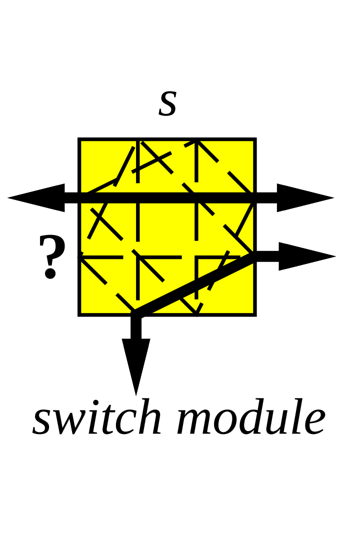
- Objective
  - **Guarantee 100% routability.**
- For high performance,
  - Minimize the maximum wire length.
  - Minimize the maximum path length.



Each channel has a capacity of 2 tracks.

Global Routing in FPGA

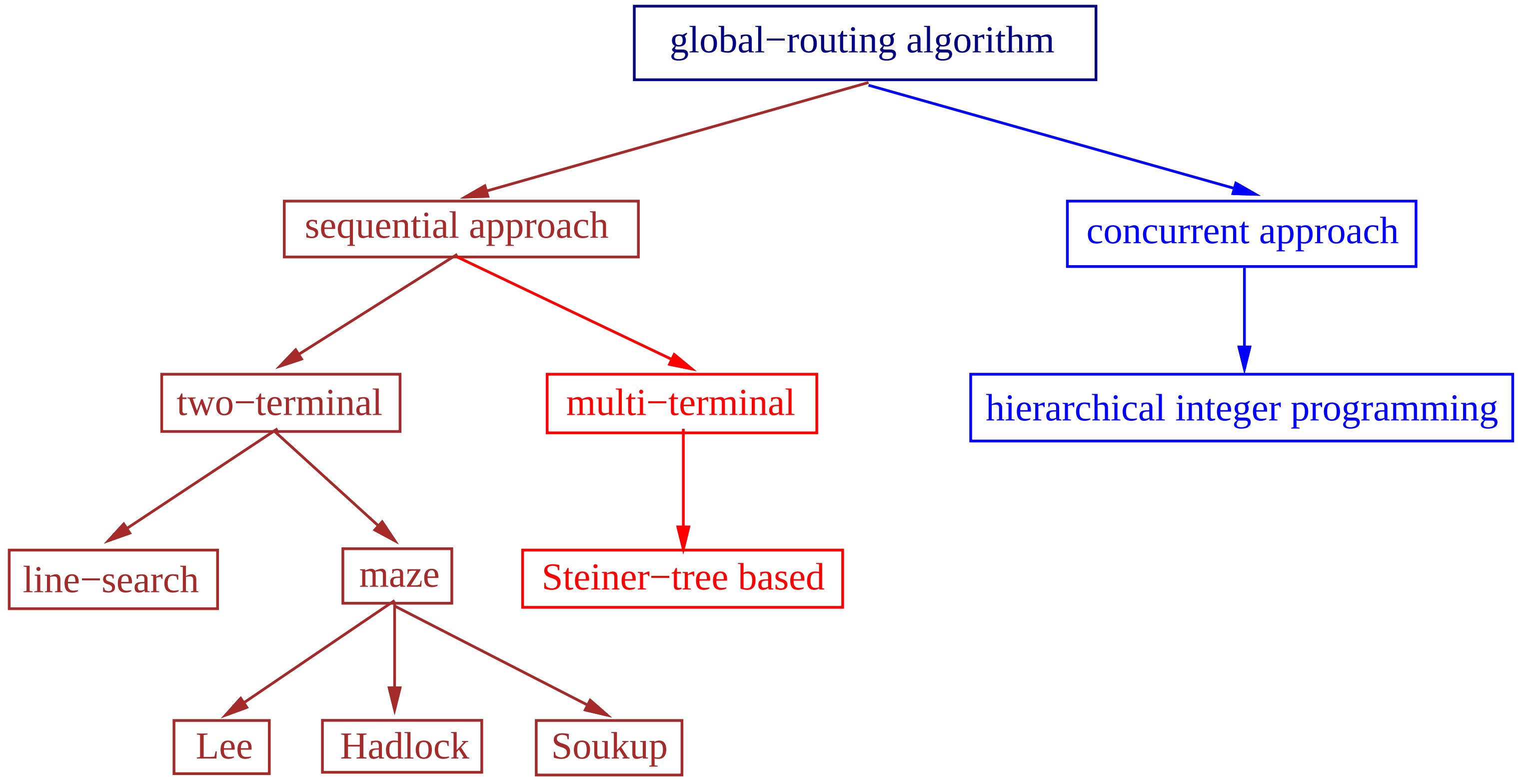
- Objective
  - Guarantee 100% routability.
  - Consider **switch-module architectural constraints.**
- For performance-driven routing,
  - **Minimize # of switches used.**
  - Minimize the maximum wire length.
  - Minimize the maximum path length.



Each channel has a capacity of 2 tracks.

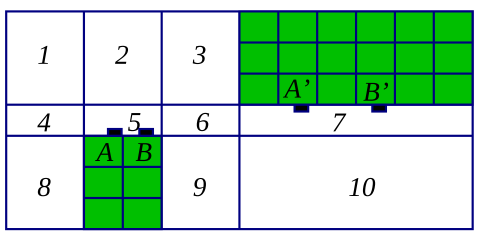
Classification of Global-Routing Algorithm

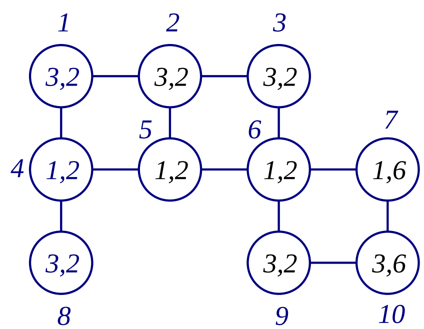
- **Sequential approach:** Assigns priority to nets; routes one net at a time based on its priority (net ordering?).
- **Concurrent approach:** All nets are considered at the same time (complexity?)

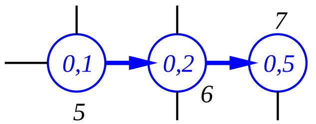


Global-Routing: Maze Routing

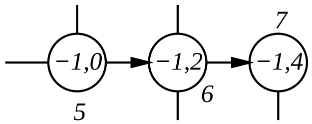
- Routing channels may be modelled by a weighted undirected graph called **channel connectivity graph**.
- Node ↔ channel; edge ↔ two adjacent channels; capacity: $(width, length)$



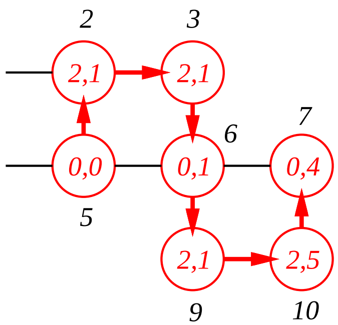




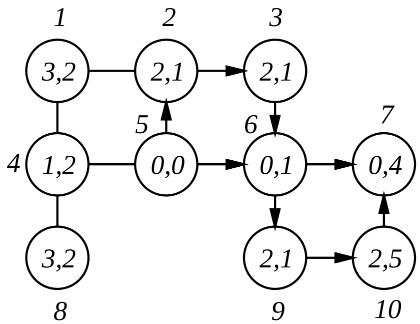
route A−A' via 5−6−7



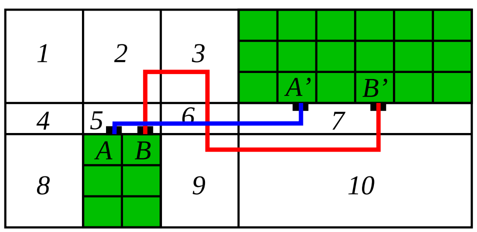
route B−B' via 5−6−7



route B−B' via 5−2−3−6−9−10−7



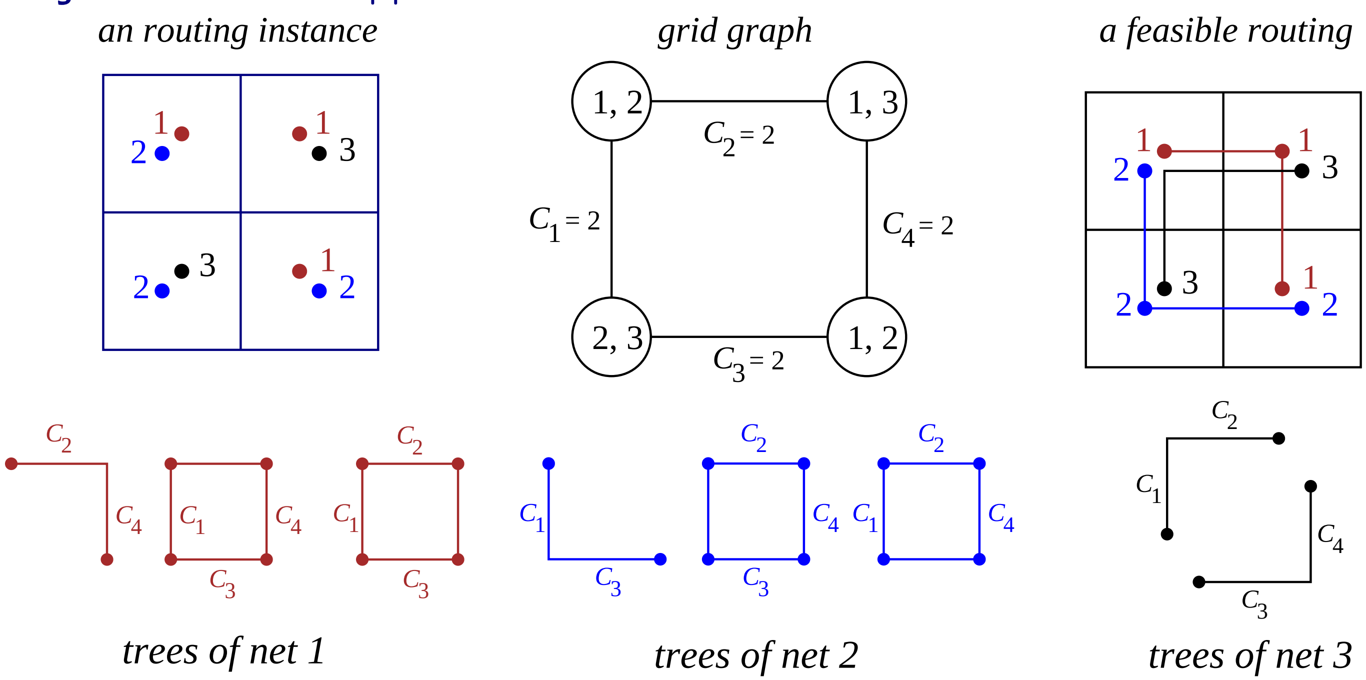
updated channel graph



maze routing for nets A and B

Global Routing by Integer Programming

- Suppose that for each net i, there are n_i possible trees $t_1^i, t_2^i, \ldots, t_{n_i}^i$ to route the net.
- Constraint I: For each net i, only one tree t_j^i will be selected.
- Constraint II: The capacity of each cell boundary c_i is not exceeded.
- Minimize the total tree cost.
- **Question:** Feasible for practical problem sizes?
  - **Key:** Hierarchical approach!



An Integer-Programming Example

Boundary	t_1^1	t_2^1	t_3^1	t_1^2	t_2^2	t_3^2	t_1^3	t_2^3
B1	0	1	1	1	0	1	1	0
B2	1	0	1	0	1	1	1	0
B3	0	1	1	1	1	0	0	1
B4	1	1	0	0	1	1	0	1

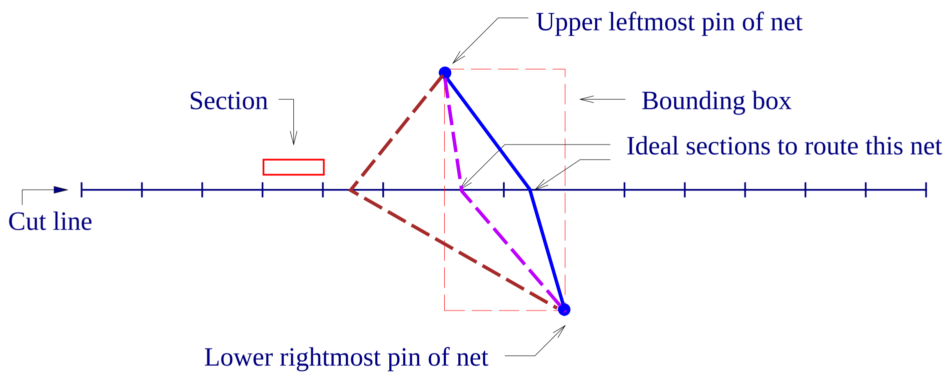
- $g_{i,j}$: cost of tree $t_j^i \Rightarrow g_{1,1} = 2, g_{1,2} = 3, g_{1,3} = 3, g_{2,1} = 2, g_{2,2} = 3, g_{2,3} = 3, g_{3,1} = 2, g_{3,2} = 2$.

Minimize $2x_{1,1} + 3x_{1,2} + 3x_{1,3} + 2x_{2,1} + 3x_{2,2} + 3x_{2,3} + 2x_{3,1} + 2x_{3,2}$
subject to

$$
\begin{array}{rcll}
x_{1,1} + x_{1,2} + x_{1,3} & = & 1 & (Constraint\ I : t^1) \\
x_{2,1} + x_{2,2} + x_{2,3} & = & 1 & (Constraint\ I : t^2) \\
x_{3,1} + x_{3,2} & = & 1 & (Constraint\ I : t^3) \\
x_{1,2} + x_{1,3} + x_{2,1} + x_{2,3} + x_{3,1} & \leq & 2 & (Constraint\ II : B1) \\
x_{1,1} + x_{1,3} + x_{2,2} + x_{2,3} + x_{3,1} & \leq & 2 & (Constraint\ II : B2) \\
x_{1,2} + x_{1,3} + x_{2,1} + x_{2,2} + x_{3,2} & \leq & 2 & (Constraint\ II : B3) \\
x_{1,1} + x_{1,2} + x_{2,2} + x_{2,3} + x_{3,2} & \leq & 2 & (Constraint\ II : B4) \\
x_{i,j} & = & 0, 1, 1 \leq i, j \leq 3 &
\end{array}
$$

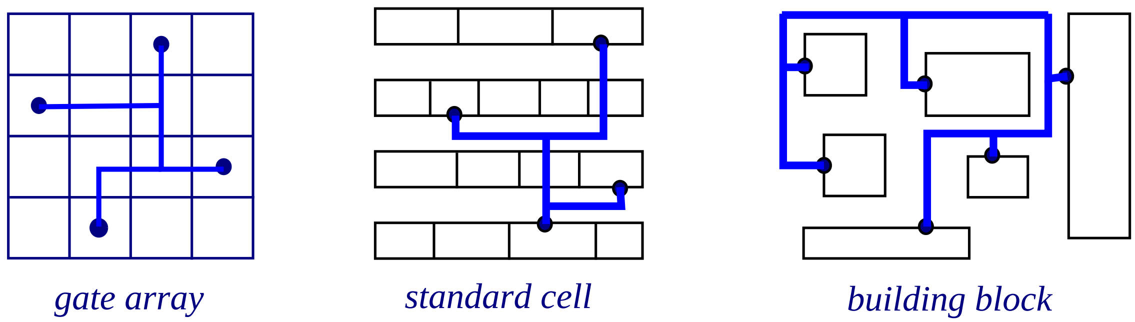
Hierarchical Global Routing

- Marek-Sadowska, "Router planner for custom chip design," ICCAD, 1986.
- At each level of the hierarchy, an attempt is made to minimize the cost of nets crossing cut lines.
- At the lowest level of the hierarchy, the layout surface is divided into $R \times R$ grid regions with boundary capacity equal to C tracks.
- Let R_l be the # of grid regions of a given cut line l; a cut line can be divided into $M = \frac{R_l}{C}$ sections.
- Global routing can be formulated as a linear assignment problem:
  - $x_{i,j} = 1$ if net i is assigned to section j; $x_{i,j} = 0$ otherwise.
  - Each net crosses the cut line exactly once: $\sum_{j=1}^{M} x_{ij} = 1, 1 \leq i \leq N$.
  - Capacity constraint of each section: $\sum_{i=1}^{N} x_{ij} \leq C, 1 \leq j \leq M$.
  - w_{ij}: cost of assigning net i to section j. Minimize $\sum_{i=1}^{N} \sum_{j=1}^{M} w_{ij} x_{ij}$.



The Routing-Tree Problem

- **Problem:** Given a set of pins of a net, interconnect the pins by a "routing tree."



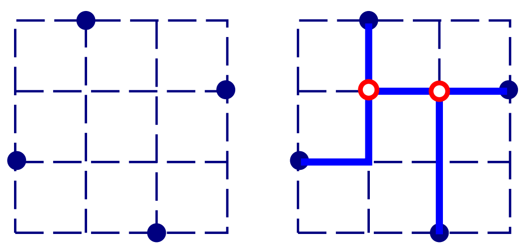
- **Minimum Rectilinear Steiner Tree (MRST) Problem:** Given n points in the plane, find a minimum-length tree of rectilinear edges which connects the points.
- $MRST(P) = MST(P \cup S)$, where P and S are the sets of original points and Steiner points, respectively.

minimum spanning tree
MST

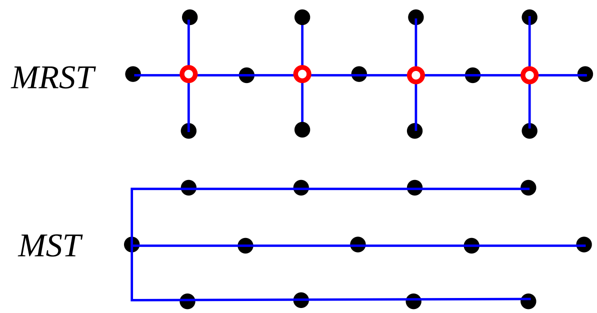
MRST

Theoretic Results for the MRST Problem

- **Hanan's Thm:** There exists an MRST with all Steiner points (set S) chosen from the intersection points of horizontal and vertical lines drawn points of P.
  - Hanan, "On Steiner's problem with rectilinear distance," SIAM J. Applied Math., 1966.
- **Hwang's Theorem:** For any point set P, $\frac{Cost(MST(P))}{Cost(MRST(P))} \leq \frac{3}{2}$.
  - Hwang, "On Steiner minimal tree with rectilinear distance," SIAM J. Applied Math., 1976.
- Best existing approximation algorithm: Performance bound $\frac{61}{48}$ by Foessmeier et al.
  - Foessmeier et al, "Fast approximation algorithm for the rectilinear Steiner problem," Wilhelm Schickard-Institut für Informatik, TR WSI-93-14, 93.
  - Zelikovsky, "An $\frac{11}{6}$ approximation algorithm for the network Steiner problem," Algorithmica., 1993.



Hanan grid



Cost(MST)/Cost(MRST) −> 3/2

A Simple Performance Bound

- Easy to show that $\frac{Cost(MST(P))}{Cost(MRST(P))} \leq 2$.
- Given any MRST T on point set P with Steiner point set S, construct a spanning tree T' on P as follows:
  1. Select any point in T as a root.
  2. Perform a depth-first traversal on the rooted tree T.
  3. Construct T' based on the traversal.